Problem 1

The Rosseland mean opacity κ is defined to be:

$$\kappa\rho = \frac{\int_0^\infty \frac{\partial B_\nu}{\partial T} d\nu}{\int_0^\infty \frac{1}{\kappa_\nu \rho} \frac{\partial B_\nu}{\partial T} d\nu},$$ where B_ν is the Planck function $B_\nu(T) = \frac{2h\nu^3}{c^2}\left(e^{h\nu/kT} - 1\right)^{-1}$.

Calculate the Rosseland mean opacity for the case of free-free absorption, in which a photon is absorbed by a free electron in the Coulomb field of a nucleus. The frequency dependent free-free absorption coefficient for pure hydrogen is:

$\kappa_\nu \rho = 1.32 \times 10^{56} \frac{\rho^2 g_{ff}}{\nu^3 T^{1/2}}\left(1 - e^{-h\nu/kT}\right) cm^{-1}$, where g_{ff} is a quantum mechanical correction called the *Gaunt factor*, which you may assume to be constant for the purposes of this problem.

a. Derive an expression for $\frac{\partial B_\nu}{\partial T}$.

b. Introduce a dimensionless variable $x = h\nu/kT$. Write the expression $\frac{1}{\rho\kappa_\nu}\frac{\partial B_\nu}{\partial T}$ for free-free emission in terms of x and plot the resulting function. Use this plot to argue that the Rosseland mean is determined largely by κ_ν, when ν is a few times kT/h.

c. Show that the Rosseland mean opacity obeys *Kramer's law*, $\kappa \propto \rho T^{-3.5}$ **.**

Problem 2

a. Go to the OPAL project website: physci.llnl.gov/Research/OPAL/existing.html (link on course website), download the opacity table for solar composition stars $(X = 0.7, Y = 0.28, Z = 0.02)$ (listed as Grevasse & Noels – table GN93hz, and in that file, table #73). The table lists the total opacity as a function of $\log T$ (rows) for different values of $\log R$ (columns). Note: R is *not* radius, but a quantity that is related to the density and temperature as follows: $R = \rho/\left(T/10^6 K\right)^3$. Use this table to plot the opacity as a function of $\log T$ for the following values of $\log R$: -8, -3.5, -0.5 (on the same plot).

b. Use the approximations for the opacity due to different sources: electron scattering, free-free absorption, bound-free absorption, and H- opacity (equations 4.60, 4.64, 4.63, and 4.65 in HKT) to calculate the total opacity as a function of temperature for these same values of $\log R$ (*since* $\log R$ *depends on temperature as well as density, you will have to find the appropriate value of density for each value of temperature*). You can simply add the first three opacities, but the H- opacity takes over when it becomes smaller than the other three. You can approximate the total opacity as

$\kappa_{tot} = \left(\left(\kappa_{H^-}\right)^{-1} + \left(\kappa_e + \kappa_{ff} + \kappa_{bf}\right)^{-1}\right)^{-1}$. Plot your approximation for the total opacity over your points from the opacity table to see how good (or poor) the approximations are.