Problem 1

The energy-averaged cross section is given by $\langle \sigma v \rangle = C S_0 \tau^2 e^{-\tau}$, where C and S_0 are constants, τ is given by $\tau = \frac{19.721 W^{1/3}}{\left(T/10^7 K\right)^{1/3}}$ and $W = Z_j^2 Z_k^2 \frac{A_j A_k}{A_j + A_k}$, where Z, A are the

charge and mass numbers of species j and k. The reaction rate (number of reactions per unit time, per unit volume) is given by $r_{j,k} = X_j X_k r'_{j,k}$, where X_j, X_k are the mass

fractions of species
$$j$$
 and k , and $r'_{j,k} = \frac{\rho^2}{\left(1 + \delta_{jk}\right) A_j A_k m_p^2} \langle \sigma v \rangle$.

Consider the reactions:

$$C_6^{12} + H_1^1 \rightarrow C_6^{13}$$

$$C_6^{13} + H_1^1 \rightarrow N_7^{14}$$

with corresponding reaction rates $r_{1,12}$, $r_{1,13}$, $r_{1,14}$, $r_{1,15}$

$$N_7^{14} + H_1^1 \rightarrow N_7^{15}$$

$$N_7^{15} + H_1^1 \rightarrow C_6^{12} + He_2^4$$

a. Derive expressions for the above 4 reaction rates as functions of C, S_0 , ρ , and T.

b. Evaluate the temperature exponent $\frac{\partial (\ln \langle \sigma v \rangle)}{\partial \ln T}$ for the above 4 equations at $T = 3 \times 10^7 K$.

Problem 2

- **a.** Write down a set of coupled differential equations for d/dt of the mass fractions C^{12} , C^{13} , N^{14} , N^{15} , H^1 , He^4 (I am using these symbols to signify mass fractions, instead of the standard X_j symbol), assuming that the reactions in problem 4 are the only ones occurring (e.g., N^{14} is only produced by the 2nd reaction and destroyed by the 3^{rd} one).
- **b.** Derive expressions for the relative abundances of C^{12} , C^{13} , N^{15} (in equilibrium) as functions of the N^{14} abundance.
- **c.** Given that the total number of C + N nuclei are conserved, derive an expression for the absolute equilibrium abundances C^{12} , C^{13} , N^{14} , N^{15} as functions of the reaction rates and the total number of C + N nuclei.

(NOTE: for this problem, you can take the reaction rates as known functions of temperature, i.e., you don't need to evaluate the expressions for $r'_{1,12}$, etc)