Problem 1

a. Use the equations of stellar structure to find the mass-luminosity relation and the mass-radius relation of Main Sequence stars. You may use dimensional analysis and drop all constants to find simple proportionalities. For example, the mass conservation equation can reduce to $\rho \sim M/R^3$. Assume the equation of state for an ideal gas $(P \sim \rho T)$, nuclear fusion with the CNO cycle $(\varepsilon \sim \rho T^{15})$, and opacity is dominated by electron Thomson scattering $(\kappa = const)$.

b. Make simple assumptions and estimate how the lifetime of such a Main Sequence star scales with the star's mass.

c. Use the Stefan-Boltzmann relation to find the proportionality between luminosity L and effective temperature T_{eff}

Problem 2

On the course website (http://people.vanderbilt.edu/~a.berlind/teaching/252_fa09/), you will find a file called "cluster.dat". It contains four columns with measurements of 75 stars in a cluster that you made last time you went observing. The first two columns are Right Ascension and Declination (the star's coordinates), the third column is the star's V-band magnitude V, and the last column is the $(B-V)$ color. Make a color-magnitude plot for the cluster using this data.

Problem 3

You were lucky enough to find a visual binary system in the cluster. After observing the system for many years, you found an angular binary separation of $\alpha = 0.388''$ and an orbital period of $P = 53.98$ years. You also measured the two stars' V-band magnitudes to be $V_1 = 8.44$ and $V_2 = 9.21$. Using the mass-luminosity relation you found from theory (normalized to the sun), determine the system's distance, parallax, masses, and absolute magnitudes M_V for the two stars. Iterate your solution until it converges (you should get convergence after 3-4 iterations so you don't need a computer to do this). You can use the fact that the absolute V-band magnitude for the sun is 4.83, and bolometric corrections are approximately constant for most stars.

Problem 4 – for graduate students or ambitious undergraduates

Now that you have a distance to the cluster, convert the 75 V-band magnitudes into absolute magnitudes. You can now make a M_V vs. $(B-V)$ plot and compare it to the theoretical $L-T_{eff}$ relation you found above to see how well the theory does. To do this, you need to convert the $L-T_{eff}$ relation into a $M_V-(B-V)$ relation. You can follow these steps:

a. Make a finely spaced (e.g., every 100K) table of values for T_{eff}, running from 3500K to 40,000K (corresponding to the temperature sequence from M stars to O stars). For each value of temperature compute a luminosity using the $L - T_{eff}$ relation (again, normalized to the sun: $L = L_{sun}$, $T_{eff} = 5{,}778K$).

b. For each temperature, compute the predicted $(B-V)$ color by convolving the blackbody spectrum for that temperature with the B and V-band filter response functions to compute the expected flux in each band. You can find these functions in the file "BV_filters.dat", also on the course website. The file has 3 columns: wavelength in nm, transmittance of B filter, transmittance of V filter (transmittance is the fraction of light that passes through the filter). The blackbody spectrum as a function of wavelength is: $B_T(\lambda) = \frac{2hc^2}{\lambda^5}\frac{1}{\exp(hc/\lambda kT - 1)}$, and the flux in a given filter is $\int_0^\infty B(\lambda)F(\lambda)d\lambda$, where $F(\lambda)$ is the filter transmittance function. (*Note: all colors are defined to be zero for A0 stars with* $T_{eff} = 9500K$ *so you need to subtract from all colors the value for that temperature*)

c. Convert the luminosities to absolute V-band magnitudes, using the fact that the absolute bolometric magnitude of the sun is 4.75 and the average bolometric correction for stars like the sun is approximately 0.2.

d. Now plot your theoretical $M_V - (B-V)$ on top of your observed one to see how well they agree. List the main assumptions you made in your theoretical calculation.

Problem 5 - optional for extra credit

If the above problems take you less time than I anticipate and you feel like challenging yourself more, look at the spectra of 5 stars on the course website (labeled star1 – star5) and classify them as best you can into spectroscopic classes, looking up whatever information you need. The files list wavelength in Angstroms and flux.