**Problem 1**

Write a computer program to solve the Lane-Emden equation:

\[ \frac{1}{\xi^2} \frac{d}{d\xi} \left( \xi^2 \frac{d\theta}{d\xi} \right) = -\theta^n, \]  

for a polytrope of index \( n \). The equation can be re-written in the more useful form:

\[ \theta'' = -\frac{2}{\xi} \theta' - \theta^n \]

Start your integration at the center of the star (\( \xi = 0 \)) using the following initial conditions: \( \theta = 1 \), \( \theta' = 0 \), and stop the integration at the surface, when \( \theta = 0 \).

Make the following plots:
- \( \theta \) vs. \( \xi \)
- \( \theta' \) vs. \( \xi \)
- \( \theta'' \) vs. \( \xi \)

In each plot, show curves for polytropic indices of \( n = 3/2 \) and \( n = 3 \). Calculate the outer radius of the star \( \xi_1 \), at which \( \theta = 0 \), and the derivative \( \theta'(\xi_1) \) at this radius, for these two cases. Attach a printout of your code.

*(check your answer: you should get \( \xi_1 = 3.65 \) for a \( n = 3/2 \) polytrope)*

**Problem 2**

Assume the polytrope for a white dwarf, which is made up of a non-relativistic, degenerate gas \( (P = K\rho^{\gamma/3}) \). Write down expressions that give you the radius of the white dwarf \( R \), the scale radius \( r_n \), the central density \( \rho_c \), and the central pressure \( P_c \), assuming that you know the white dwarf mass \( M \), the dimensionless radius \( \xi \), the derivative \( \theta'(\xi_1) \), the equation of state constant \( K \), and physical constants.

**Problem 3**

Plug numbers into your equations from Problem 2 to get values for \( R \), \( \rho_c \), and \( P_c \), assuming a \( M = 1M_{\odot} \) white dwarf made up of fully ionized Carbon and Oxygen (i.e., the electron mean molecular weight is \( \mu_e = 2 \)). Use the values of \( \xi_1 \) and \( \theta'(\xi_1) \) that you got in Problem 1. Show the values you got.

Now that you have \( \rho_c \) and \( P_c \), use your results from Problem 1 to make plots of the density \( \rho \) and pressure \( P \) of the white dwarf, as a function of physical radius \( r \).

What mass white dwarfs have a non-relativistic core?