

Problem 1

Write a computer program to solve the Lane-Emden equation:

$\frac{1}{\xi^2} \frac{d}{d\xi} \left[\xi^2 \frac{d\theta}{d\xi} \right] = -\theta^n$, for a polytrope of index n . The equation can be re-written in the

more useful form: $\theta'' = -\frac{2}{\xi} \theta' - \theta^n$

Start your integration at the center of the star ($\xi=0$) using the following initial conditions: $\theta=1$, $\theta'=0$, and stop the integration at the surface, when $\theta=0$.

Make the following plots:

- θ vs. ξ
- θ' vs. ξ
- θ'' vs. ξ

In each plot, show curves for polytropic indices of $n=3/2$ and $n=3$. Calculate the outer radius of the star ξ_1 , at which $\theta=0$, and the derivative $\theta'(\xi_1)$ at this radius, for these two cases. Attach a printout of your code.

(check your answer: you should get $\xi_1 = 3.65$ for a $n=3/2$ polytrope)

Problem 2

Assume the polytrope for a white dwarf, which is made up of a non-relativistic, degenerate gas ($P = K\rho^{5/3}$). Write down expressions that give you the radius of the white dwarf R , the scale radius r_n , the central density ρ_c , and the central pressure P_c , assuming that you know the white dwarf mass M , the dimensionless radius ξ_1 , the derivative $\theta'(\xi_1)$, the equation of state constant K , and physical constants.

Problem 3

Plug numbers into your equations from Problem 2 to get values for R , ρ_c , and P_c , assuming a $M=1M_{sun}$ white dwarf made up of fully ionized Carbon and Oxygen (i.e., the electron mean molecular weight is $\mu_e = 2$). Use the values of ξ_1 and $\theta'(\xi_1)$ that you got in Problem 1. Show the values you got.

Now that you have ρ_c and P_c , use your results from Problem 1 to make plots of the density ρ and pressure P of the white dwarf, as a function of physical radius r .

What mass white dwarfs have a non-relativistic core?