

**Problem 1**

**a.** Use the equations of stellar structure to find the mass-luminosity relation and the mass-radius relation of Main Sequence stars. You may use Homology and drop all constants to find simple proportionalities. For example, the mass conservation equation can reduce to  $\rho \sim M/R^3$ . Assume the equation of state for an ideal gas ( $P \sim \rho T$ ), nuclear fusion with the CNO cycle ( $\epsilon \sim \rho T^{15}$ ), and opacity is dominated by electron Thomson scattering ( $\kappa = \text{const}$ ).

**b.** Make simple assumptions and estimate how the lifetime of such a Main Sequence star scales with the star's mass.

**c.** Use the Stefan-Boltzmann relation to find the proportionality between luminosity  $L$  and effective temperature  $T_{\text{eff}}$

**Problem 2**

On the course website ([http://astro.phy.vanderbilt.edu/~berlinaa/teaching/8030\\_fa15/](http://astro.phy.vanderbilt.edu/~berlinaa/teaching/8030_fa15/)), you will find a file called "cluster.dat". It contains four columns with measurements of 75 stars in a cluster that you made last time you went observing. The first two columns are Right Ascension and Declination (the star's coordinates), the third column is the star's  $V$ -band magnitude  $V$ , and the last column is the  $(B - V)$  color. Make a color-magnitude plot for the cluster using this data.

**Problem 3**

You were lucky enough to find a visual binary system in the cluster. After observing the system for many years, you found an angular binary separation of  $\alpha = 0.388''$  and an orbital period of  $P = 53.98$  years. You also measured the two stars'  $V$ -band magnitudes to be  $V_1 = 8.44$  and  $V_2 = 9.21$ . Using the mass-luminosity relation you found from theory (normalized to the sun), determine the system's distance, parallax, masses, and absolute magnitudes  $M_V$  for the two stars. Iterate your solution until it converges (you should get convergence after 3-4 iterations so you don't need a computer to do this). You can use the fact that the absolute  $V$ -band magnitude for the sun is 4.83, and bolometric corrections are approximately constant for most stars.

**Problem 4**

Now that you have a distance to the cluster, convert the 75  $V$ -band magnitudes into absolute magnitudes. You can now make a  $M_V$  vs.  $(B - V)$  plot and compare it to the theoretical  $L - T_{\text{eff}}$  relation you found above to see how well the theory does. To do this, you need to convert the  $L - T_{\text{eff}}$  relation into a  $M_V - (B - V)$  relation. You can follow these steps:

**a.** Make a finely spaced (e.g., every 100K) table of values for  $T_{\text{eff}}$ , running from 3500K to 40,000K (corresponding to the temperature sequence from M stars to O stars). For

each value of temperature compute a luminosity using the  $L - T_{\text{eff}}$  relation (again, normalized to the sun:  $L = L_{\text{sun}}, T_{\text{eff}} = 5,778\text{K}$ ).

**b.** For each temperature, compute the predicted  $(B - V)$  color by convolving the blackbody spectrum for that temperature with the  $B$  and  $V$ -band filter response functions to compute the expected flux in each band. You can find these functions in the file “BV\_filters.dat”, also on the course website. The file has 3 columns: wavelength in nm, transmittance of  $B$  filter, transmittance of  $V$  filter (transmittance is the fraction of light that passes through the filter). The blackbody spectrum as a function of wavelength is:

$$B_T(\lambda) = \frac{2hc^2}{\lambda^5} \frac{1}{\exp(hc/\lambda kT) - 1}, \text{ and the flux in a given filter is } \int_0^{\infty} B(\lambda)S(\lambda)d\lambda, \text{ where}$$

$S(\lambda)$  is the filter transmittance function. (*Note: all colors are defined to be zero for A0 stars with  $T_{\text{eff}} = 9500\text{K}$  so you need to subtract from all colors the value for that temperature*)

**c.** Convert the luminosities to absolute  $V$ -band magnitudes, using the fact that the absolute bolometric magnitude of the sun is 4.75 and the average bolometric correction for stars like the sun is approximately 0.2.

**d.** Now plot your theoretical  $M_V - (B - V)$  on top of your observed one to see how well they agree. List the main assumptions you made in your theoretical calculation.

### **Problem 5**

Using your work from problems 1 and 4, estimate the age of this cluster.

### **Problem 6 - optional for extra credit**

If the above problems take you less time than I anticipate and you feel like challenging yourself more, look at the spectra of 5 stars on the course website (labeled star1 – star5) and classify them as best you can into spectroscopic classes, looking up whatever information you need. The files list wavelength in Angstroms and flux.