Problem Set 3: Due Thursday, October 27 AST 8030, Fall 2015

Problem 1
Write a computer program to solve the Lane-Emden equation:
$\frac{1}{\xi^2}\frac{d}{d\xi}\left[\xi^2\frac{d\theta}{d\xi}\right] = -\theta^n$, for a polytrope of index n. The equation can be re-written in the more useful form: $\theta'' = -\frac{2}{\xi}\theta' - \theta^n$
Start your integration at the center of the star ($\xi = 0$) using the following initial conditions: $\theta = 1$, $\theta' = 0$, and stop the integration at the surface, when $\theta = 0$.
Make the following plots:
- θ vs. ξ
- θ' vs. ξ
- θ'' vs. ξ

In each plot, show curves for polytropic indices of $n = 3/2$ and $n = 3$. Calculate the outer radius of the star ξ_1, at which $\theta = 0$, and the derivative $\theta'(\xi_1)$ at this radius, for these two cases. Attach a printout of your code.
(*check your answer*: *you should get* $\xi_1 = 3.65$ *for a* $n = 3/2$ *polytrope*)

Problem 2
Assume the polytrope for a white dwarf, which is made up of a non-relativistic, degenerate gas ($P = K\rho^{5/3}$). Write down expressions that give you the radius of the white dwarf R, the scale radius r_n, the central density ρ_c, and the central pressure P_c, assuming that you know the white dwarf mass M, the dimensionless radius ξ_1, the derivative $\theta'(\xi_1)$, the equation of state constant K, and physical constants.

Problem 3
Plug numbers into your equations from Problem 2 to get values for R, ρ_c, and P_c, assuming a $M = 1M_{sun}$ white dwarf made up of fully ionized Carbon and Oxygen (i.e., the electron mean molecular weight is $\mu_e = 2$). Use the values of ξ_1 and $\theta'(\xi_1)$ that you got in Problem 1. Show the values you got.
Now that you have ρ_c and P_c, use your results from Problem 1 to make plots of the density ρ and pressure P of the white dwarf, as a function of physical radius r.
What mass white dwarfs have a non-relativistic core?