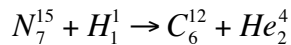
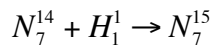
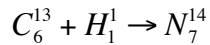
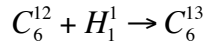


**Problem 1**

The energy-averaged cross section is given by  $\langle\sigma v\rangle = CS_0\tau^2 e^{-\tau}$ , where  $C$  and  $S_0$  are constants,  $\tau$  is given by  $\tau = \frac{19.721W^{1/3}}{(T/10^7\text{K})^{1/3}}$  and  $W = Z_j^2 Z_k^2 \frac{A_j A_k}{A_j + A_k}$ , where  $Z, A$  are the charge and mass numbers of species  $j$  and  $k$ . The reaction rate (number of reactions per unit time, per unit volume) is given by  $r_{j,k} = X_j X_k r'_{j,k}$ , where  $X_j, X_k$  are the mass fractions of species  $j$  and  $k$ , and  $r'_{j,k} = \frac{\rho^2}{(1 + \delta_{jk}) A_j A_k m_p^2} \langle\sigma v\rangle$ .

Consider the reactions:



with corresponding reaction rates  $r_{1,12}, r_{1,13}, r_{1,14}, r_{1,15}$

- a. Derive expressions for the above 4 reaction rates as functions of  $C, S_0, \rho, T$ , and the mass fractions  $X_i$ .
- b. Evaluate the temperature exponent  $\frac{\partial(\ln\langle\sigma v\rangle)}{\partial \ln T}$  for the above 4 equations at  $T = 3 \times 10^7 \text{K}$ .

**Problem 2**

- a. Write down a set of coupled differential equations for  $d/dt$  of the mass fractions  $C^{12}, C^{13}, N^{14}, N^{15}, H^1, He^4$  (I am using these symbols to signify mass fractions, instead of the standard  $X_j$  symbol), assuming that the reactions in problem 4 are the only ones occurring (e.g.,  $N^{14}$  is only produced by the 2nd reaction and destroyed by the 3rd one).
- b. Derive expressions for the relative abundances of  $C^{12}, C^{13}, N^{15}$  (in equilibrium) as functions of the  $N^{14}$  abundance.
- c. Given that the total number of  $C + N$  nuclei are conserved, derive an expression for the absolute equilibrium abundances  $C^{12}, C^{13}, N^{14}, N^{15}$  as functions of the reaction rates and the total number of  $C + N$  nuclei.

(NOTE: for this problem, you can take the reaction rates as known functions of temperature, i.e., you don't need to evaluate the expressions for  $r'_{1,12}$ , etc)