Problem 1

The Rosseland mean opacity κ is defined to be:

$\kappa\rho = \dfrac{\int_0^\infty \frac{\partial B_\nu}{\partial T} d\nu}{\int_0^\infty \frac{1}{\kappa_\nu \rho}\frac{\partial B_\nu}{\partial T} d\nu}$, where B_ν is the Planck function $B_\nu(T) = \dfrac{2h\nu^3}{c^2}\left(e^{h\nu/kT} - 1\right)^{-1}$.

Calculate the Rosseland mean opacity for the case of free-free absorption, in which a photon is absorbed by a free electron in the Coulomb field of a nucleus. The frequency dependent free-free absorption coefficient for pure hydrogen is:

$\kappa_\nu \rho = 1.32\times10^{56}\dfrac{\rho^2 g_{ff}}{\nu^3 T^{1/2}}\left(1-e^{-h\nu/kT}\right)cm^{-1}$, where g_{ff} is a quantum mechanical correction called the *Gaunt factor*, which you may assume to be constant for the purposes of this problem.

a. Derive an expression for $\dfrac{\partial B_\nu}{\partial T}$.

b. Introduce a dimensionless variable $x = h\nu/kT$. Write the expression $\dfrac{1}{\rho\kappa_\nu}\dfrac{\partial B_\nu}{\partial T}$ for free-free emission in terms of x: $const \times \rho^\alpha T^\beta f(x)$ and plot the resulting function $f(x)$. Use this plot to argue that the Rosseland mean opacity is determined mainly by the frequency range when ν is a few times kT/h.

c. Show that the Rosseland mean opacity obeys *Kramer's law*, $\kappa \propto \rho T^{-3.5}$ *(hint: you do not need to evaluate the integrals)***.**

Problem 2

a. Download the OPAL opacity table (link on course website) for solar composition stars $(X = 0.7, Y = 0.28, Z = 0.02)$. The table lists the log of the total opacity as a function of $\log T$ (rows) for different values of $\log R$ (columns). Note: R is *not* radius, but a quantity that is related to the density and temperature as follows: $R = \rho/\left(T/10^6 K\right)^3$. The first column lists values of $\log T$, the first row lists values of $\log R$, and all other values are log of opacity. The bottom right corner of the table was empty so I have filled it with values of -99999. Use this table to plot the opacity as a function of $\log T$ for the following values of $\log R$: -8, -3.5, -0.5 (put the three curves on the same plot).

b. Use the approximations for the opacity due to different sources: electron scattering, free-free absorption, bound-free absorption, and H- opacity (equations 4.60, 4.64, 4.63, and 4.65 in HKT) to calculate the total opacity as a function of temperature for these same values of $\log R$ (*since* $\log R$ *depends on temperature as well as density, you will have to find the appropriate value of density for each value of temperature*). You can simply add the first three opacities, but the H- opacity takes over when it becomes smaller than the other three. You can approximate the total opacity as $\kappa_{tot} = \left(\left(\kappa_{H^-} \right)^{-1} + \left(\kappa_e + \kappa_{ff} + \kappa_{bf} \right)^{-1} \right)^{-1}$. Plot your approximation for the total opacity over your points from the opacity table to see how good (or poor) the approximations are.

Problem 3

Download the file `stellar_profile.dat` from the course website. This file contains three columns: (m/M), $\log T$, $\log \rho$ where each row corresponds to a spherical shell within a star like the sun and the three columns represent (a) the total mass interior to the shell (in units of the total star mass), (b) the log of the temperature of the shell (in Kelvin), and (c) the log of the density of the shell (in g/cm^3). Assume that the star has solar composition $(X = 0.7, Y = 0.28, Z = 0.02)$ and is completely ionized.

Undergrads: compute the Rosseland mean opacity in each shell using the approximations in problem 2b. Plot κ_R as a function of mass.

Grads: compute the Rosseland mean opacity in each shell by interpolating the opacity table in two dimensions. Plot κ_R as a function of mass.