Problem 1

Write a computer program to solve the Lane-Emden equation:

$\frac{1}{\xi^2}\frac{d}{d\xi}\left[\xi^2\frac{d\theta}{d\xi}\right]=-\theta^n$, for a polytrope of index n. The equation can be re-written in the more useful form: $\theta''=-\frac{2}{\xi}\theta'-\theta^n$

Start your integration at the center of the star ($\xi=0$) using the following initial conditions: $\theta=1$, $\theta'=0$, and stop the integration at the surface, when $\theta=0$.

Make the following plots:

- θ vs. ξ
- θ' vs. ξ
- θ'' vs. ξ

In each plot, show curves for polytropic indices of $n=3/2$ and $n=3$. Calculate the outer radius of the star ξ_1, at which $\theta=0$, and the derivative $\theta'(\xi_1)$ at this radius, for these two cases. Attach a printout of your code.

(*check your answer*: *you should get* $\xi_1=3.65$ *for a* $n=3/2$ *polytrope*)

Problem 2

Assume the polytrope for a white dwarf, which is made up of a non-relativistic, degenerate gas ($P=K\rho^{5/3}$). Write down expressions that give you the radius of the white dwarf R, the scale radius r_n, the central density ρ_c, and the central pressure P_c, assuming that you know the white dwarf mass M, the dimensionless radius ξ_1, the derivative $\theta'(\xi_1)$, the equation of state constant K, and physical constants.

Problem 3

Plug numbers into your equations from Problem 2 to get values for R, ρ_c, and P_c, assuming a $M=1M_{sun}$ white dwarf made up of fully ionized Carbon and Oxygen (i.e., the electron mean molecular weight is $\mu_e=2$). Use the values of ξ_1 and $\theta'(\xi_1)$ that you got in Problem 1. Show the values you got.

Now that you have ρ_c and P_c, use your results from Problem 1 to make plots of the density ρ and pressure P of the white dwarf, as a function of physical radius r.

What mass white dwarfs have a non-relativistic core?