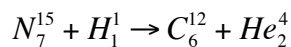
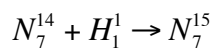
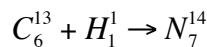
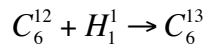


Problem 1

The energy-averaged cross section is given by $\langle\sigma v\rangle = CS_0\tau^2e^{-\tau}$, where C and S_0 are constants, τ is given by $\tau = \frac{19.721W^{1/3}}{(T/10^7K)^{1/3}}$ and $W = Z_j^2Z_k^2\frac{A_jA_k}{A_j + A_k}$, where Z, A are the charge and mass numbers of species j and k . The reaction rate (number of reactions per unit time, per unit volume) is given by $r_{j,k} = X_jX_kr'_{j,k}$, where X_j, X_k are the mass fractions of species j and k , and $r'_{j,k} = \frac{\rho^2}{(1 + \delta_{jk})A_jA_km_p^2}\langle\sigma v\rangle$.

Consider the reactions:



with corresponding reaction rates $r_{1,12}, r_{1,13}, r_{1,14}, r_{1,15}$

- a. Derive expressions for the above 4 reaction rates as functions of C, S_0, ρ, T , and the mass fractions X_i .
- b. Evaluate the temperature exponent $\frac{\partial(\ln\langle\sigma v\rangle)}{\partial\ln T}$ for the above 4 equations at $T = 3 \times 10^7 K$.

Problem 2

- a. Write down a set of coupled differential equations for d/dt of the mass fractions $C^{12}, C^{13}, N^{14}, N^{15}, H^1, He^4$ (I am using these symbols to signify mass fractions, instead of the standard X_j symbol), assuming that the reactions in problem 4 are the only ones occurring (e.g., N^{14} is only produced by the 2nd reaction and destroyed by the 3rd one).
- b. Derive expressions for the relative abundances of C^{12}, C^{13}, N^{15} (in equilibrium) as functions of the N^{14} abundance.
- c. Given that the total number of $C + N$ nuclei are conserved, derive an expression for the absolute equilibrium abundances $C^{12}, C^{13}, N^{14}, N^{15}$ as functions of the reaction rates and the total number of $C + N$ nuclei. By “total number of $C + N$ nuclei”, I mean the number density of all nuclei: $n_{CN} = n_{12} + n_{13} + n_{14} + n_{15}$.
(NOTE: for this problem, you can take the reaction rates as known functions of temperature, i.e., you don't need to evaluate the expressions for $r'_{1,12}$, etc)