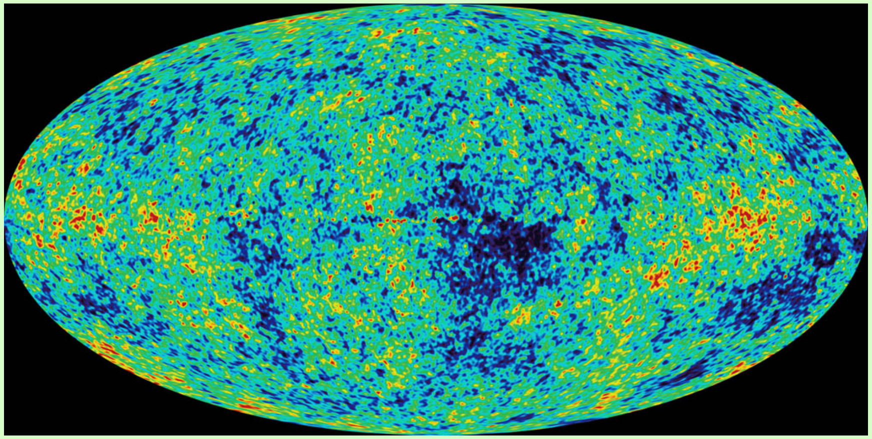
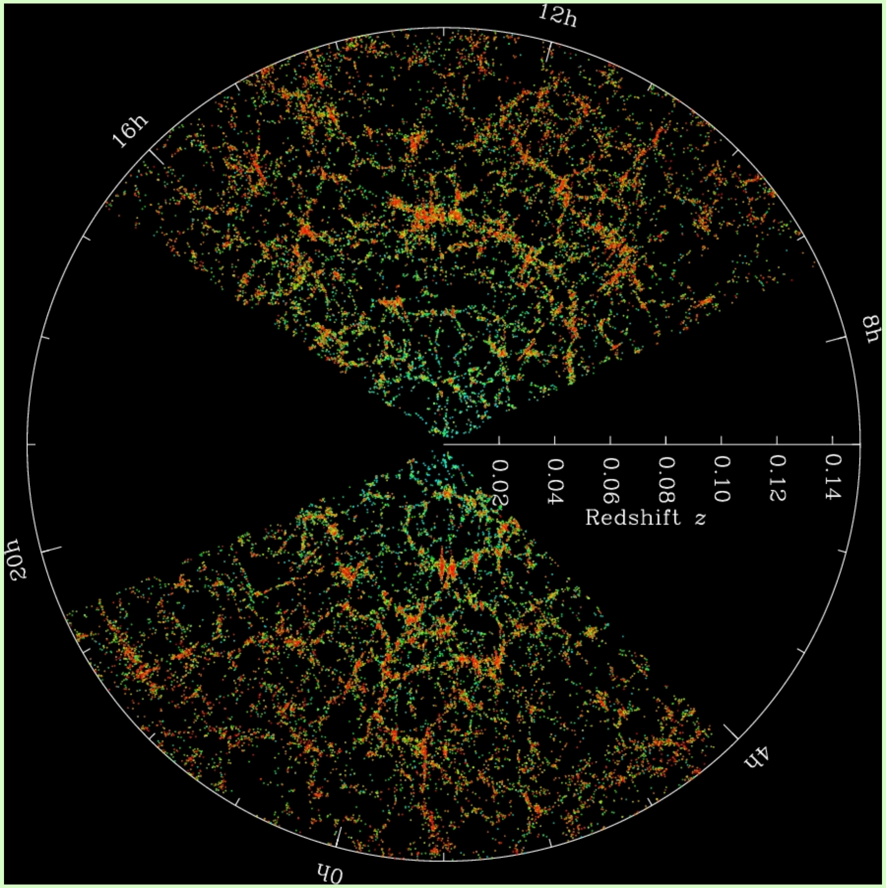
Probes of the Mass Density Field



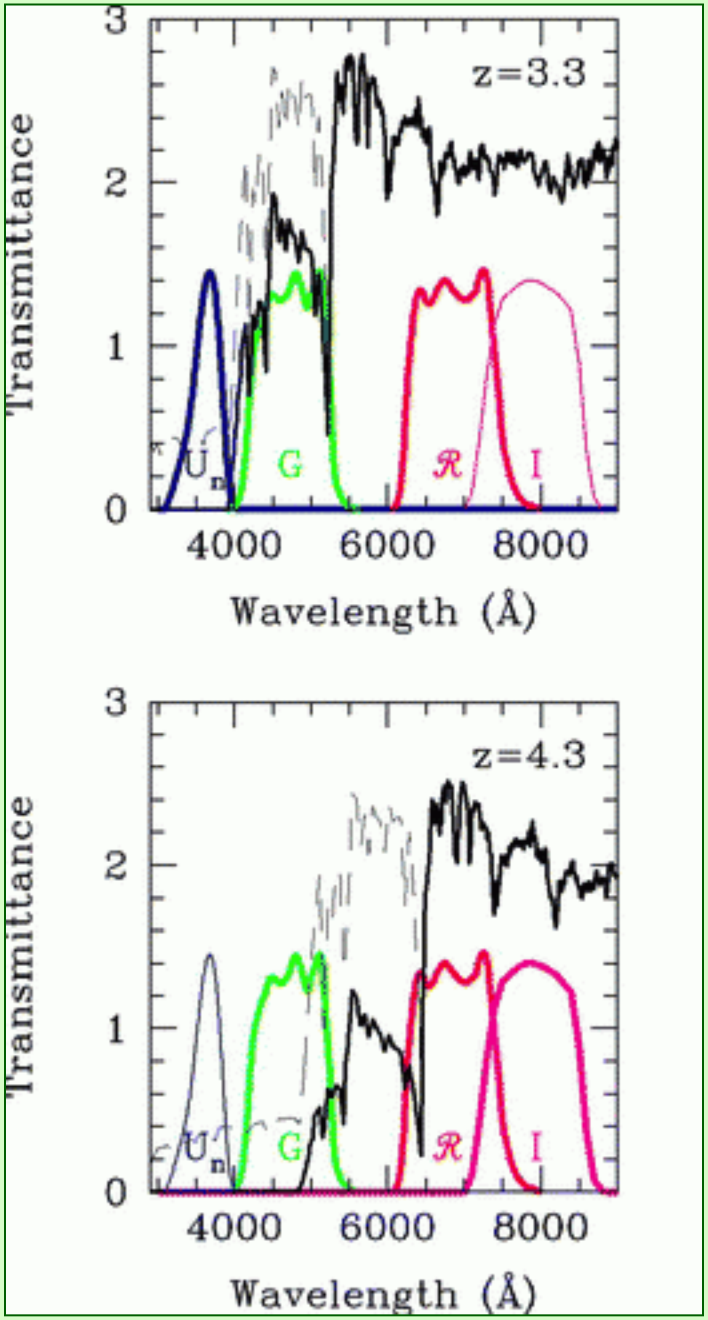
z~1000

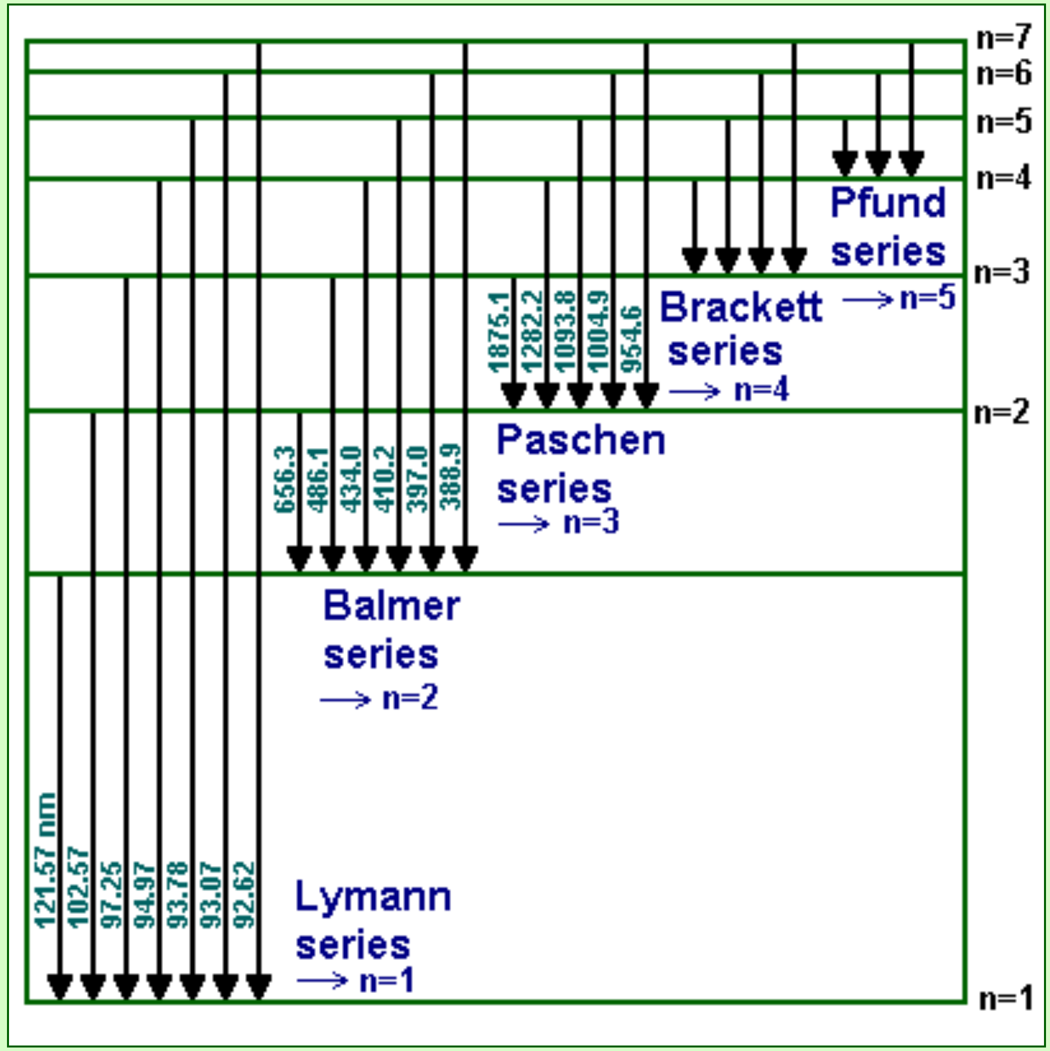


z~0-1

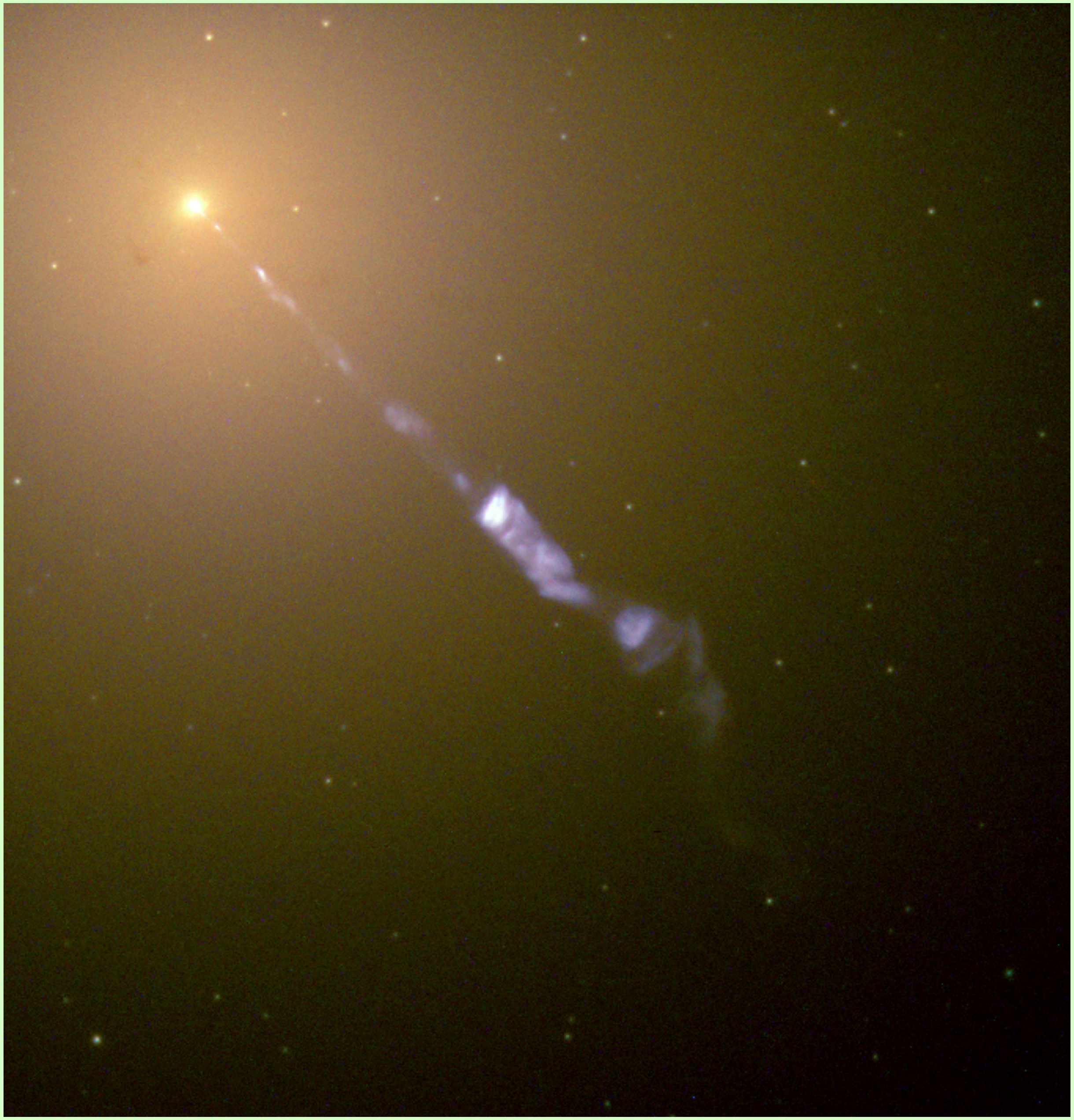
What about intermediate redshifts?

Lyman Break Galaxies



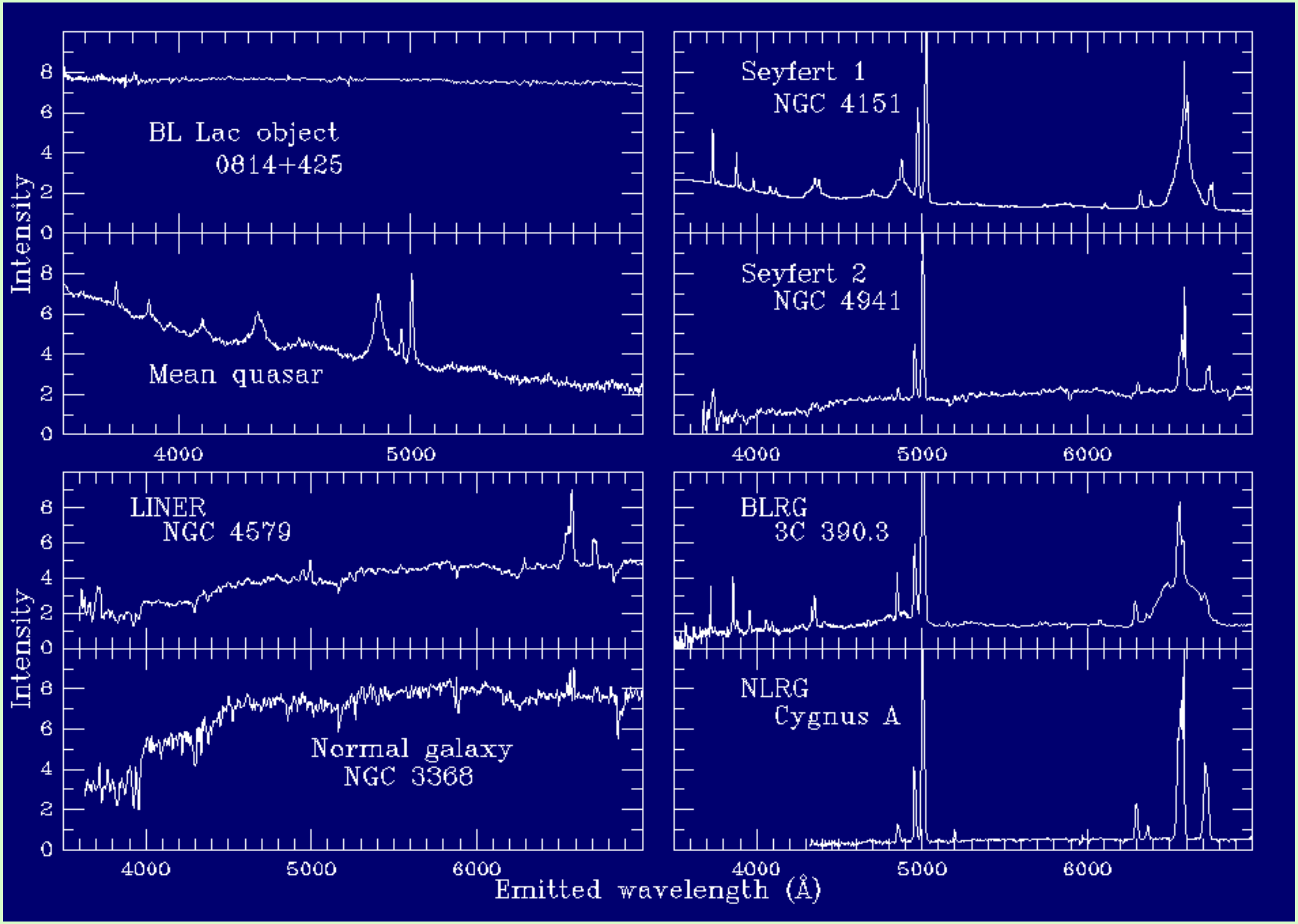


Quasars



Active Galactic Nuclei

AGN - Types



BL Lac object
0814+425
Seyfert 1
NGC 4151
Mean quasar
Seyfert 2
NGC 4941
LINER
NGC 4579
BLRG
3C 390.3
Normal galaxy
NGC 3368
NLRG
Cygnus A
Intensity
Emitted wavelength (Å)

AGN - Types

Differences between active galaxy types and normal galaxies.

Galaxy Type	Active Nuclei	Emission Lines		X-rays	Excess of		Strong Radio	Jets	Variable	Radio loud
		Narrow	Broad		UV	Far-IR				
Normal	no	weak	none	weak	none	none	none	none	no	no
Starburst	no	yes	no	some	no	yes	some	no	no	no
Seyfert I	yes	yes	yes	some	some	yes	few	no	yes	no
Seyfert II	yes	yes	no	some	some	yes	few	yes	yes	no
Quasar	yes	yes	yes	some	yes	yes	some	some	yes	10%
Blazar	yes	no	some	yes	yes	no	yes	yes	yes	yes
BL Lac	yes	no	none/faint	yes	yes	no	yes	yes	yes	yes
OVV	yes	no	stronger than BL Lac	yes	yes	no	yes	yes	yes	yes
Radio galaxy	yes	some	some	some	some	yes	yes	yes	yes	yes

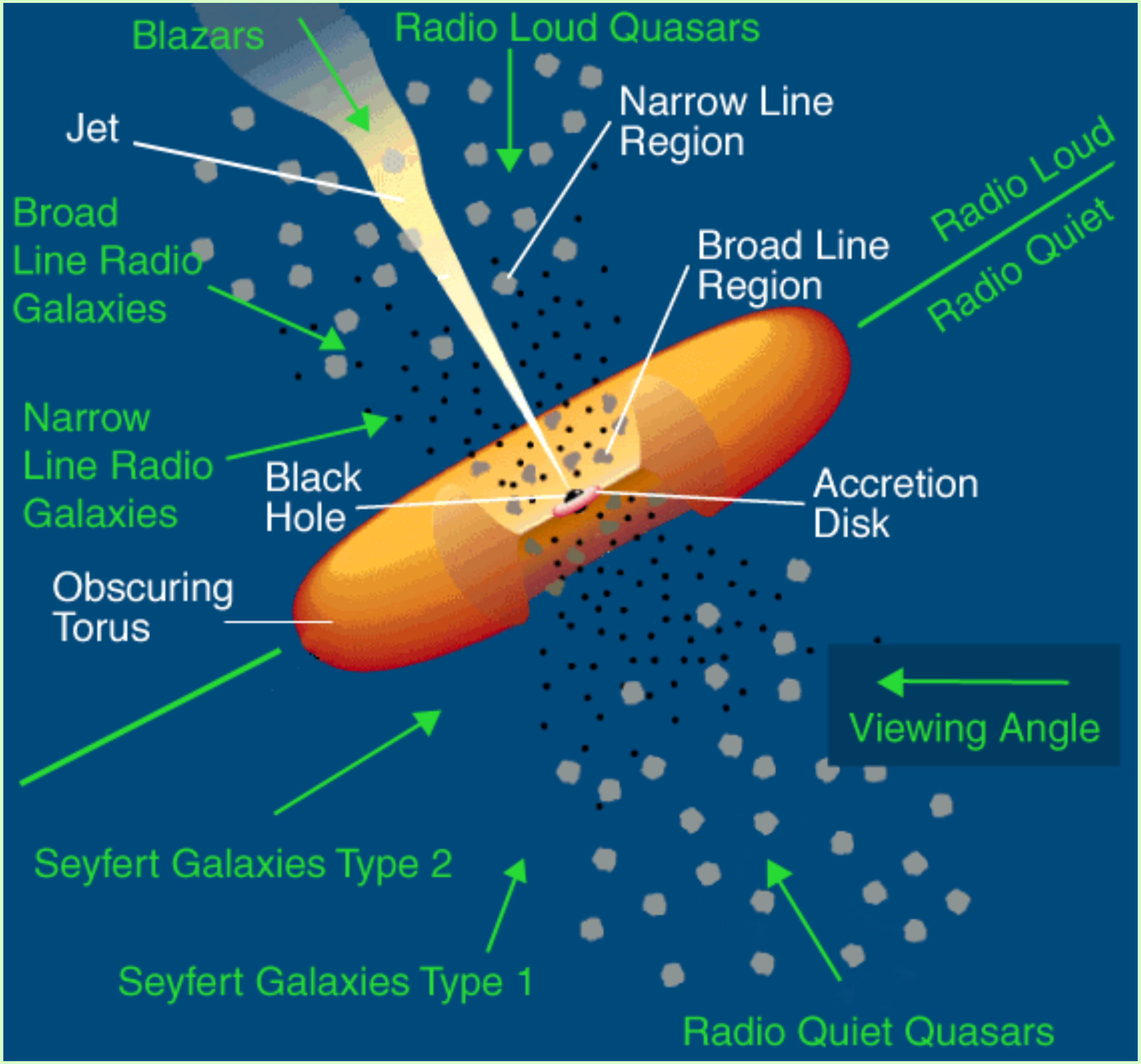
AGN – Unified Model

Hypothesis

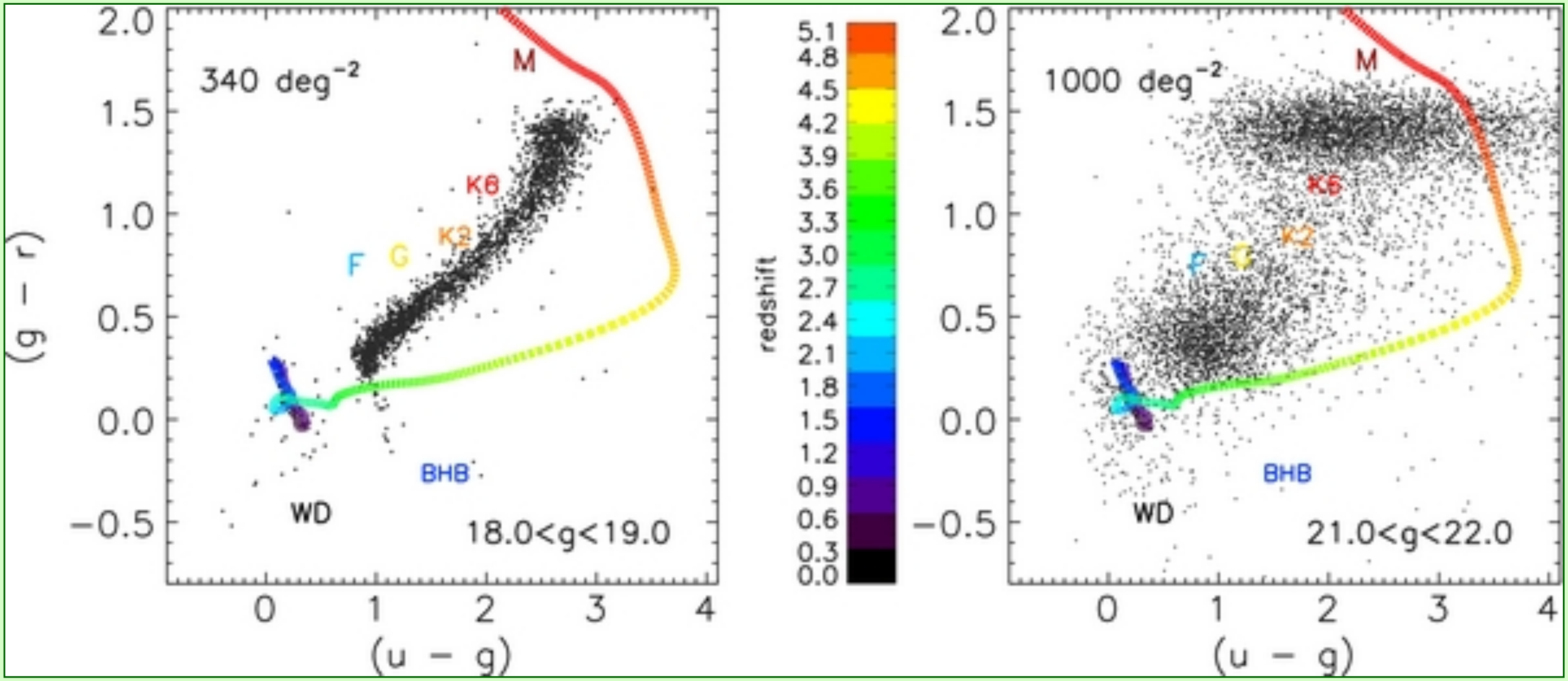
All active galactic nuclei are supermassive black holes at the centers of galaxies being fed by an accretion disk. Different types are just differences in:

- Black hole mass
- Accretion rate
- Type of galaxy
- Viewing angle

AGN – Unified Model

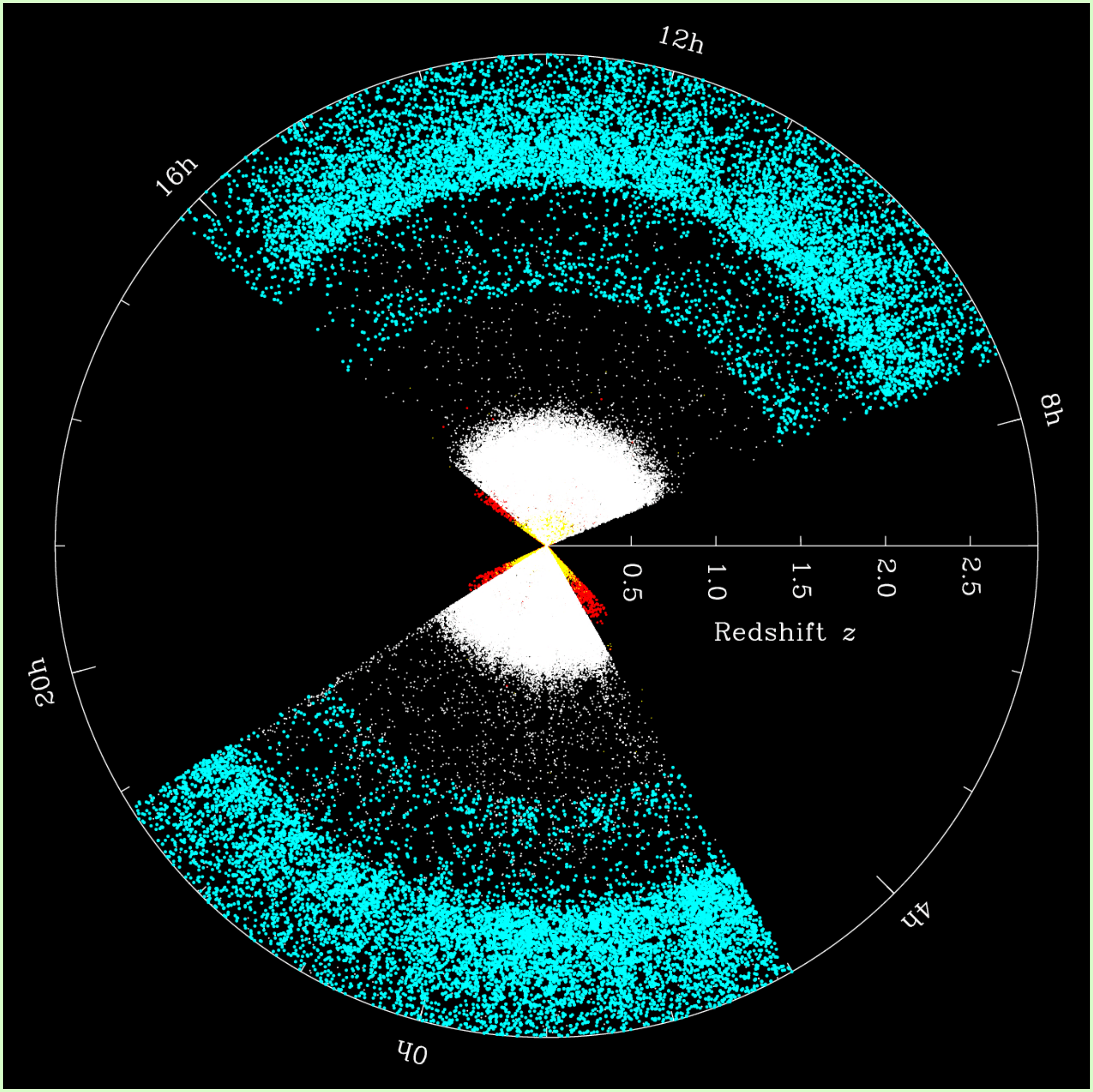


Quasars

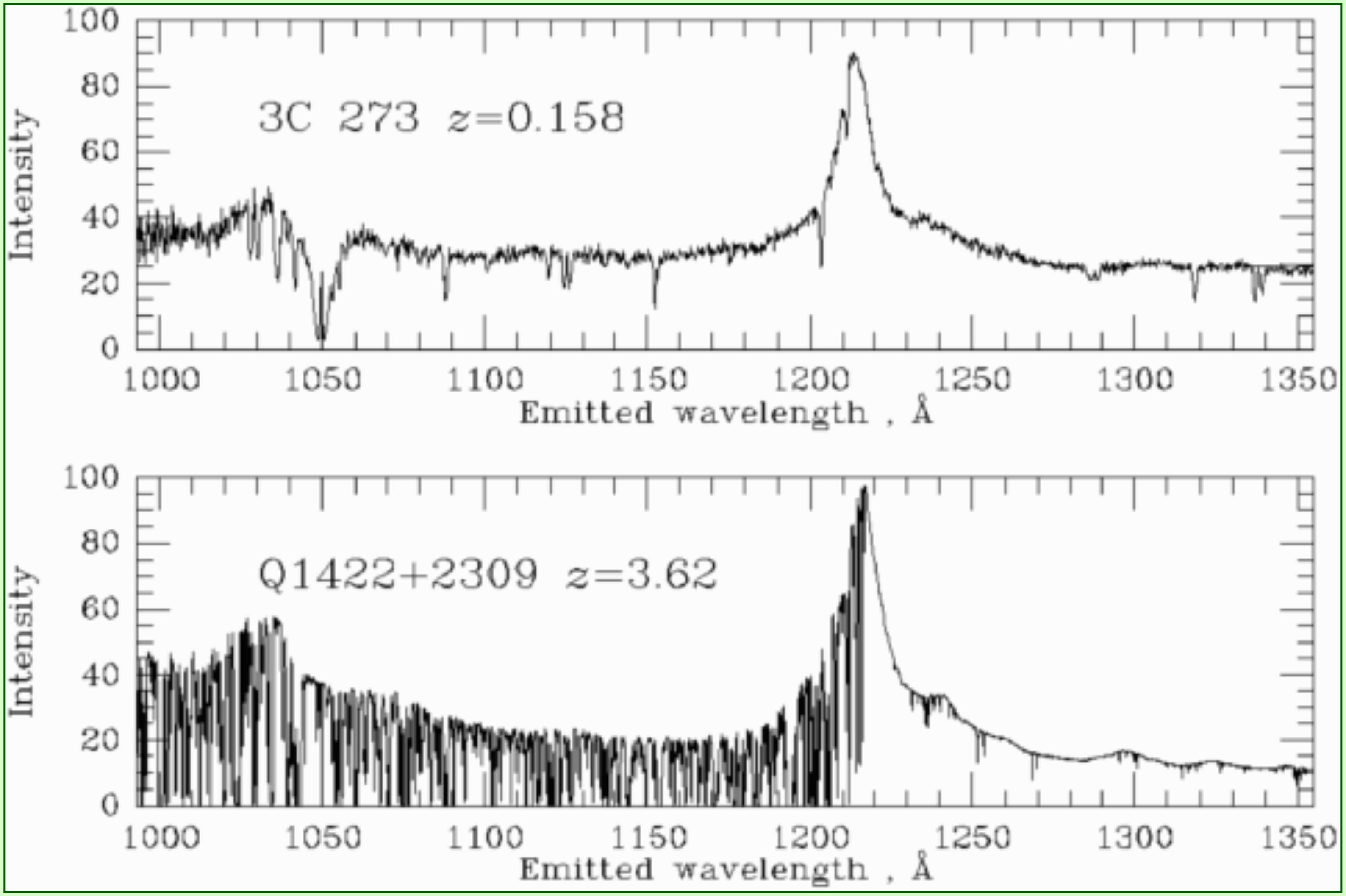


Ross et al. (2012)

Quasars



The Lyman alpha Forest



The Lyman alpha Forest

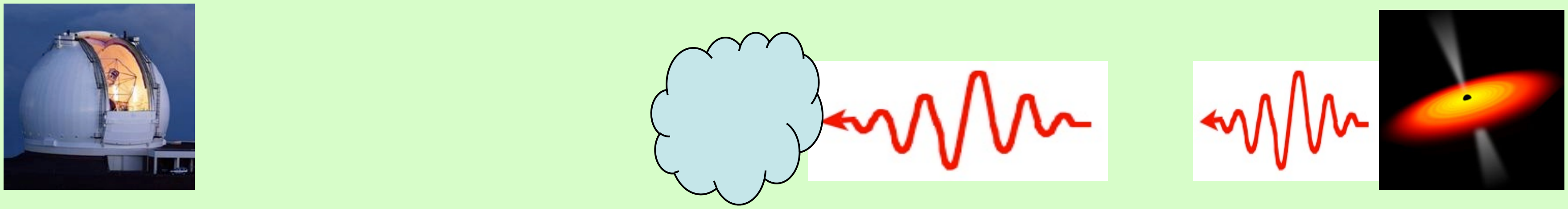
$z=0$ z_{QSO}

Emission wavelength: λ_e

$$\boxed{\frac{\lambda_o}{\lambda_e} = \frac{1+z_e}{1+z_o}}$$

Observation wavelength: $\lambda_o = \lambda_e\left(1+z_e\right)$

The Lyman alpha Forest

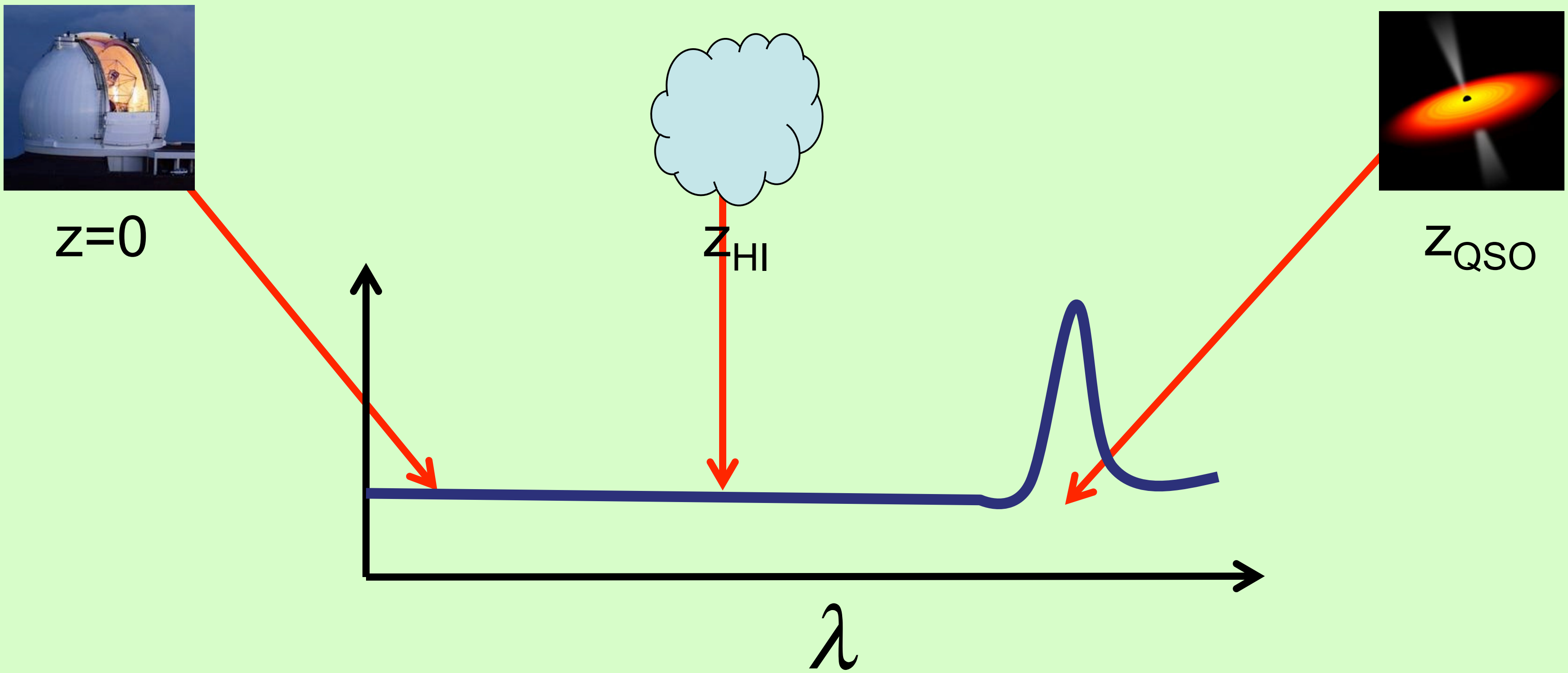


$z=0$ z_{HI} z_{QSO}

- A Lyman-α photon (1216 Å) emitted by the quasar has a longer wavelength by the time it encounters the HI cloud and so it will not be absorbed.

- The shorter wavelength photon emitted by the quasar that has stretched to 1216 Å by the time it encounters the HI cloud can be absorbed.

The Lyman alpha Forest



- In the emitted frame of the quasar, the Ly-α forest lies between the wavelengths of 1216/(1+z) and 1216 Å
- In the observed frame, the Ly-α forest lies between the wavelengths of 1216 and 1216(1+z) Å

The Lyman alpha Forest

The BOSS spectrograph covers the range 3600-10,400 Å

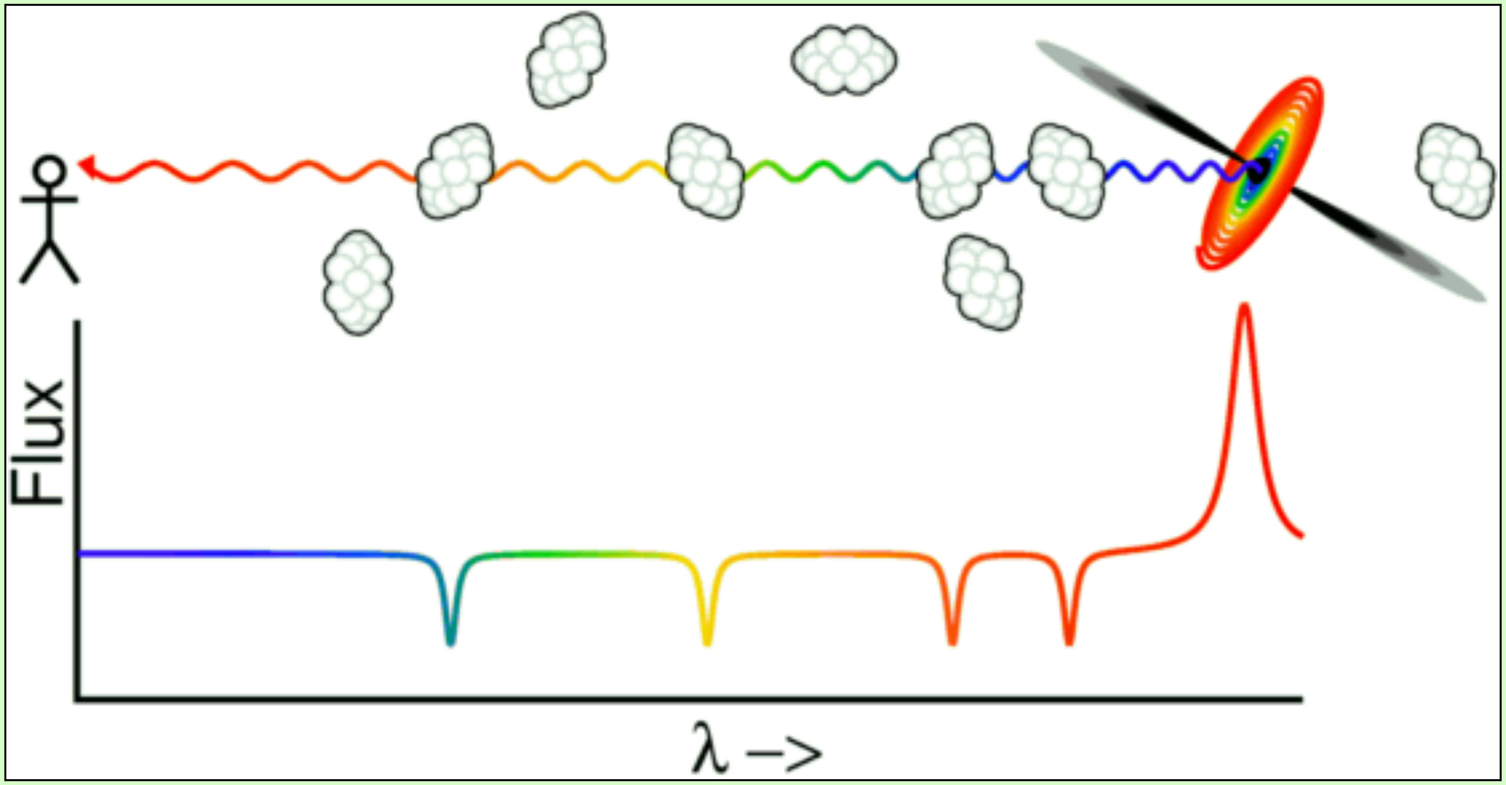
- The closest gas cloud that it can probe is at

$$\lambda_o = \lambda_e(1+z) \rightarrow z = \frac{\lambda_o}{\lambda_e} - 1 = \frac{3600}{1216} - 1 \rightarrow z = 1.96$$

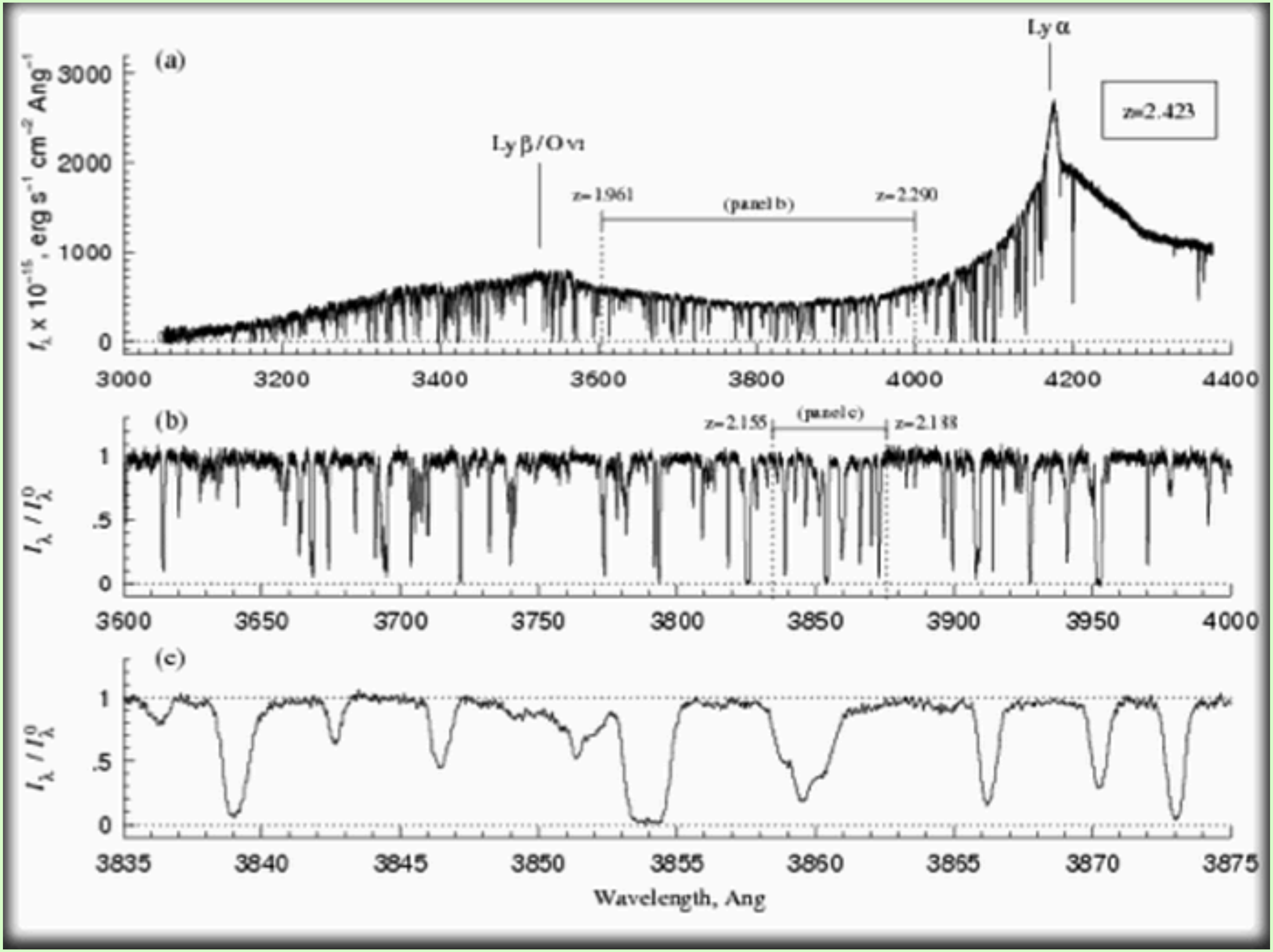
- The farthest gas cloud that it can probe is at

$$z = \frac{10,400}{1216} - 1 \rightarrow z = 7.55$$

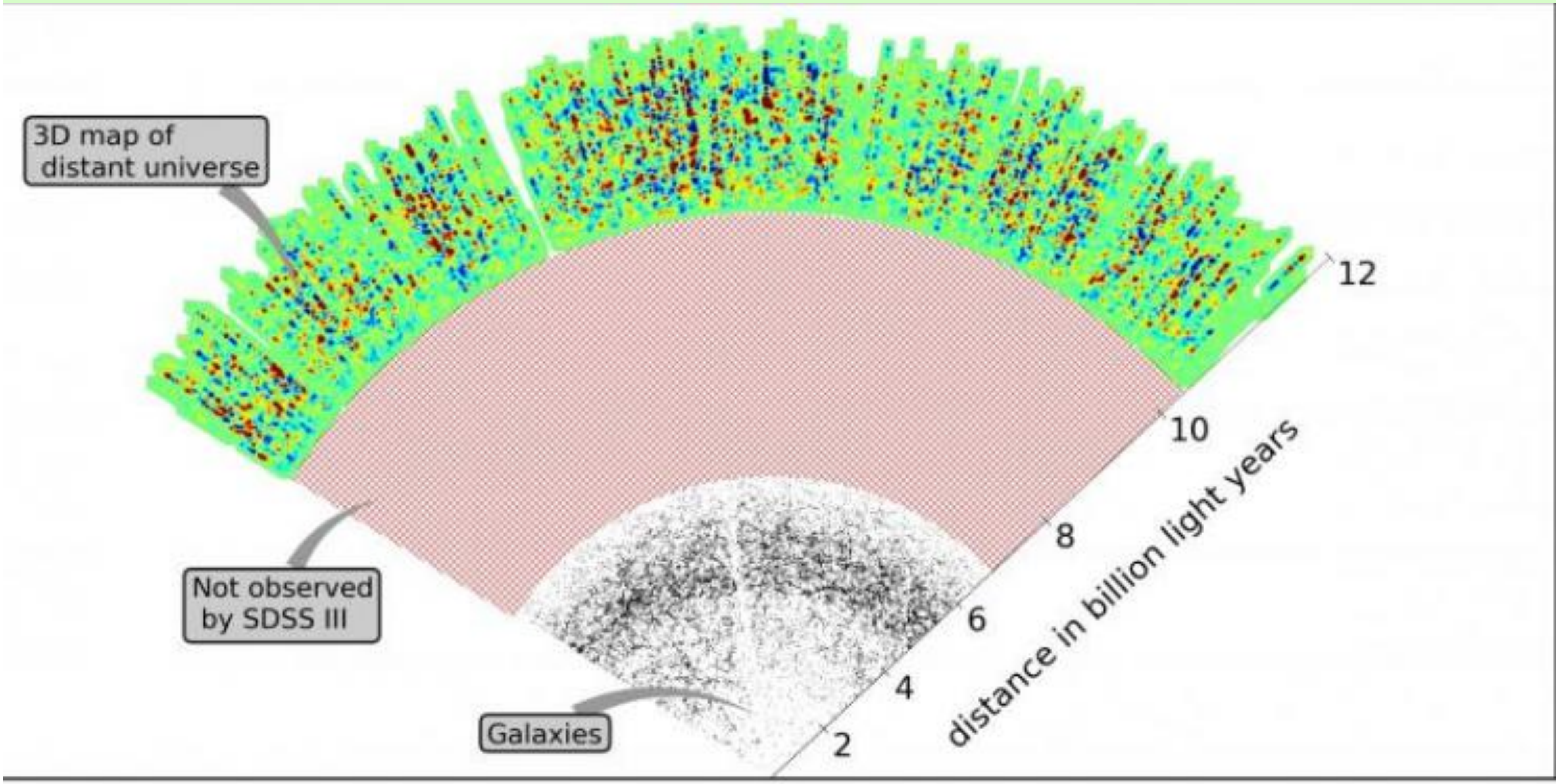
The Lyman alpha Forest



The Lyman alpha Forest



The Lyman alpha Forest

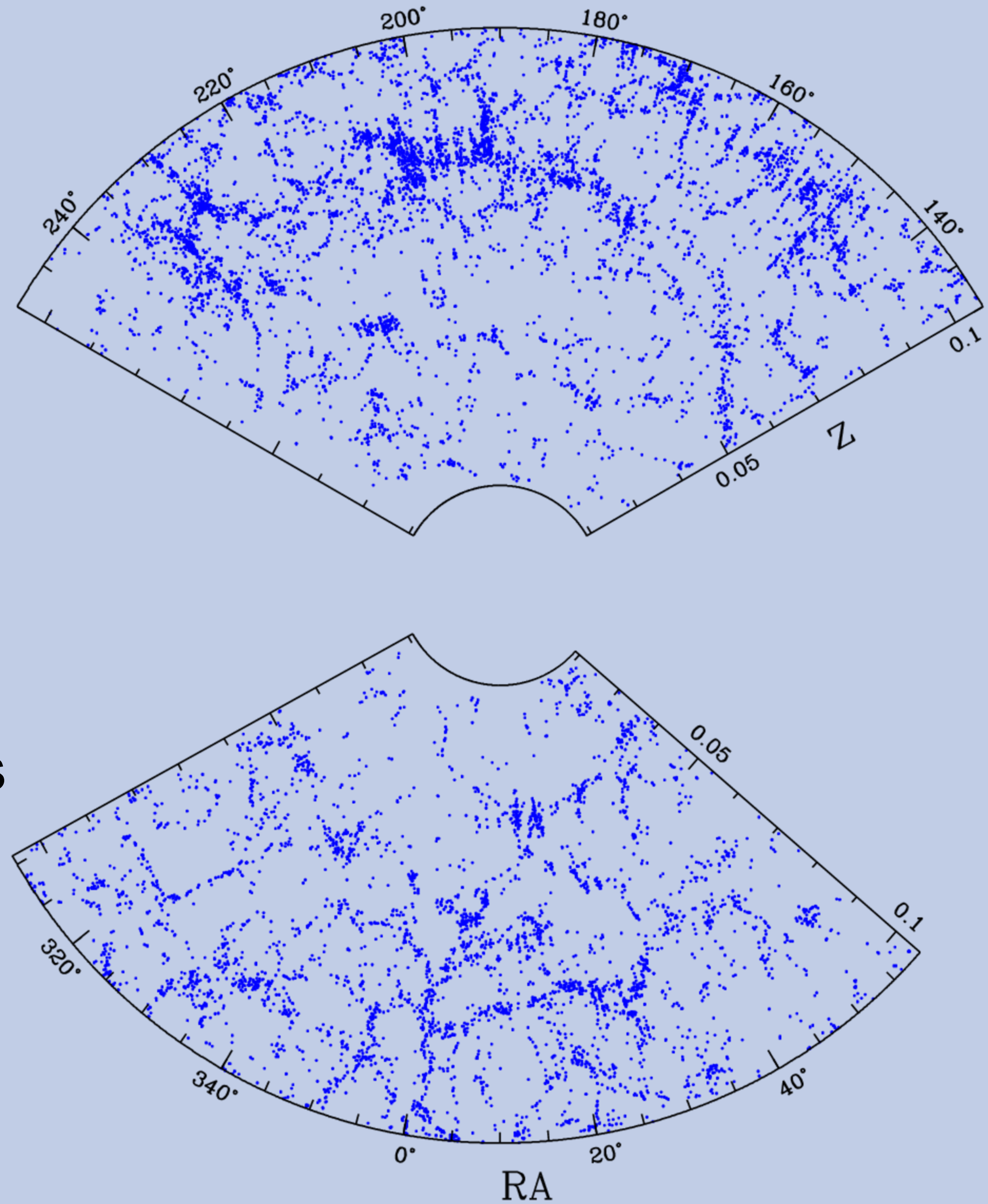


Statistics of the Galaxy Distribution

measure the environment
around individual galaxies

or

measure average statistics
for a sample of galaxies

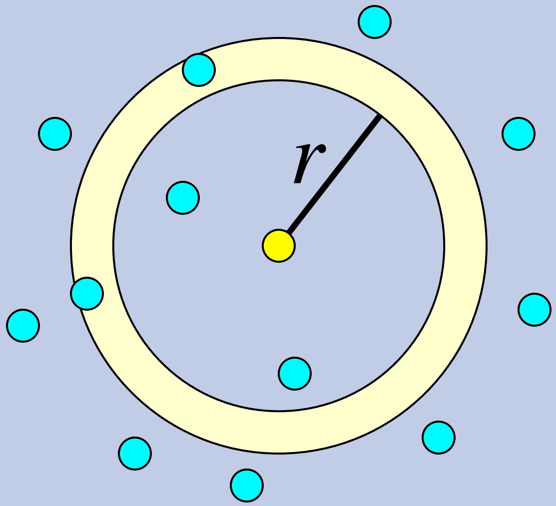


The 2-point Correlation Function

The *excess* probability that two galaxies are separated by a distance ***r*** relative to that for a random distribution.

For a random point distribution of number density ***n***, the number of points at a distance between ***r*** and ***r+dr*** from any one point is:

$$n \cdot dV = n \cdot 4\pi r^2 dr$$



The total number density of pairs at this separation is then:

$$\frac{n}{2} \times n \cdot 4\pi r^2 dr = n^2 2\pi r^2 dr$$

The 2-point Correlation Function

For a point distribution that is not random, the number density of pairs
At this separation is:

$$n^2 2\pi r^2 dr\left[1+\xi(r)\right]$$

The correlation function is then equal to:

$$\xi(r)=\frac{n_{\text{pairs, data}}(r)}{n_{\text{pairs, rand}}(r)}-1$$

$$\xi(r)=\frac{DD(r)}{RR(r)}-1$$

The 2-point Correlation Function

Complications

- When the sample volume is complex, calculate number of random pairs using an actual generated random data set that occupies the same volume as the data.

- In practice, we often use other estimators. For example, the most commonly used estimator for galaxy samples is the Landy-Szalay estimator:

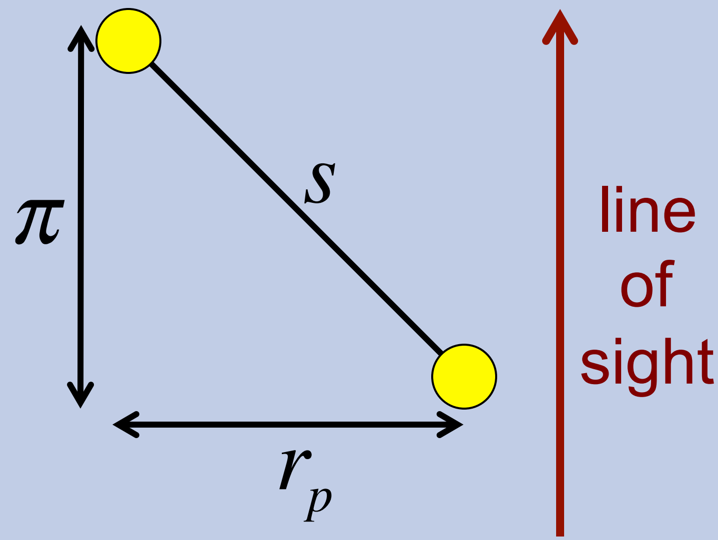
$$\xi(r) = \frac{DD - 2DR + RR}{RR}$$

- Must normalize the DD, DR, and RR terms when the number of data and random points are not the same.

Complications

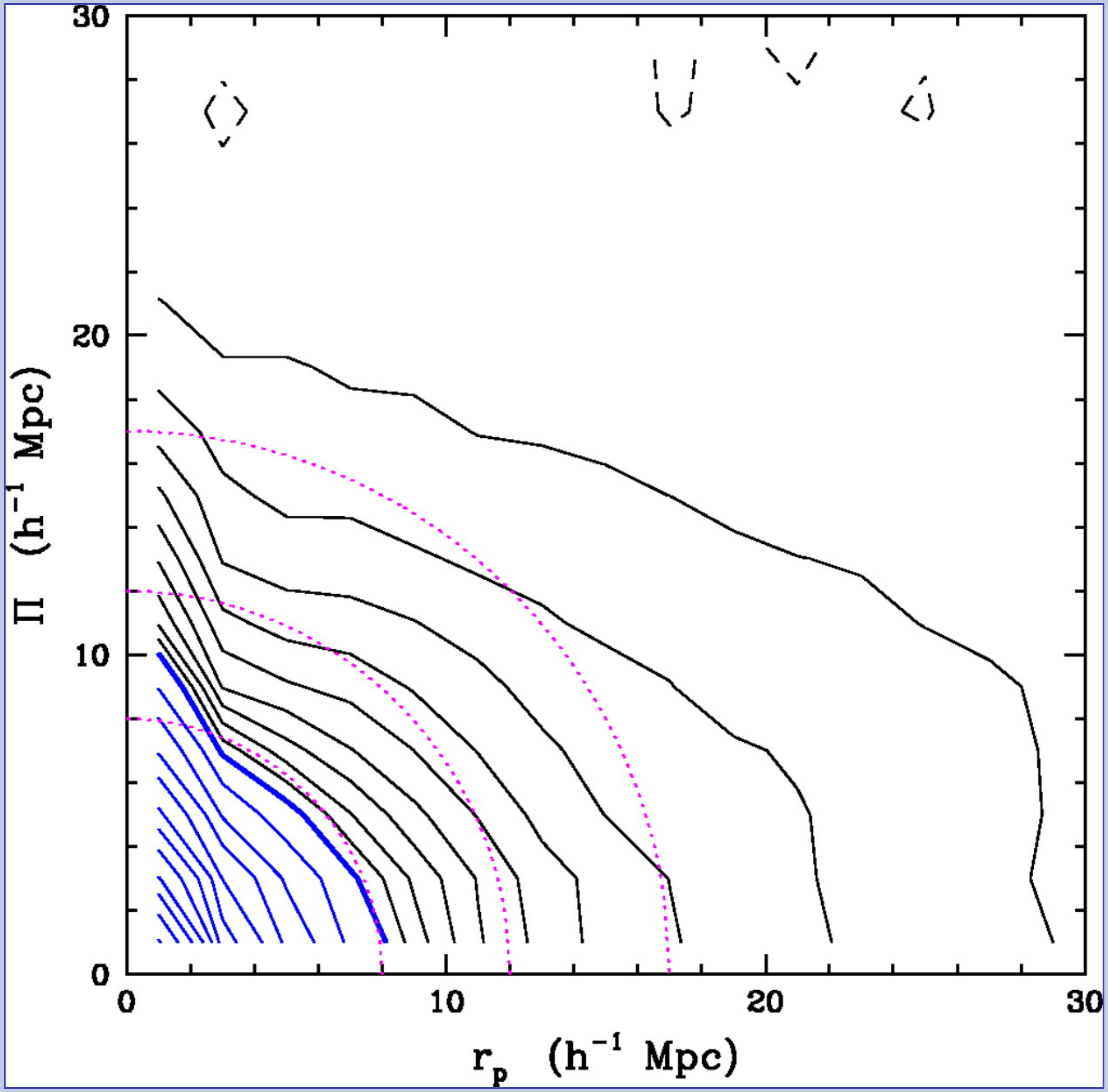
- To deal with redshift distortions, we usually measure a *projected* correlation function.

$$\xi\left(r_p,\pi\right)$$



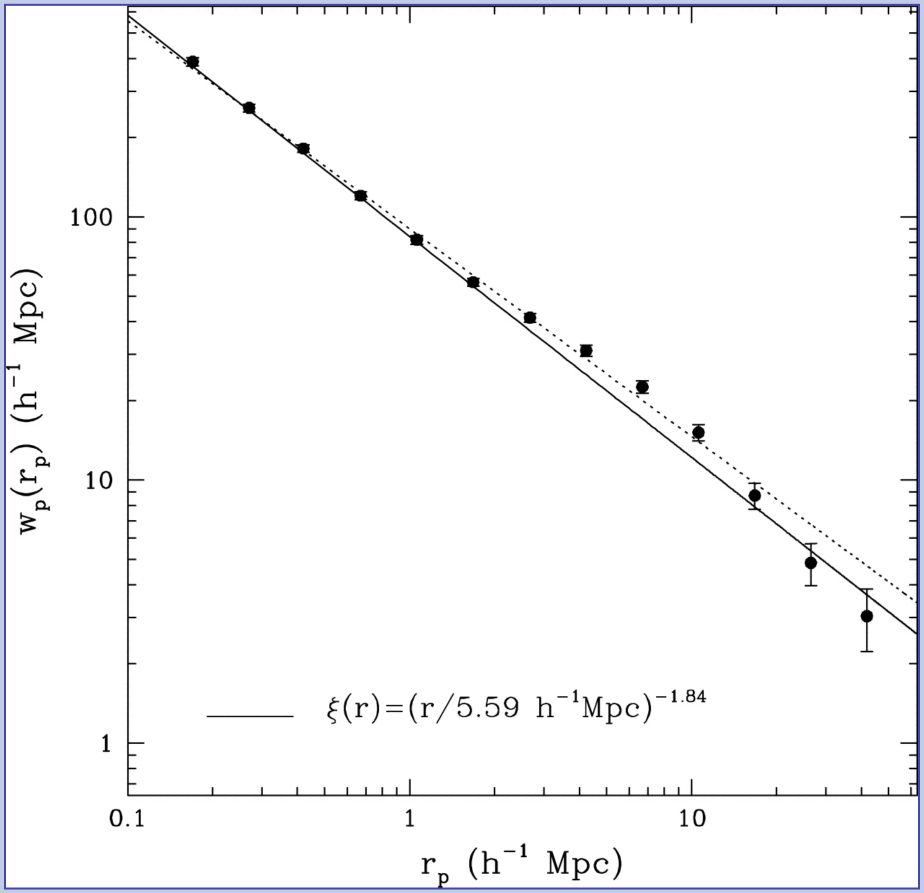
$$w_p\left(r_p\right)=2\int_0^{\pi_{\max}}\xi\left(r_p,\pi\right)d\pi$$

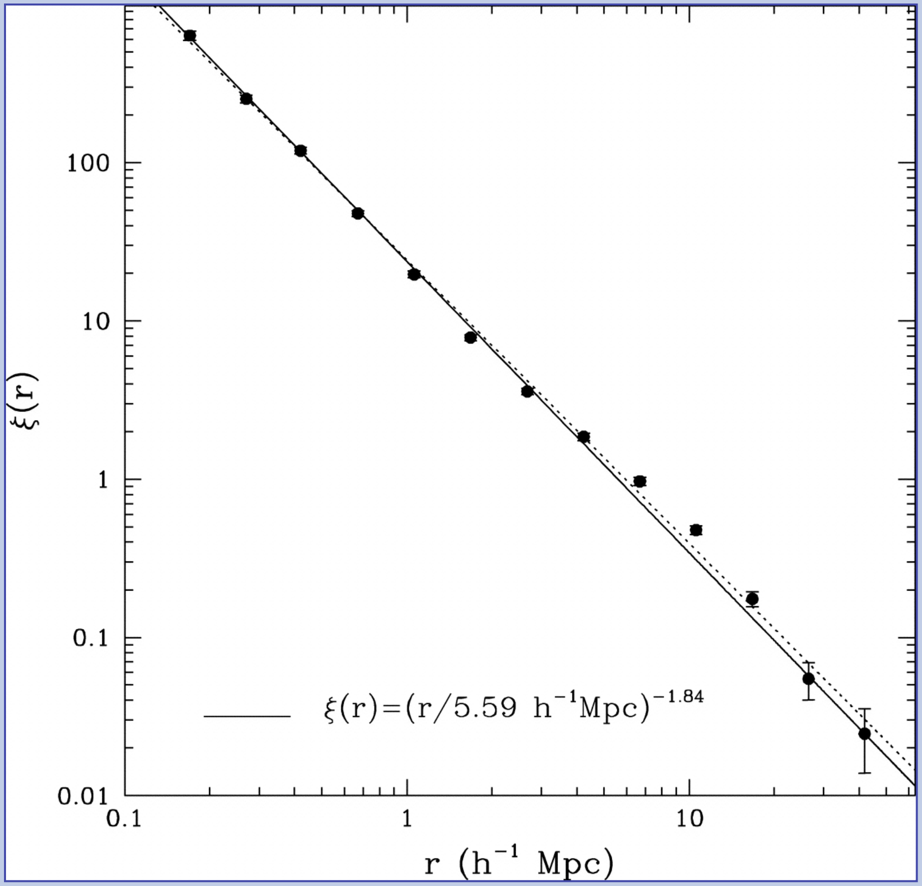
The 2-point Correlation Function



Zehavi et al. (2005)

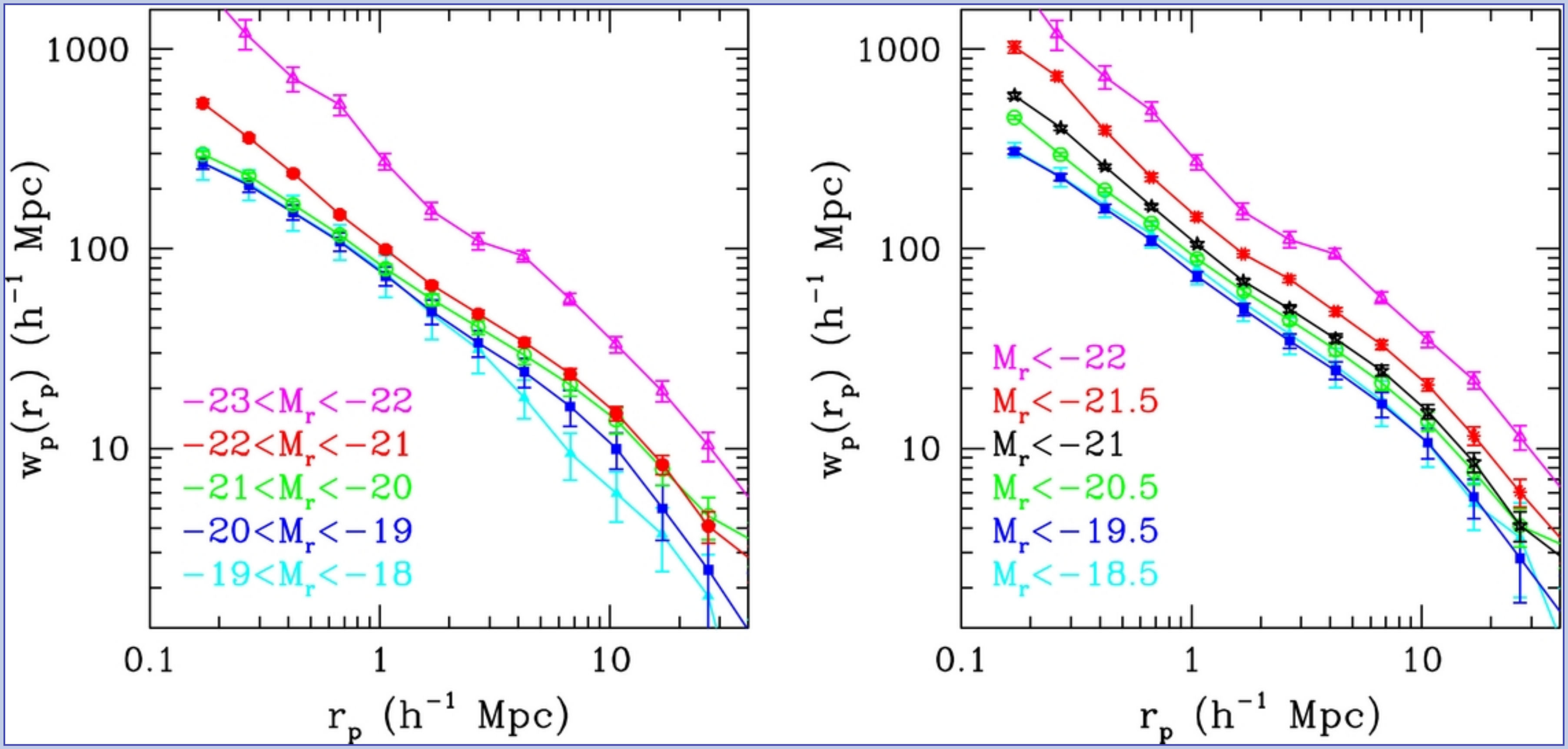
The 2-point Correlation Function





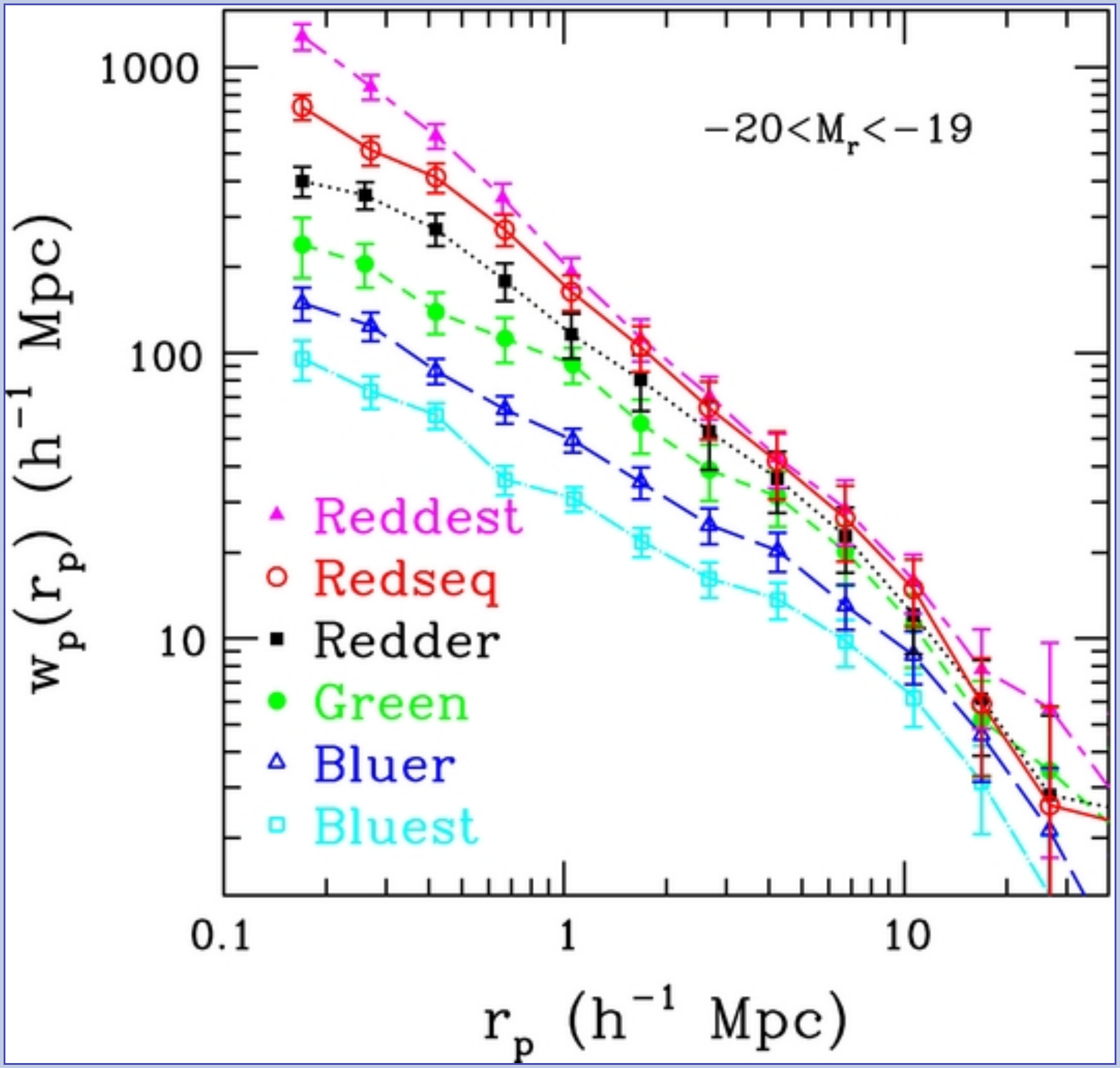
Zehavi et al. (2005)

The 2-point Correlation Function



Zehavi et al. (2011)

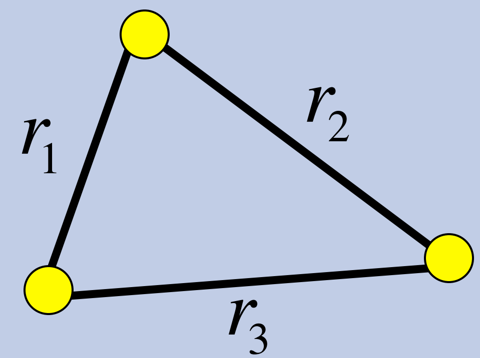
The 2-point Correlation Function

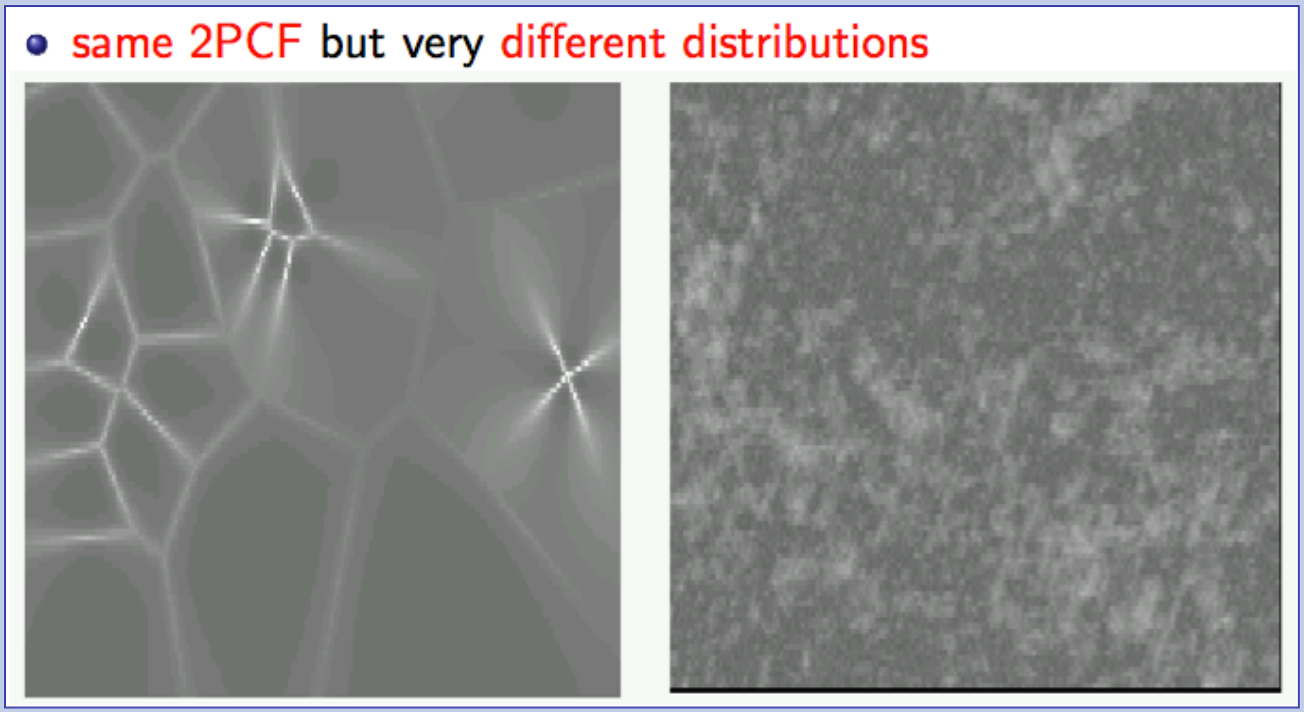


Zehavi et al. (2011)

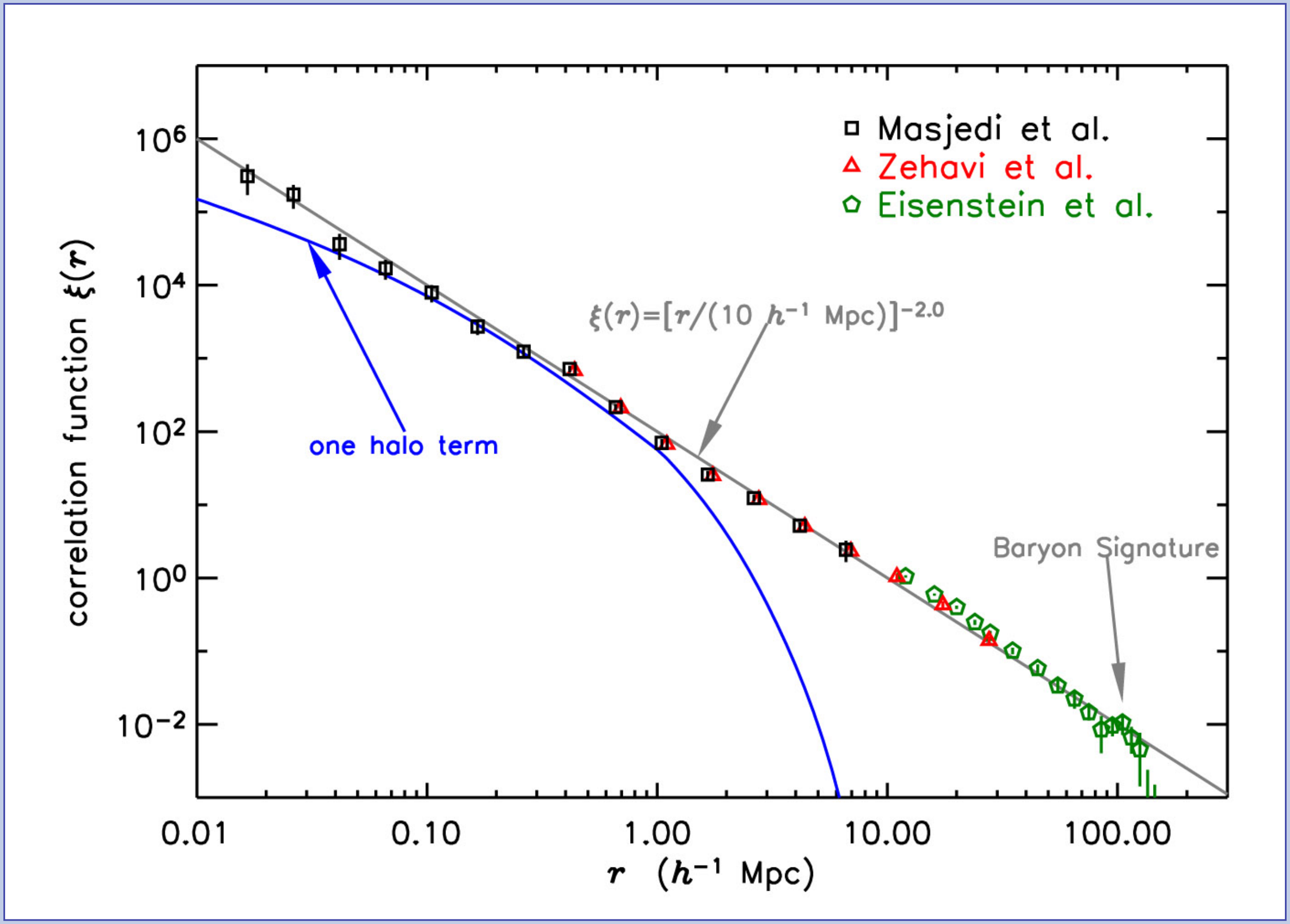
Other Correlation Functions

- Angular correlation function $\omega(\theta)$
- Cross-correlation function
- Higher order: three-point, etc





The 2-point Correlation Function



Masjedi et al. (2006)

The 2-point Correlation Function

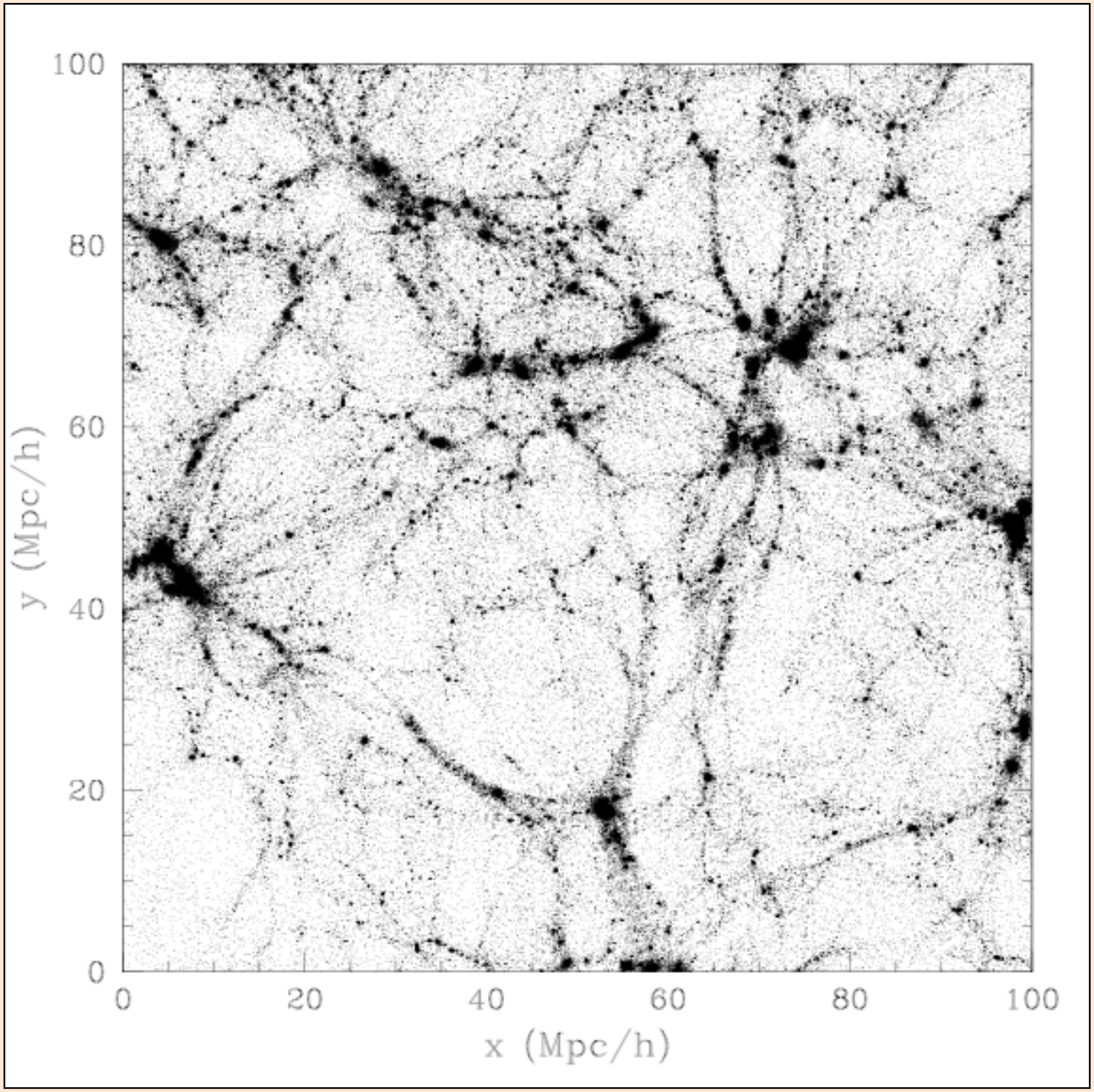
Galaxy “bias” refers to the amount of galaxy clustering relative to the clustering of the underlying dark matter

$$b = \sqrt{\frac{\xi_{\text{galaxy}}}{\xi_{\text{mass}}}}$$

This is a function of scale ***r***, but on large scales it becomes constant.

For example, the bias of Milky Way – like galaxies is ***b*** ~ 1
the bias of Luminous Red Galaxies is ***b*** ~ 2

Power Spectrum



Power Spectrum

Density field

$$\delta(\vec{x}) = \frac{\rho(\vec{x}) - \bar{\rho}}{\bar{\rho}}$$

Correlation function

$$\xi(r) = \langle \delta(\vec{x})\delta(\vec{x}+r) \rangle$$

Fourier density modes

$$\delta_{\vec{k}} = \int \delta(\vec{x}) e^{i\vec{k}\cdot\vec{x}} d^3\vec{x}$$

Power spectrum

$$P(k) = \left\langle \left|\delta_{\vec{k}}\right|^2 \right\rangle$$

$$\boxed{P(k) = \int \xi(r) e^{i\vec{k}\cdot\vec{r}} d^3\vec{x}}$$

Power Spectrum

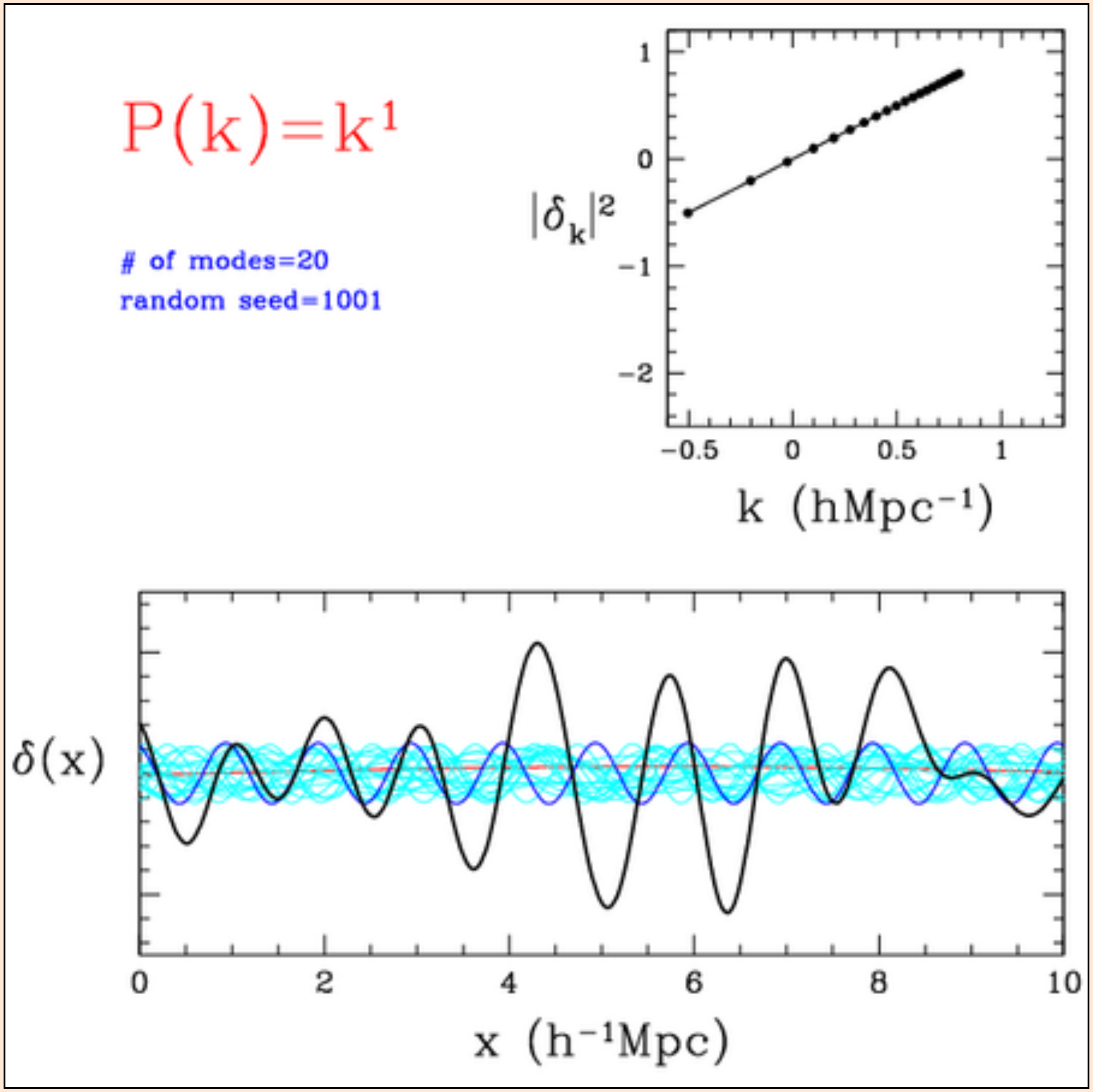
Any density field can be decomposed into an infinite set of modes (i.e., sine waves) $\delta_{\vec{k}}$

Each mode has a

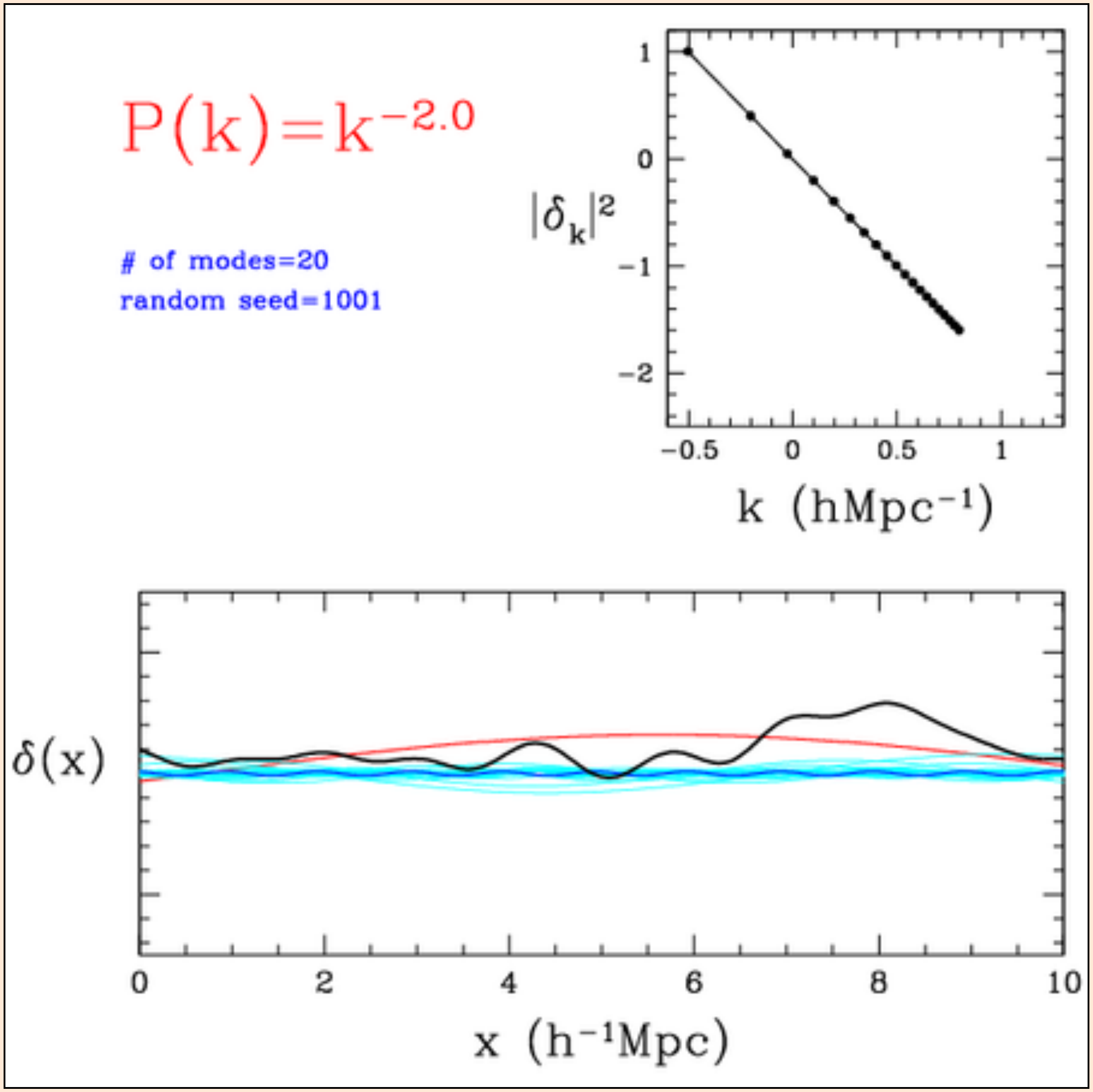
- wavelength λ or wavenumber $k = \dfrac{2\pi}{\lambda}$
- amplitude $\left|\delta_{\vec{k}}\right|$
- phase $e^{-i\theta}$

The power spectrum is the amplitude as a function of k

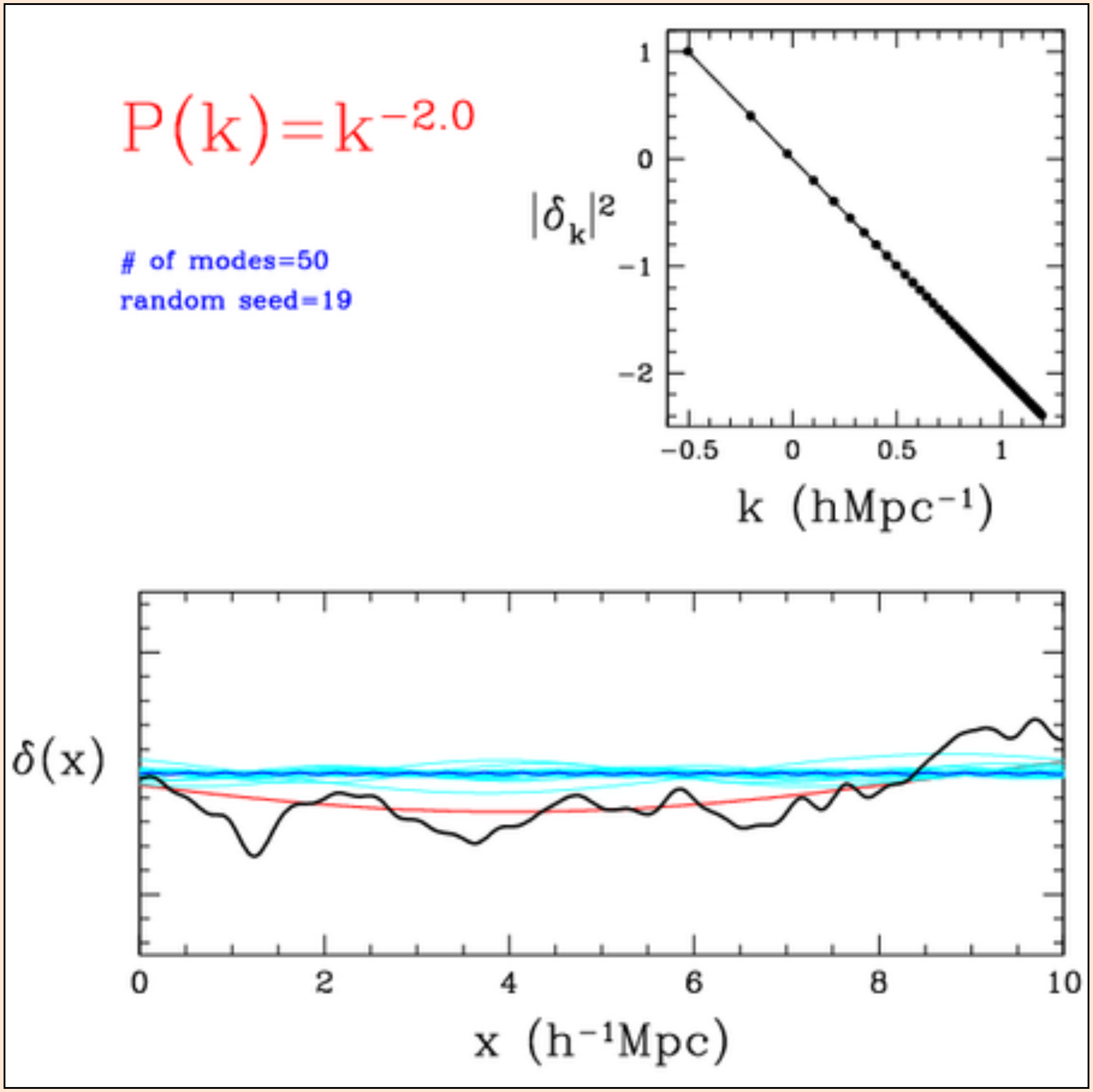
Power Spectrum



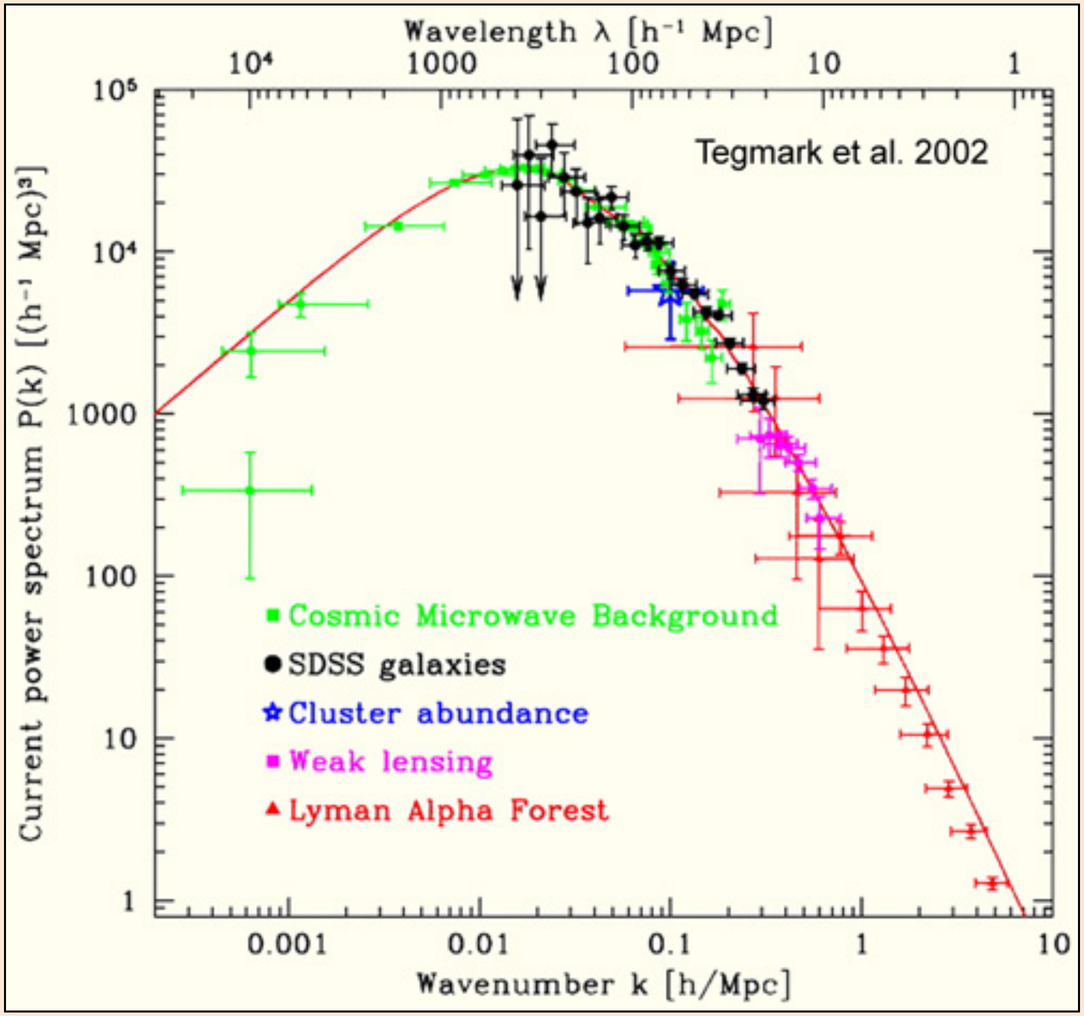
Power Spectrum



Power Spectrum



Power Spectrum



Power Spectrum

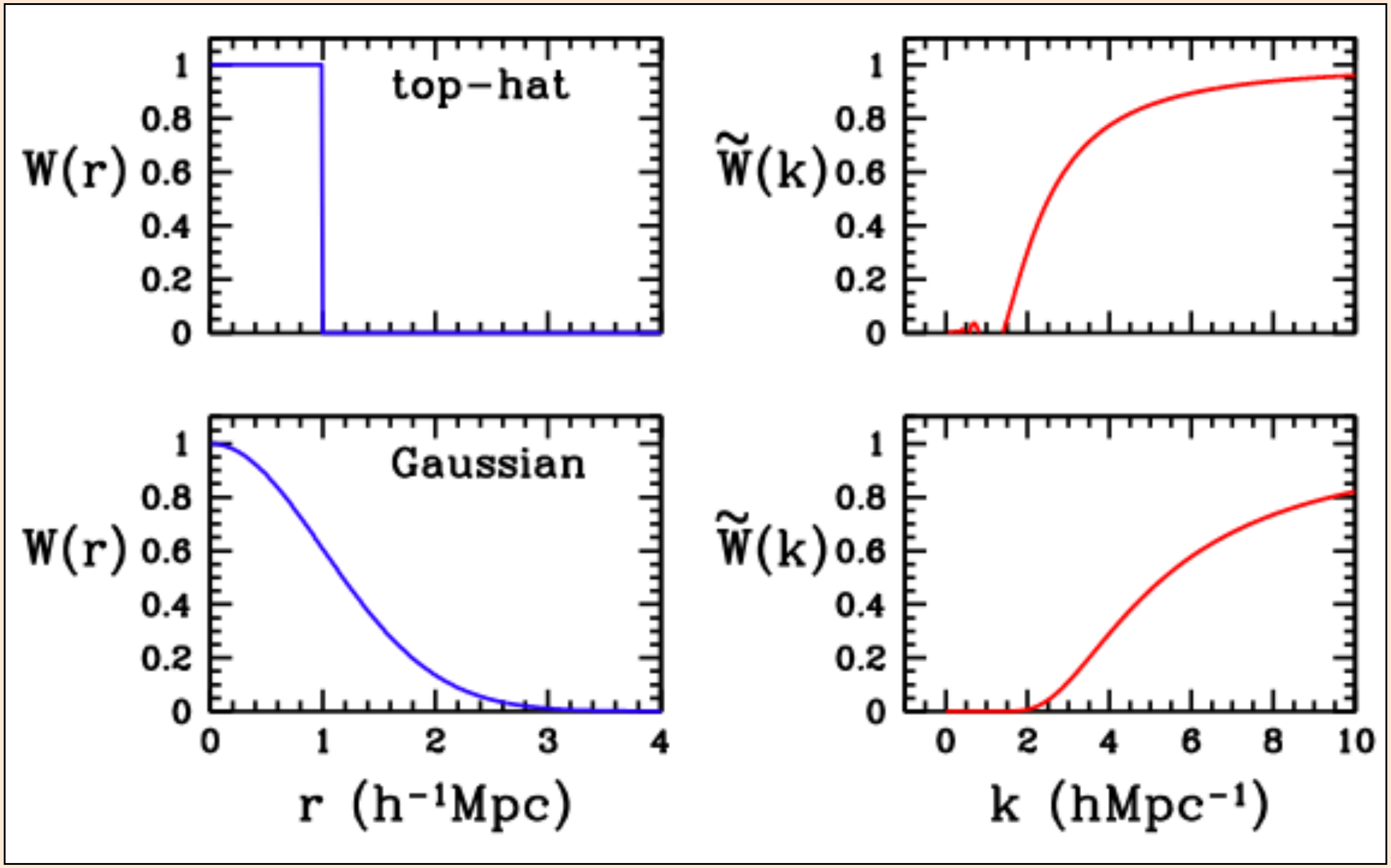
Window function: filter used to smooth density field

- Gaussian filter of scale *R* $$W_R(r) = e^{-r^2/2R^2}$$

- Top-hat filter of scale *R* $$W_R(r) = \begin{cases} 1 & r < R \\ 0 & r > R \end{cases}$$

In Fourier space: $$\tilde{W}_R(k) = \int W_R(r) e^{i\vec{k}\cdot\vec{r}} d^3r$$

Power Spectrum



Power Spectrum

Density field, smoothed with window function

$$\delta_R(\vec{x}) = \int \delta(\vec{x}') W_R(|\vec{x}' - \vec{x}|) d^3x'$$

Mean density, smoothed with window function

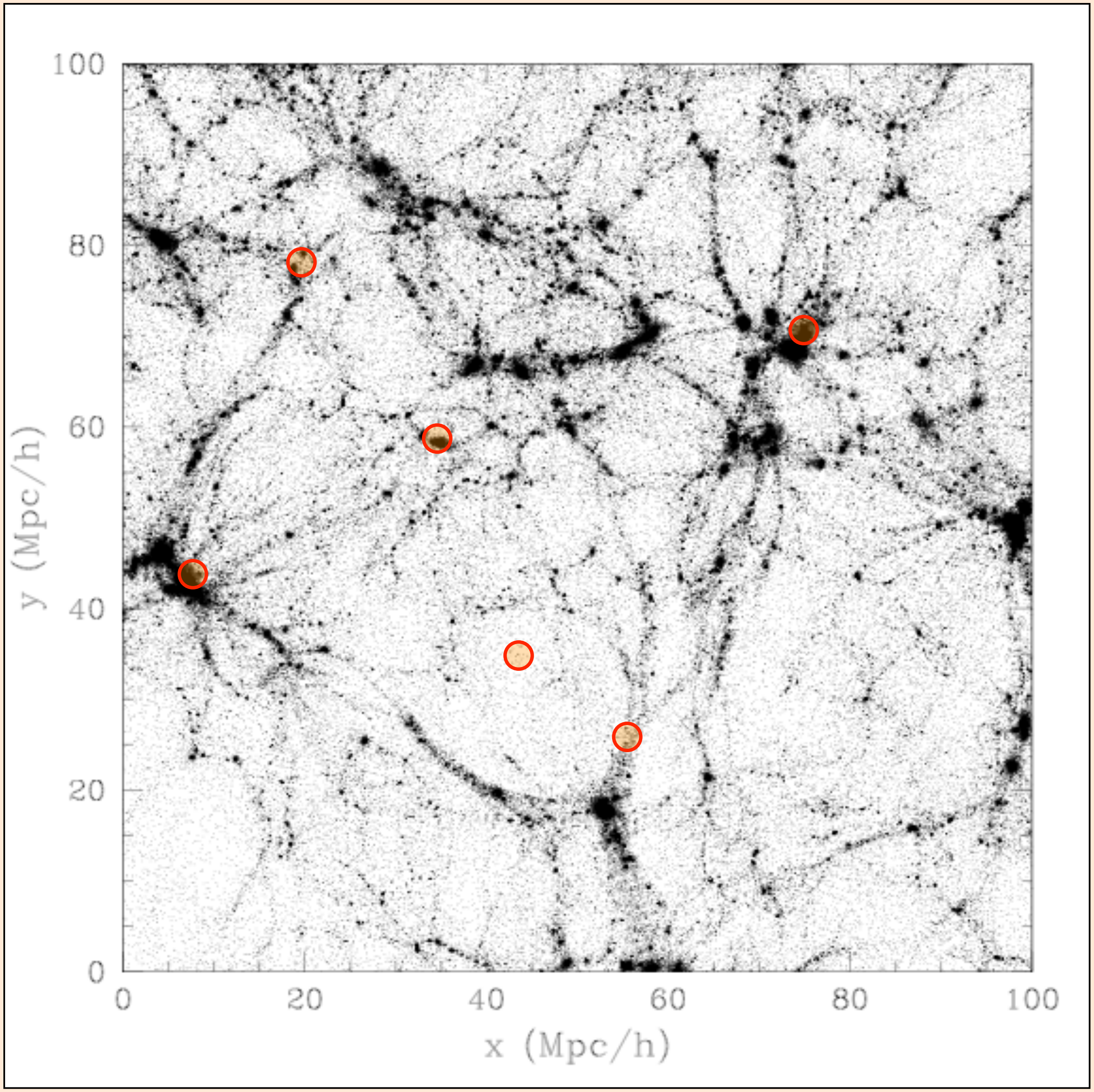
$$\bar{\delta}_R = \langle \delta_R(\vec{x}) \rangle = 0 \quad \text{since} \quad \delta = \frac{\rho - \bar{\rho}}{\bar{\rho}}$$

Variance of smoothed density field

$$\sigma_R^2 = \langle \delta_R(\vec{x})^2 \rangle$$

$$\sigma_R^2 = \int P(k) \tilde{W}_R(k)^2 d^3k$$

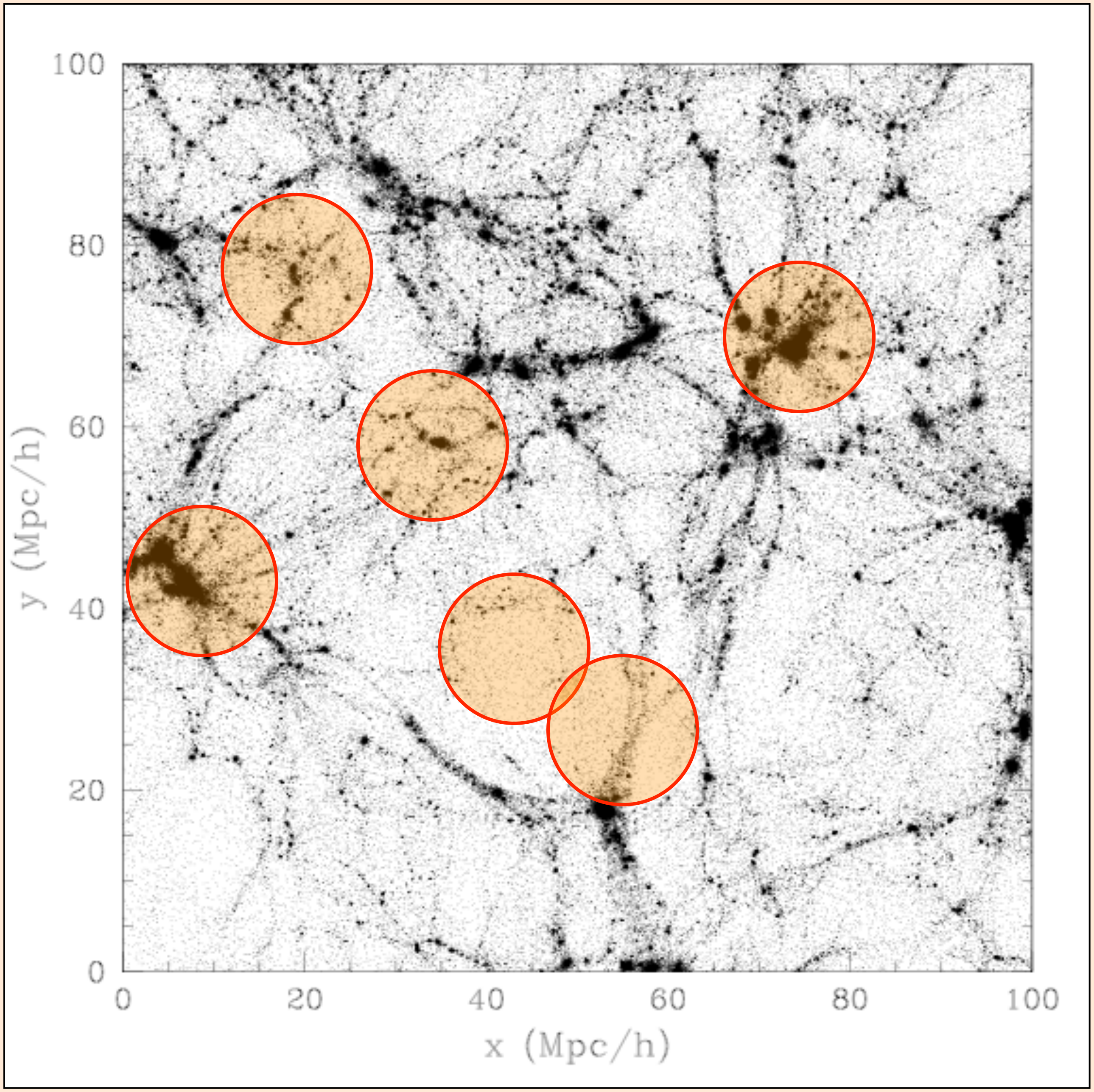
Power Spectrum



R=2 Mpc/h

Top-hat filter

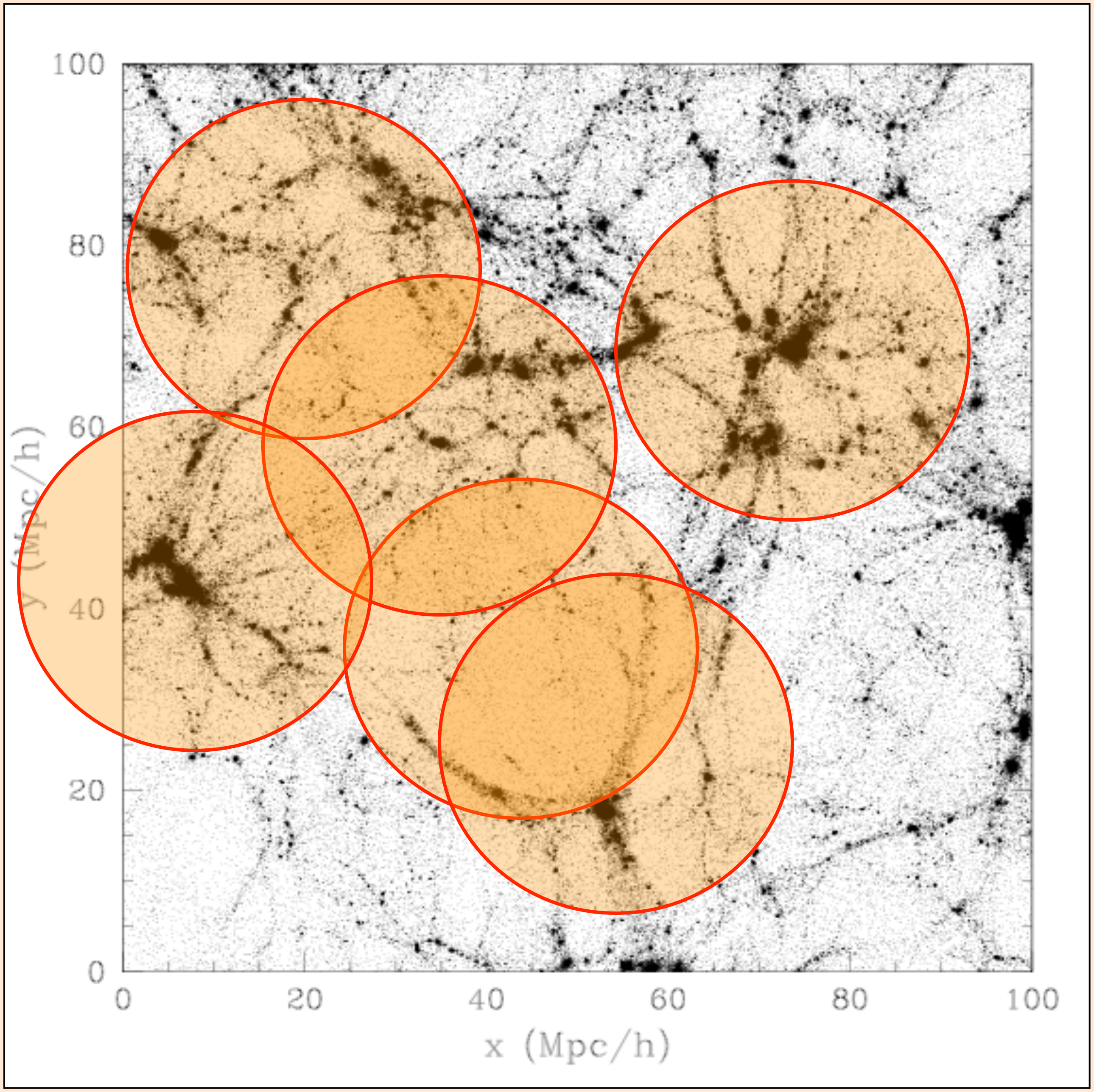
Power Spectrum



R=8 Mpc/h

Top-hat filter

Power Spectrum



R=20 Mpc/h

Top-hat filter

Power Spectrum

$$\sigma_R^2 = \frac{1}{N}\sum\left(\delta_R - \cancelto{0}{\bar{\delta}_R}\right)^2 \quad = \left\langle \delta_R^2 \right\rangle$$

The variance is large on small scales and approaches zero on large scales.

Power Spectrum

σ_R^2 Is the variance of the matter density field

It also sets the amplitude of the matter power spectrum on scale *R*

For a power spectrum with a power-law shape $P(k) \sim k^n$, defining the variance on one scale sets the amplitude on all scales. Also, any window function will do.

We choose a top-hat filter of *R*=8 Mpc/h to describe the amplitude of P(k)

Cosmological Parameters

Cosmological parameter #1: the Hubble constant

$$h$$

$$\boxed{h \approx 0.7 \pm 0.02}$$

Cosmological Parameters

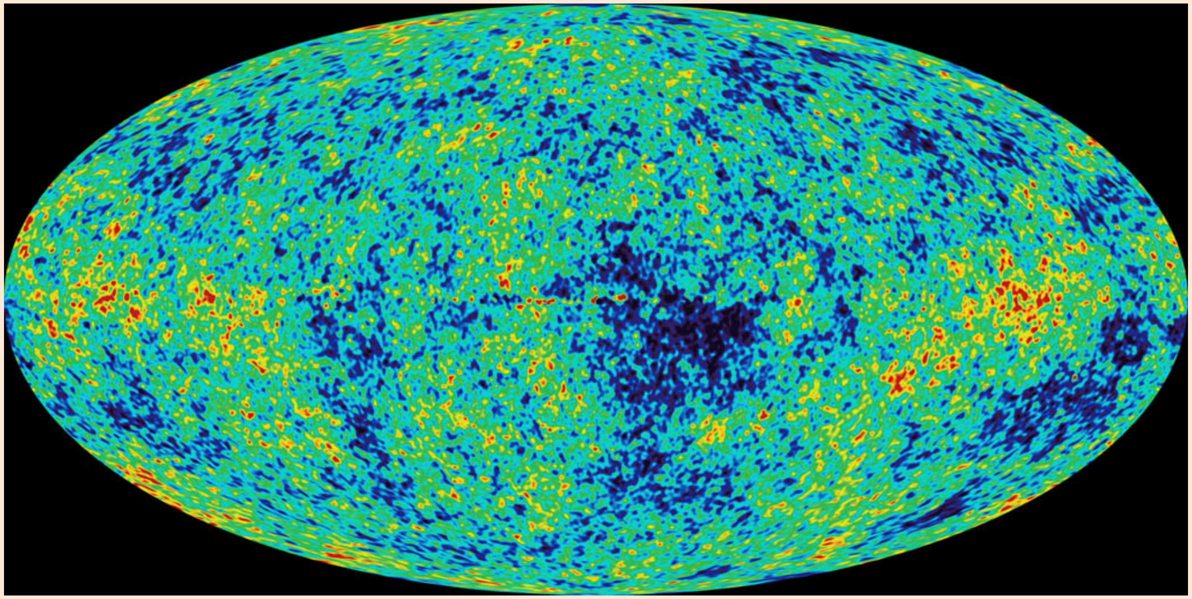
Cosmological parameter #2: the amplitude of the matter power spectrum

$$\sigma_8$$

$$\boxed{\sigma_8 \approx 0.82 \pm 0.02}$$

Power Spectrum

Primordial
power spectrum



Quantum fluctuations + inflation:

$$P(k) \propto k^n$$

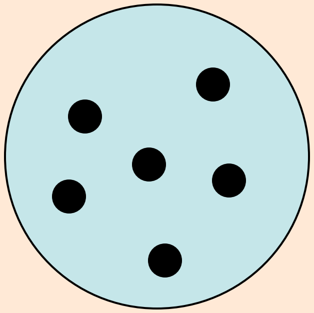
<u>Cosmological parameter #3</u>: the power spectrum index

$$n_s$$

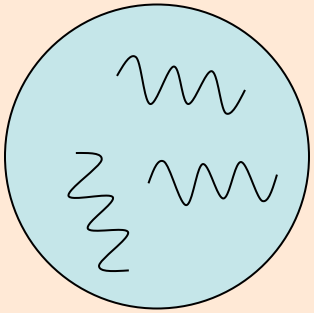
$$\boxed{n_s \approx 0.96 \pm 0.02}$$

Power Spectrum

Matter

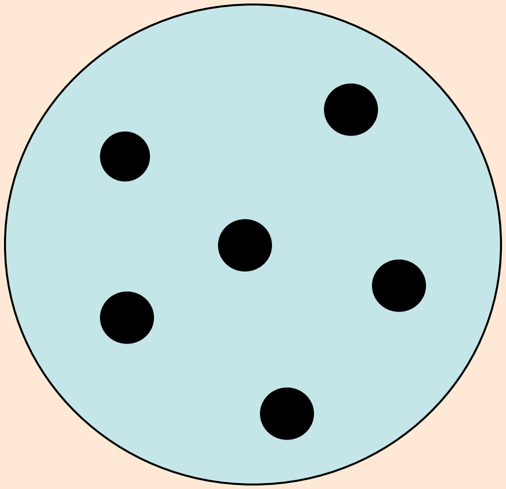


Radiation

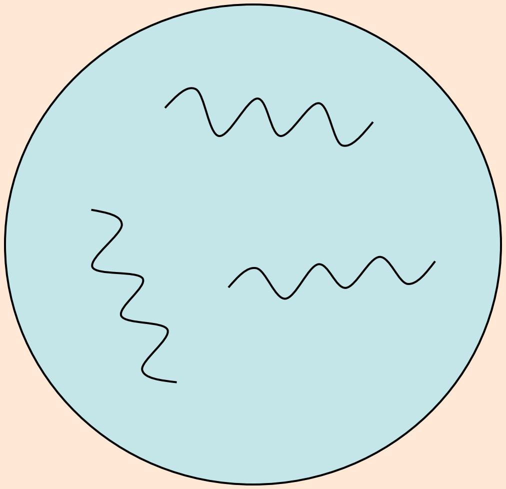


Power Spectrum

Matter

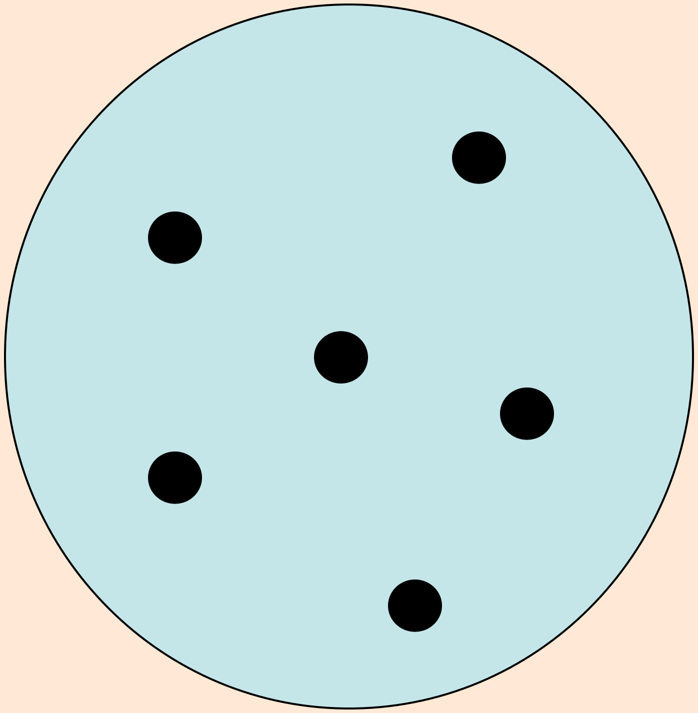


Radiation

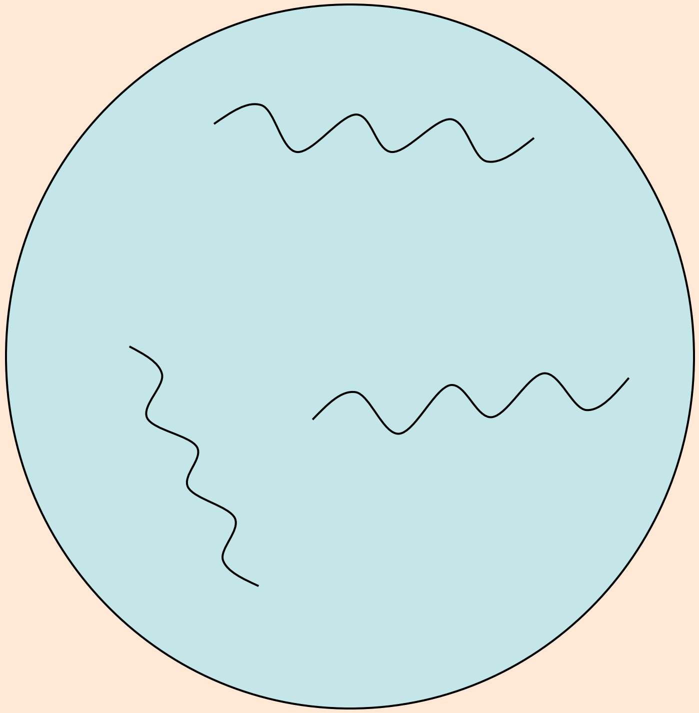


Power Spectrum

Matter



Radiation



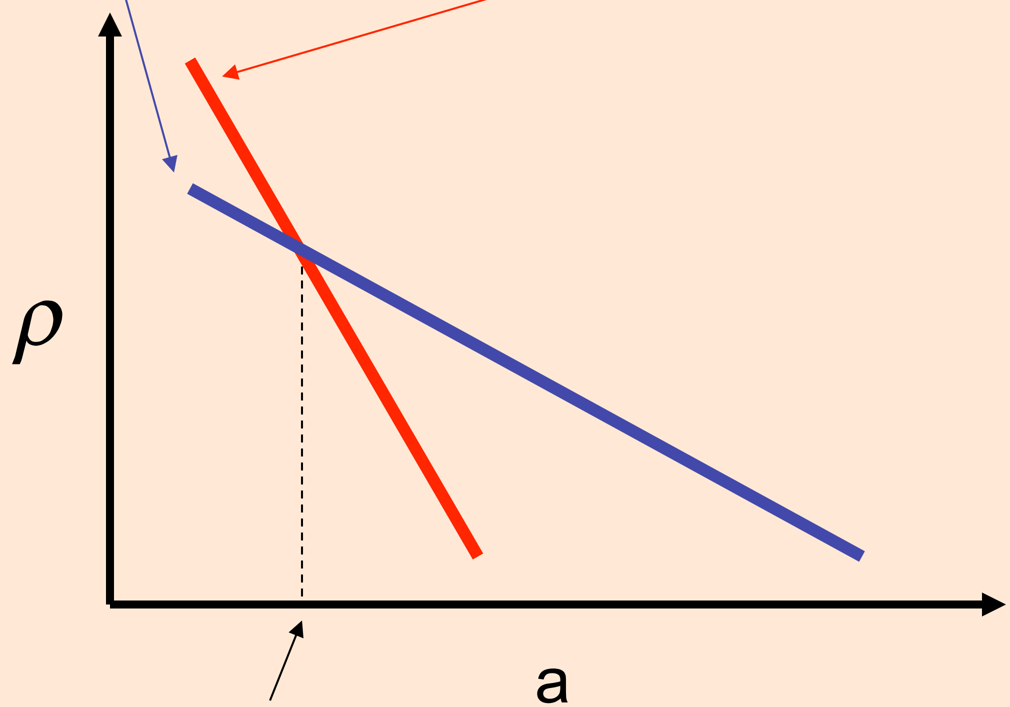
Power Spectrum

Matter

$$\rho \sim a^{-3} = (1+z)^3$$

Radiation

$$\rho \sim a^{-4} = (1+z)^4$$



Matter-radiation equality

$$a_{eq} \approx 2.8 \times 10^{-4}$$

$$z_{eq} \approx 3600$$

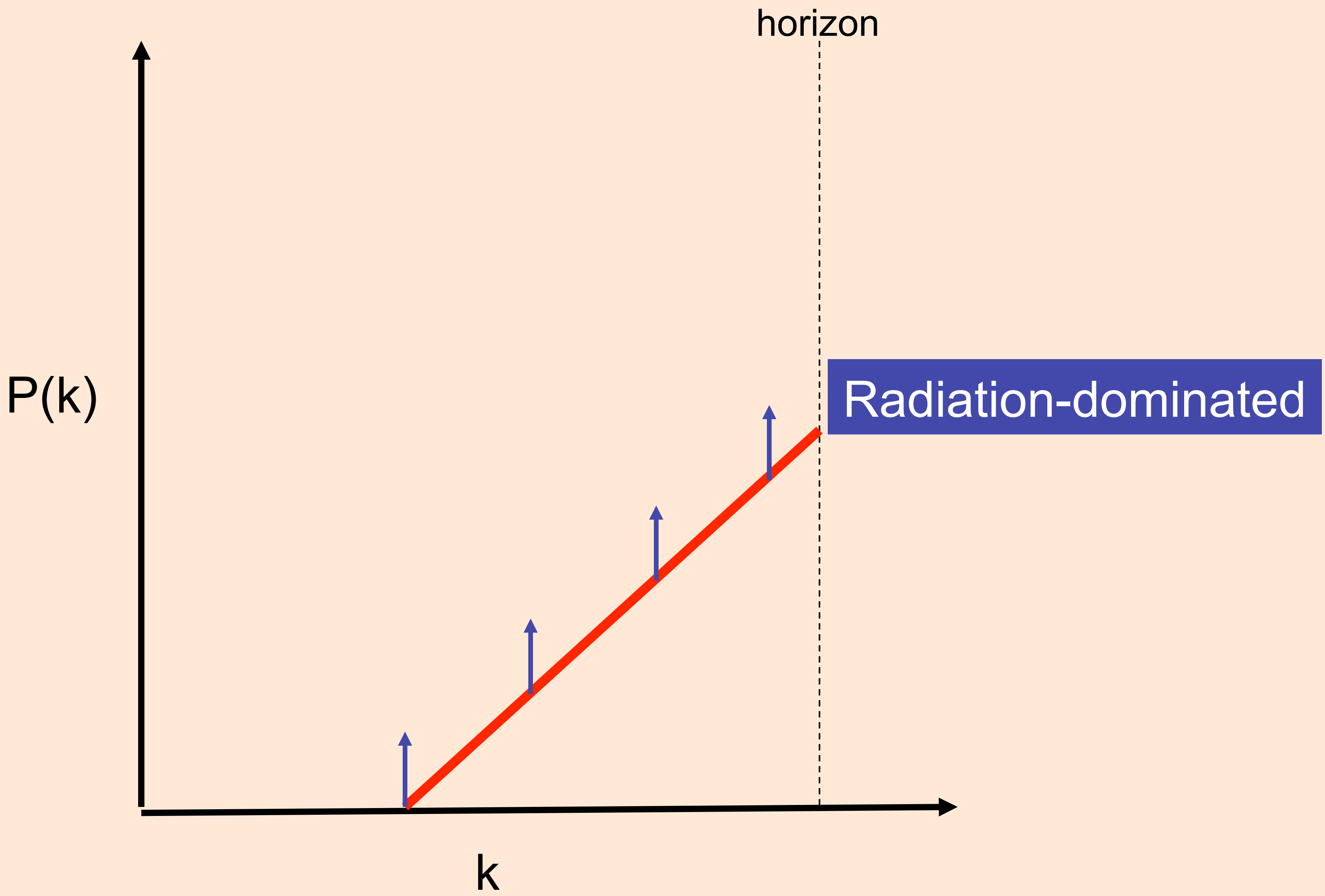
$$t_{eq} \approx 47{,}000\,yr$$

Power Spectrum

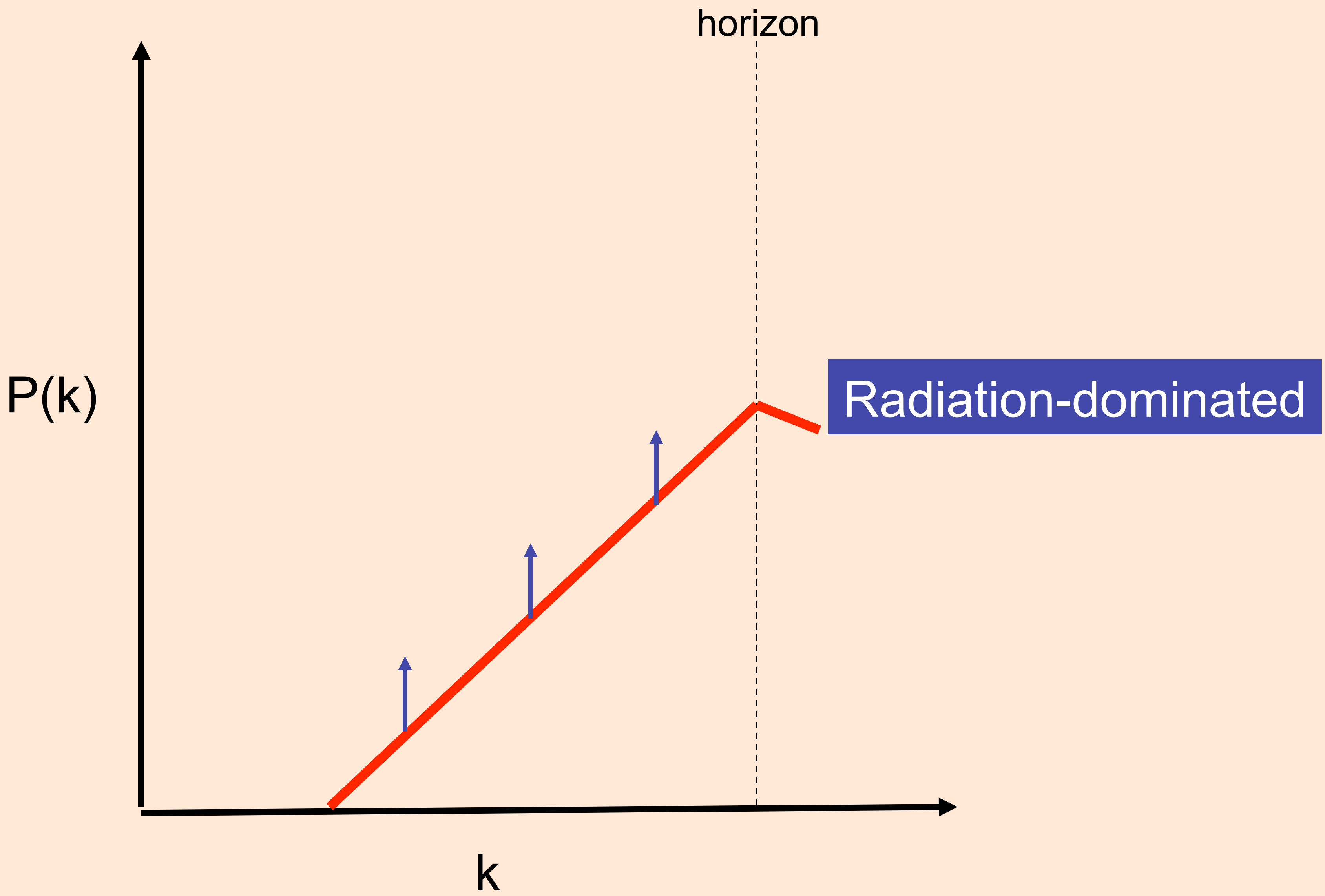
Density fluctuations cannot grow when the universe is radiation-dominated if their wavelength is smaller than the size of the horizon.

	Radiation dominated	Matter dominated
$\lambda \ll c/H_0$	Cannot grow	Grow
$\lambda \gg c/H_0$	Grow	Grow

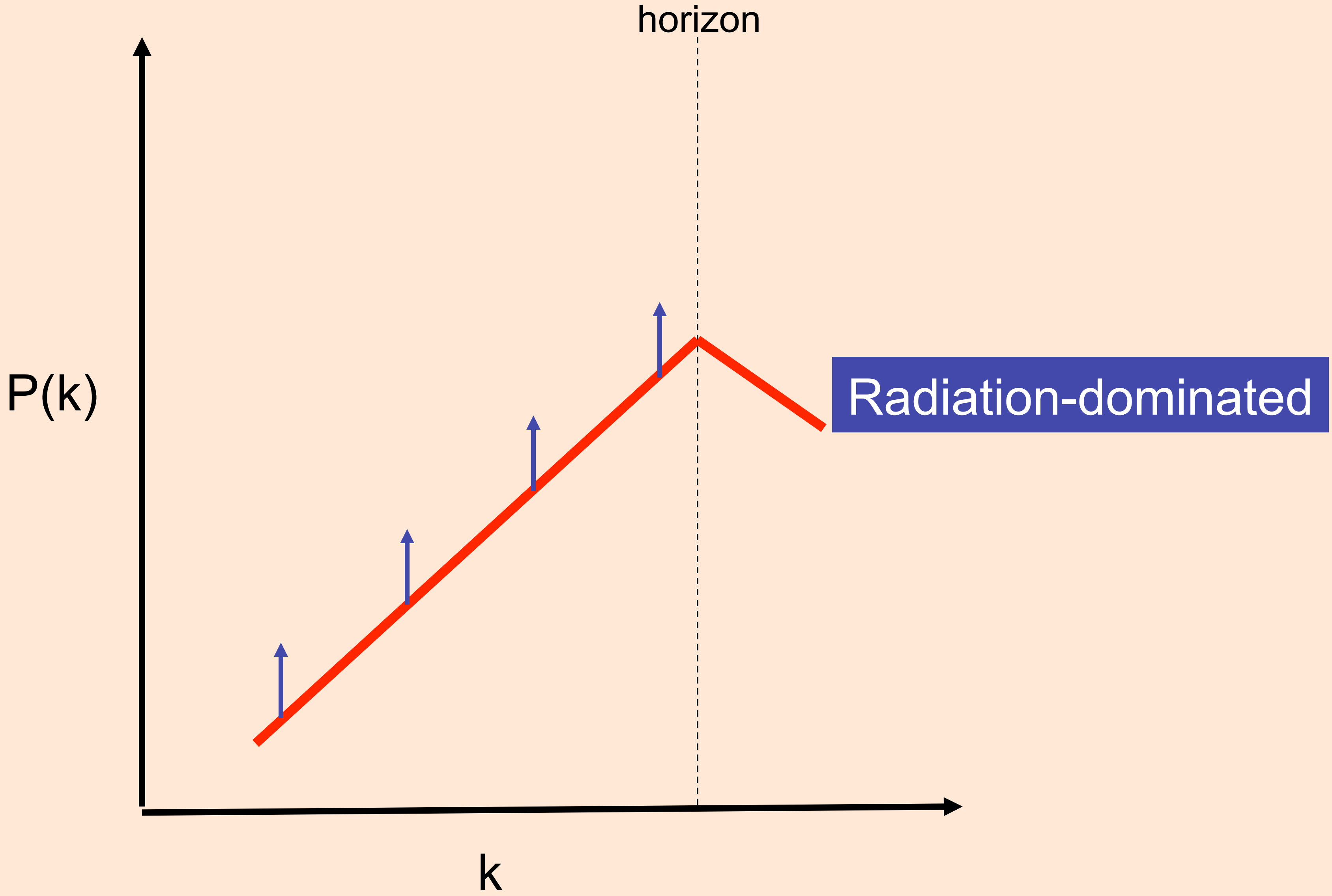
Power Spectrum



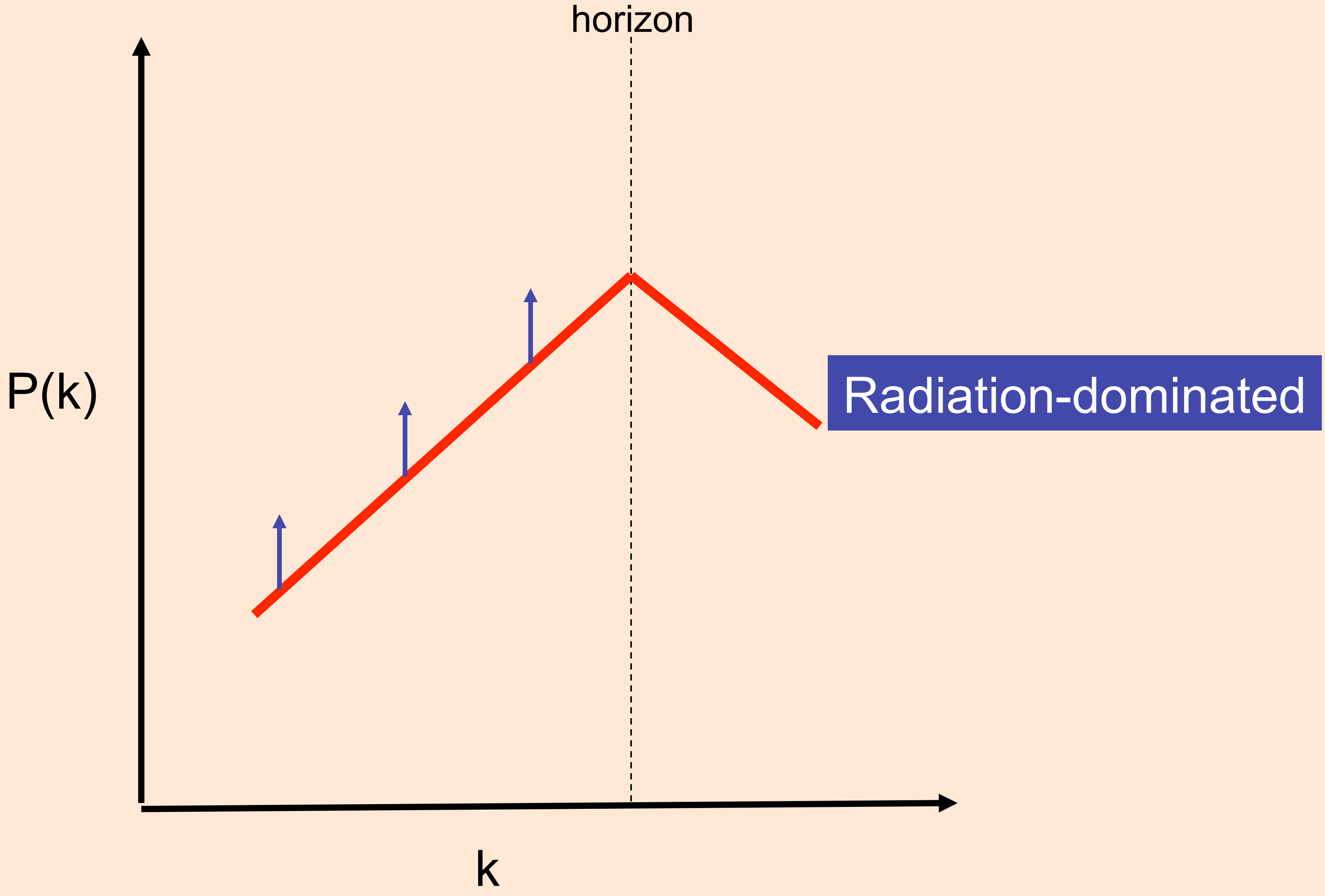
Power Spectrum



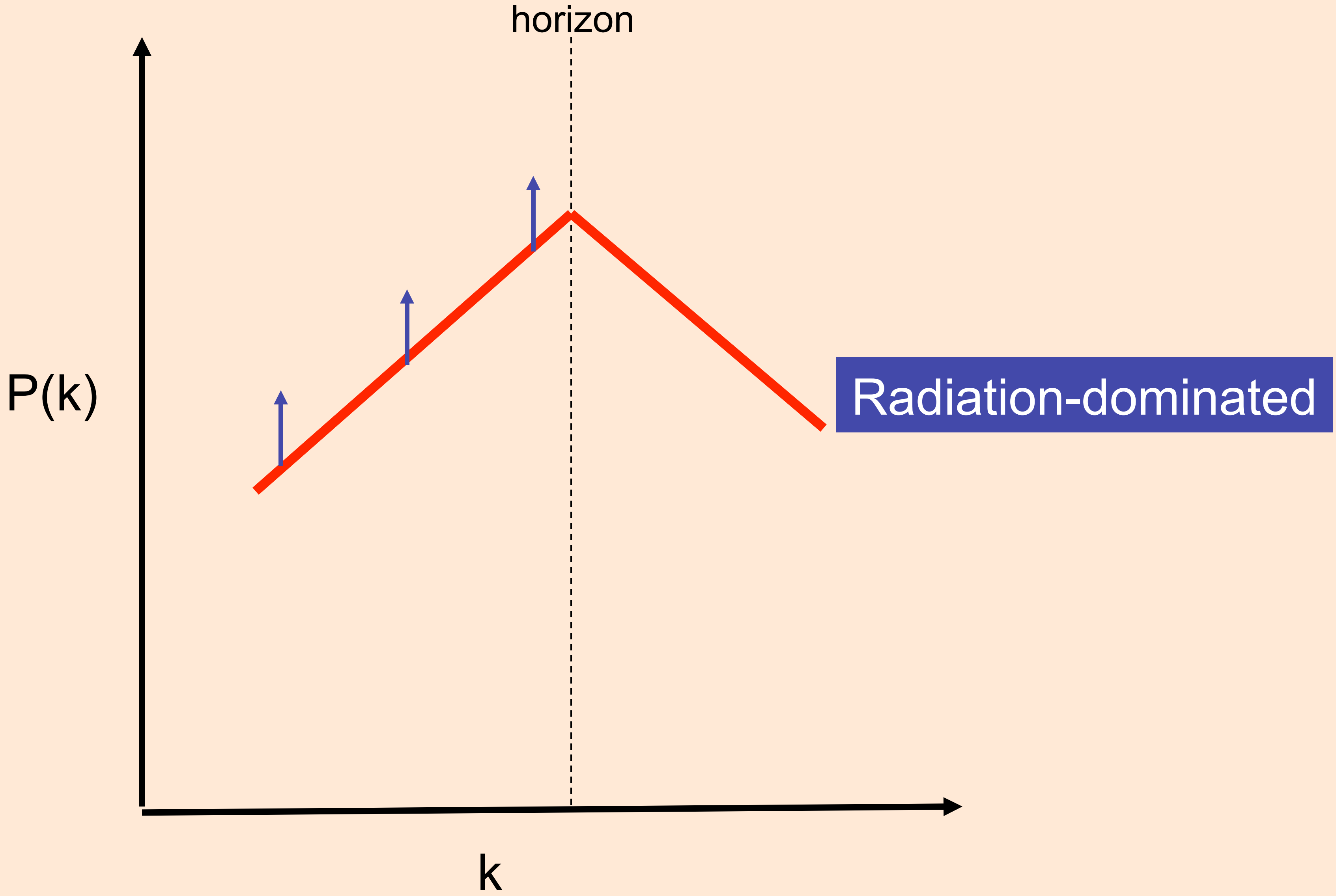
Power Spectrum



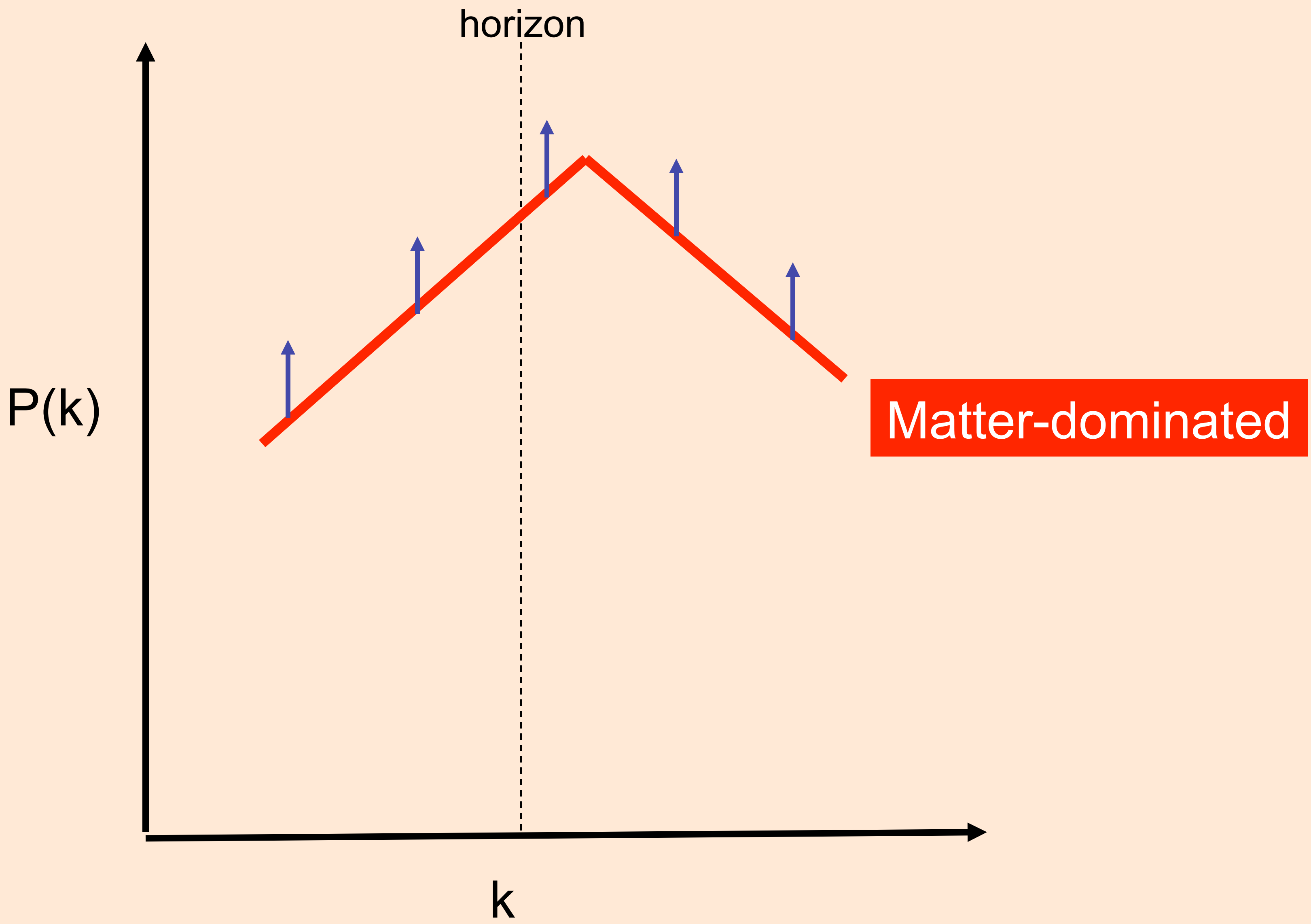
Power Spectrum



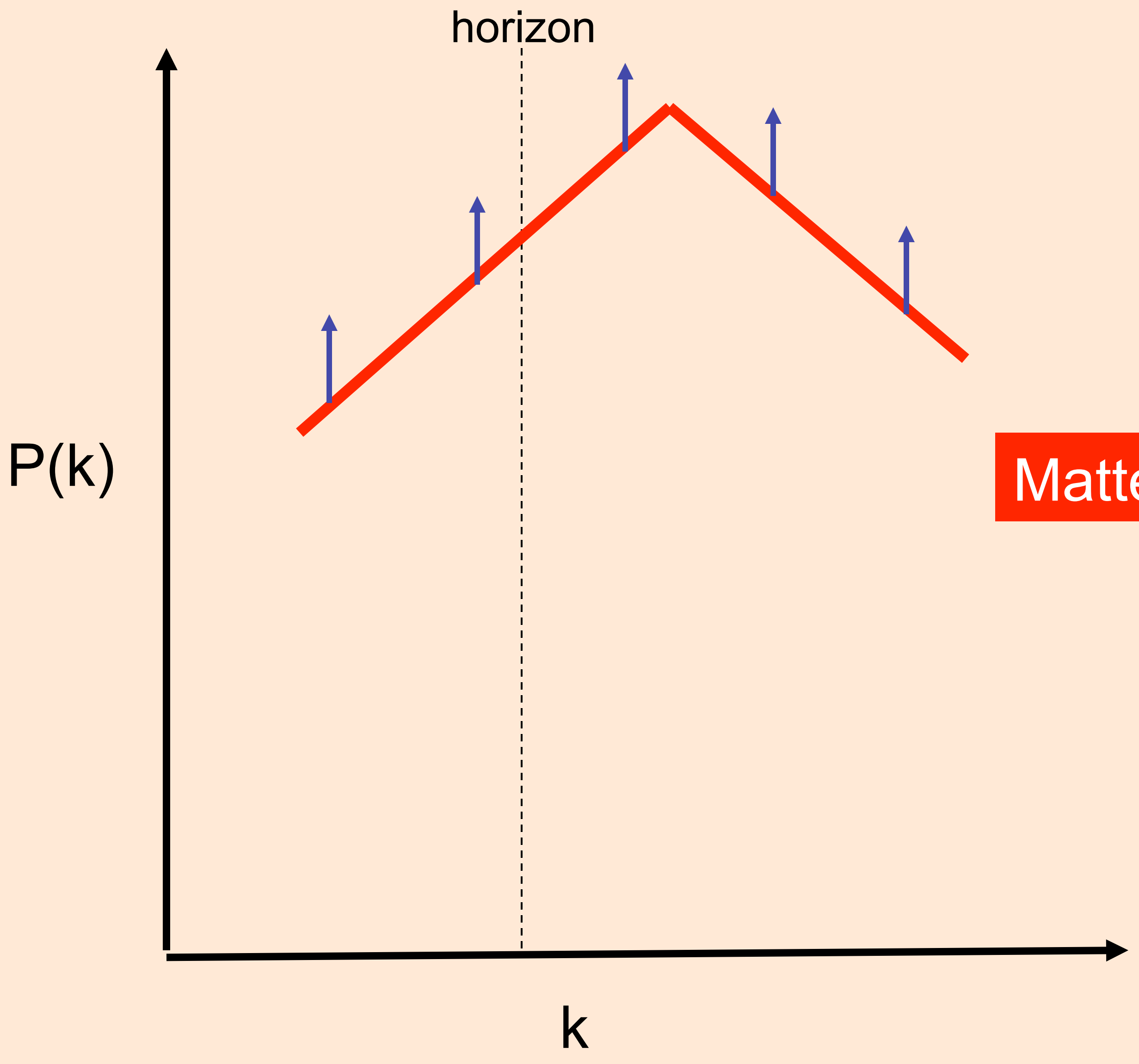
Power Spectrum



Power Spectrum

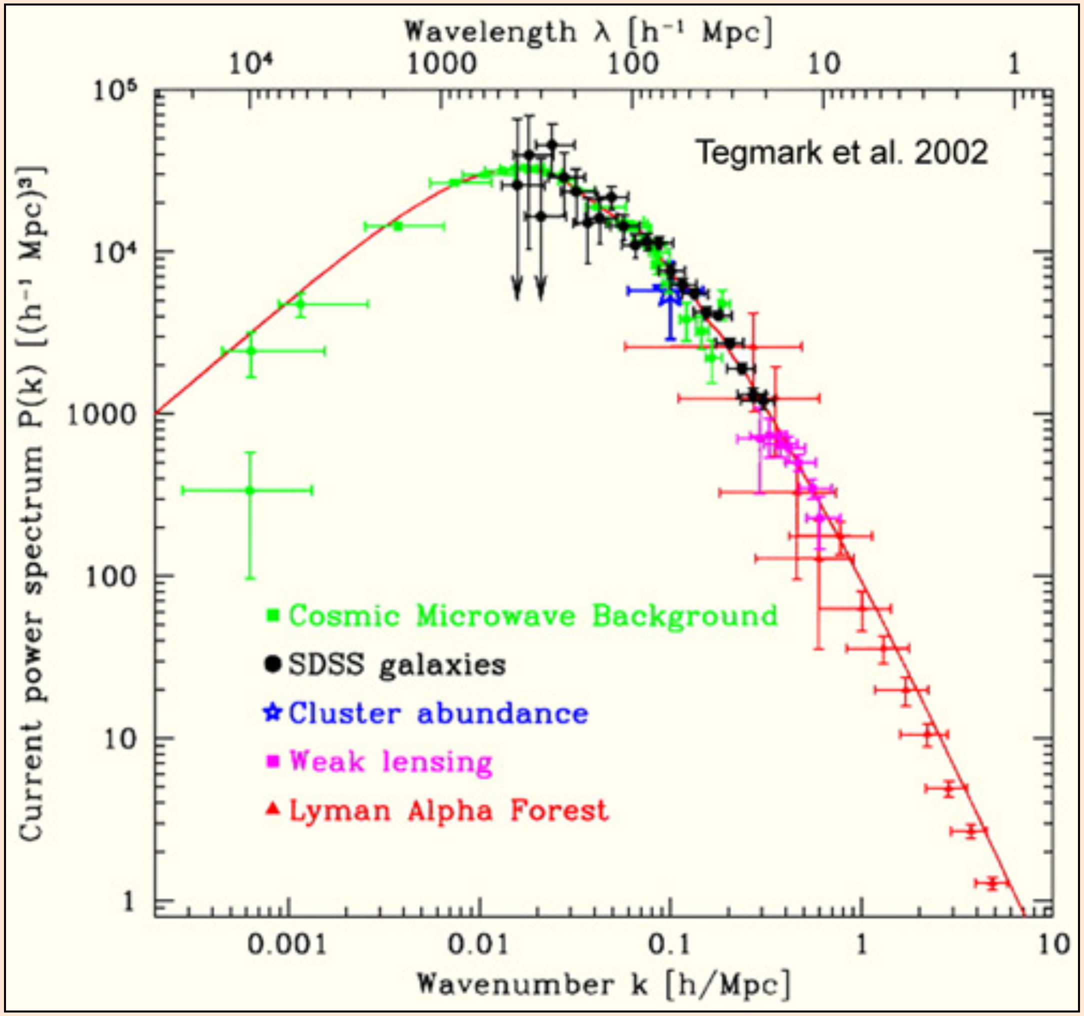


Power Spectrum



Matter-dominated

Power Spectrum



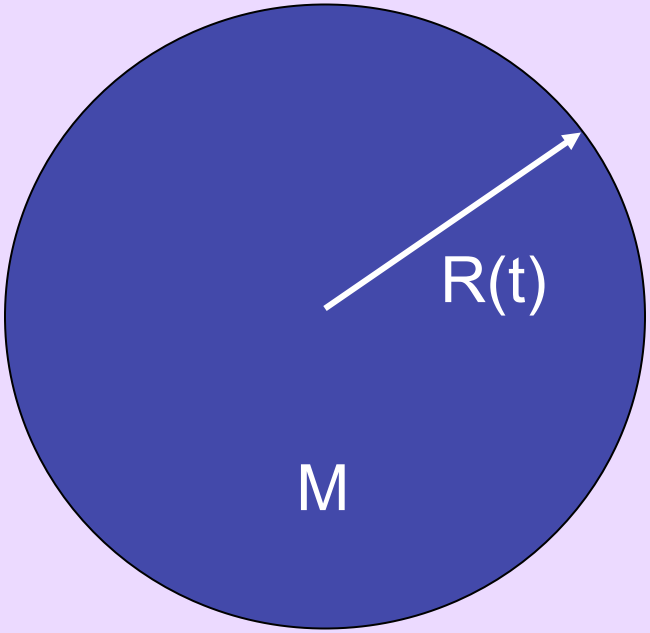
The Friedmann Equation

$$\ddot{R} = -\frac{GM}{R^2}$$

$$\ddot{R}\dot{R} = -\frac{GM\dot{R}}{R^2}$$

$$\frac{d}{dt}\left(\frac{1}{2}\dot{R}^2\right) = \frac{d}{dt}\left(\frac{GM}{R}\right)$$

$$\frac{1}{2}\dot{R}^2 = \frac{GM}{R} + K$$



Kinetic + potential energy per unit mass = constant

The Friedmann Equation

$$\frac{1}{2}\dot{R}^2 = \frac{GM}{R} + K$$

$$\boxed{M = \rho \frac{4}{3}\pi R^3}$$

$$\frac{1}{2}\dot{R}^2 = \frac{4\pi G\rho R^2}{3} + K$$

$$\frac{\dot{R}^2}{R^2} = \frac{8\pi G\rho}{3} + \frac{2K}{R^2}$$

$$\boxed{a(t) = \frac{R(t)}{R(t=0)} = \frac{R}{R_0}}$$

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G\rho}{3} + \frac{2K}{R_0^2 a^2}$$

The Friedmann Equation

$$\boxed{\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho(t) + \frac{2K}{R_0^2}\frac{1}{a(t)^2}}$$

Newtonian form of Friedman equation

General Relativity:

• Replace density with energy density

$$\rho(t) \rightarrow \frac{\varepsilon(t)}{c^2}$$

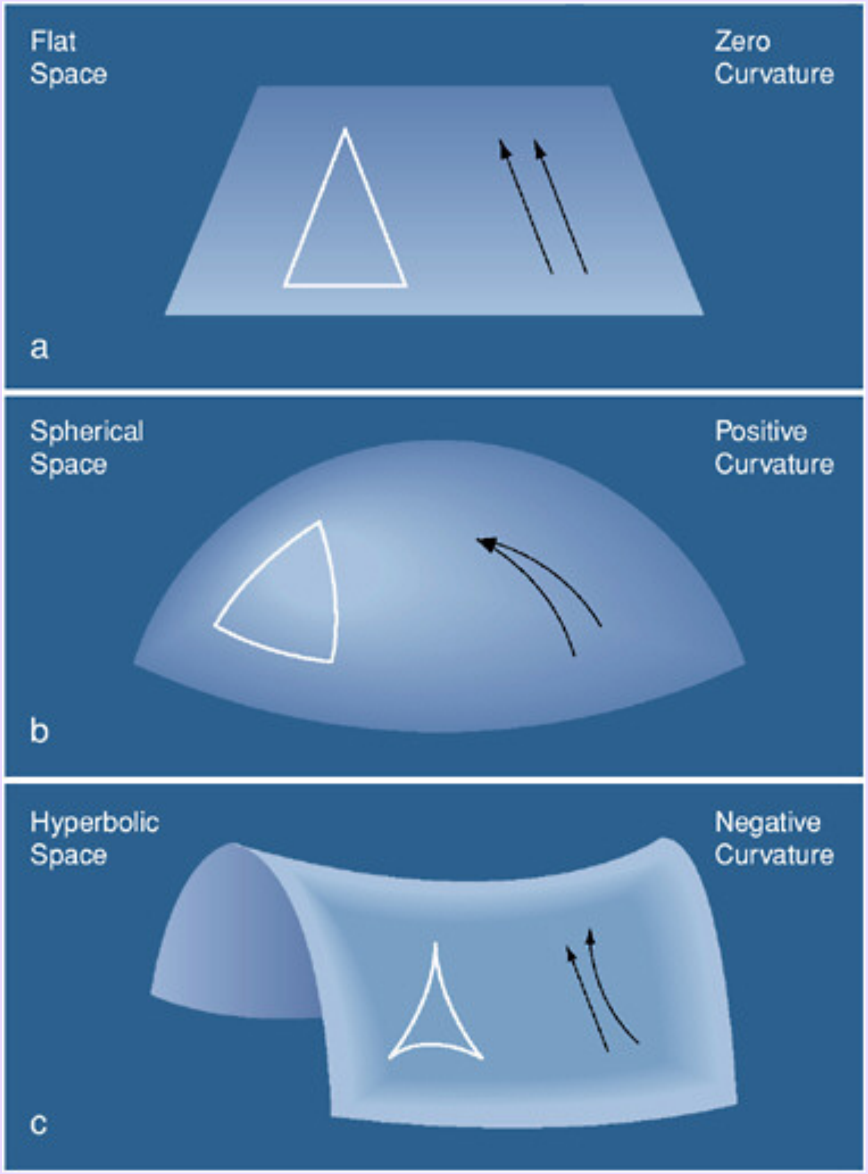
$$E = \left(m^2c^4 + p^2c^2\right)^{1/2}$$

• Constant of integration is curvature of spacetime

$$\frac{2K}{R_0^2} \rightarrow -\frac{kc^2}{R_0^2}$$

$$k = \begin{cases} -1 & \text{negative curvature} \\ 0 & \text{flat space} \\ +1 & \text{positive curvature} \end{cases}$$

The Friedmann Equation



k=0

k=+1

k=-1

The Friedmann Equation

$$\boxed{\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3c^2}\varepsilon(t) - \frac{kc^2}{R_0^2}\frac{1}{a(t)^2}}$$

$$\frac{\dot{a}}{a} \equiv H(t)$$

$$H(t)^2 = \frac{8\pi G}{3c^2}\varepsilon(t)$$

If k=0

$$\varepsilon_c(t) = \frac{3c^2}{8\pi G}H(t)^2$$

Critical density

$$\frac{\varepsilon_{c,0}}{c^2} = 2.8\times 10^{11} h^2 M_\odot \mathrm{Mpc}^{-3}$$

The Friedmann Equation

$$\boxed{\Omega(t) = \frac{\varepsilon(t)}{\varepsilon_c(t)}}$$

Energy density in units of critical density

$$H(t)^2 = H(t)^2\,\Omega(t) - \frac{kc^2}{R_0^2}\frac{1}{a(t)^2}$$

$$1 - \Omega(t) = -\frac{kc^2}{R_0^2}\frac{1}{H(t)^2\,a(t)^2}$$

$$\boxed{\Omega(t)\ \begin{cases} <1 & k=-1 \\ =1 & k=0 \\ >1 & k=+1 \end{cases}}$$

Sign of 1-Ω does not change as universe expands.

The Friedmann Equation

At the present epoch: $$H_0^2\left(1-\Omega_0\right)=-\frac{kc^2}{R_0^2}$$

Replace curvature constant in Friedman equation:

$$1-\Omega\left(t\right)=\frac{H_0^2\left(1-\Omega_0\right)}{H\left(t\right)^2 a\left(t\right)^2}$$

$$\boxed{\frac{H\left(t\right)^2}{H_0^2}\left[1-\Omega\left(t\right)\right]=\frac{\left(1-\Omega_0\right)}{a\left(t\right)^2}}$$

The Fluid Equation

$$dQ = dE + PdV$$

1st law of thermodynamics

$$\dot{E} + P\dot{V} = 0$$

$$E(t) = \varepsilon(t)V(t)$$

$$\dot{\varepsilon}V + \varepsilon\dot{V} + P\dot{V} = 0$$

$$V(t) = \frac{4}{3}\pi R(t)^3 \rightarrow$$

$$\dot{V} = \frac{4}{3}\pi 3R^2\dot{R} \rightarrow$$

$$\dot{V} = V3\frac{\dot{R}}{R} = V3\frac{\dot{a}}{a}$$

$$V\left(\dot{\varepsilon} + 3\frac{\dot{a}}{a}(\varepsilon + P)\right) = 0$$

$$\boxed{\dot{\varepsilon} + 3\frac{\dot{a}}{a}(\varepsilon + P) = 0}$$

The Acceleration Equation

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3c^2}\varepsilon(t) - \frac{kc^2}{R_0^2}\frac{1}{a(t)^2}$$

$$\dot{a}^2 = \frac{8\pi G}{3c^2}\varepsilon(t)a(t)^2 - \frac{kc^2}{R_0^2}$$

$$2\dot{a}\ddot{a} = \frac{8\pi G}{3c^2}\left(\dot{\varepsilon}a^2 + 2\varepsilon a\dot{a}\right)$$

$$\frac{\ddot{a}}{a} = \frac{4\pi G}{3c^2}\left(\dot{\varepsilon}\frac{a}{\dot{a}} + 2\varepsilon\right)$$

$$\dot{\varepsilon} + 3\frac{\dot{a}}{a}(\varepsilon + P) = 0$$

$$\dot{\varepsilon}\frac{a}{\dot{a}} = -3(\varepsilon + P)$$

The Acceleration Equation

$$\frac{\ddot{a}}{a} = \frac{4\pi G}{3c^2}\left(-3\left(\varepsilon + P\right) + 2\varepsilon\right)$$

$$\boxed{\frac{\ddot{a}}{a} = -\frac{4\pi G}{3c^2}\left(\varepsilon + 3P\right)}$$

The energy density is always positive

- If Pressure is positive then the universe must decelerate.
  (e.g., baryonic gas, photons, dark matter)

- If Pressure is negative, the universe can accelerate.
  (e.g., dark energy)

The Equations of Motion of the Universe

$$\boxed{\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3c^2}\varepsilon(t) - \frac{kc^2}{R_0^2}\frac{1}{a(t)^2}}$$

Friedmann Equation

$$\boxed{\frac{\ddot{a}}{a} = -\frac{4\pi G}{3c^2}(\varepsilon + 3P)}$$

Acceleration Equation

Two equations and three unknowns: $a(t),\ \varepsilon(t),\ P(t)$

Need a third equation: Equation of state $P = P(\varepsilon)$

$$\boxed{P = w\varepsilon}$$

The Equation of State

$$\boxed{P = w\varepsilon}$$

- Non-relativistic particles: matter (Ideal gas law)

$$P = \frac{\rho}{\mu}kT = \frac{kT}{\mu c^2}\varepsilon \qquad w = \frac{kT}{\mu c^2} \approx 0$$

- Relativistic particles: radiation

$$P = \frac{1}{3}\varepsilon \qquad w = \frac{1}{3}$$

- Mildly relativistic particles:

$$0 < w < \frac{1}{3}$$

Cosmological Constant / Dark Energy

1915 Einstein's GR equations predict a dynamic universe.

1917 But Einstein thought the Universe was static, so he introduced the "Cosmological Constant", **Λ**, to his equations of motion.

1929 When Hubble discovered the expansion of the universe, Einstein called **Λ** his "greatest blunder".

1998 SN results show that the universe is accelerating in its expansion so scientists revive **Λ**

The cosmological constant or "Dark Energy" is thought to be the energy of a vacuum, predicted by quantum mechanics.

Cosmological Constant / Dark Energy

$$\boxed{\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3c^2}\varepsilon(t) - \frac{kc^2}{R_0^2}\frac{1}{a(t)^2} + \frac{\Lambda}{3}}$$

Friedman Equation

$$\boxed{\frac{\ddot{a}}{a} = -\frac{4\pi G}{3c^2}(\varepsilon + 3P) + \frac{\Lambda}{3}}$$

Acceleration Equation

$$\boxed{\dot{\varepsilon} + 3\frac{\dot{a}}{a}(\varepsilon + P) = 0}$$

Fluid Equation:

If the energy density of Dark Energy is constant with time

$$\dot{\varepsilon} = -3\frac{\dot{a}}{a}(\varepsilon + P) = 0 \rightarrow P = -\varepsilon \rightarrow \quad \boxed{w = -1}$$

Cosmological Constant / Dark Energy

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3c^2}\varepsilon(t) - \frac{kc^2}{R_0^2}\frac{1}{a(t)^2} + \frac{\Lambda}{3}$$

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3c^2}\varepsilon(t) + \frac{8\pi G}{3c^2}\left(\frac{c^2\Lambda}{8\pi G}\right) - \frac{kc^2}{R_0^2}\frac{1}{a(t)^2}$$

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3c^2}\left(\varepsilon_m + \varepsilon_\Lambda\right) - \frac{kc^2}{R_0^2}\frac{1}{a(t)^2}$$

$$\boxed{H(t)^2\left(1 - \Omega_m - \Omega_\Lambda\right) = -\frac{kc^2}{R_0^2}\frac{1}{a(t)^2}}$$

Cosmological Parameters

Cosmological parameter #4: the matter density

$$\Omega_m$$

$$\boxed{\Omega_m \approx 0.28 \pm 0.01}$$

Cosmological parameter #5: the baryon density

$$\Omega_b$$

$$\boxed{\Omega_b \approx 0.047 \pm 0.001}$$

Cosmological Parameters

Cosmological parameter #6: the dark energy density

$$\Omega_\Lambda$$

$$\boxed{\Omega_\Lambda \approx 0.72 \pm 0.01}$$

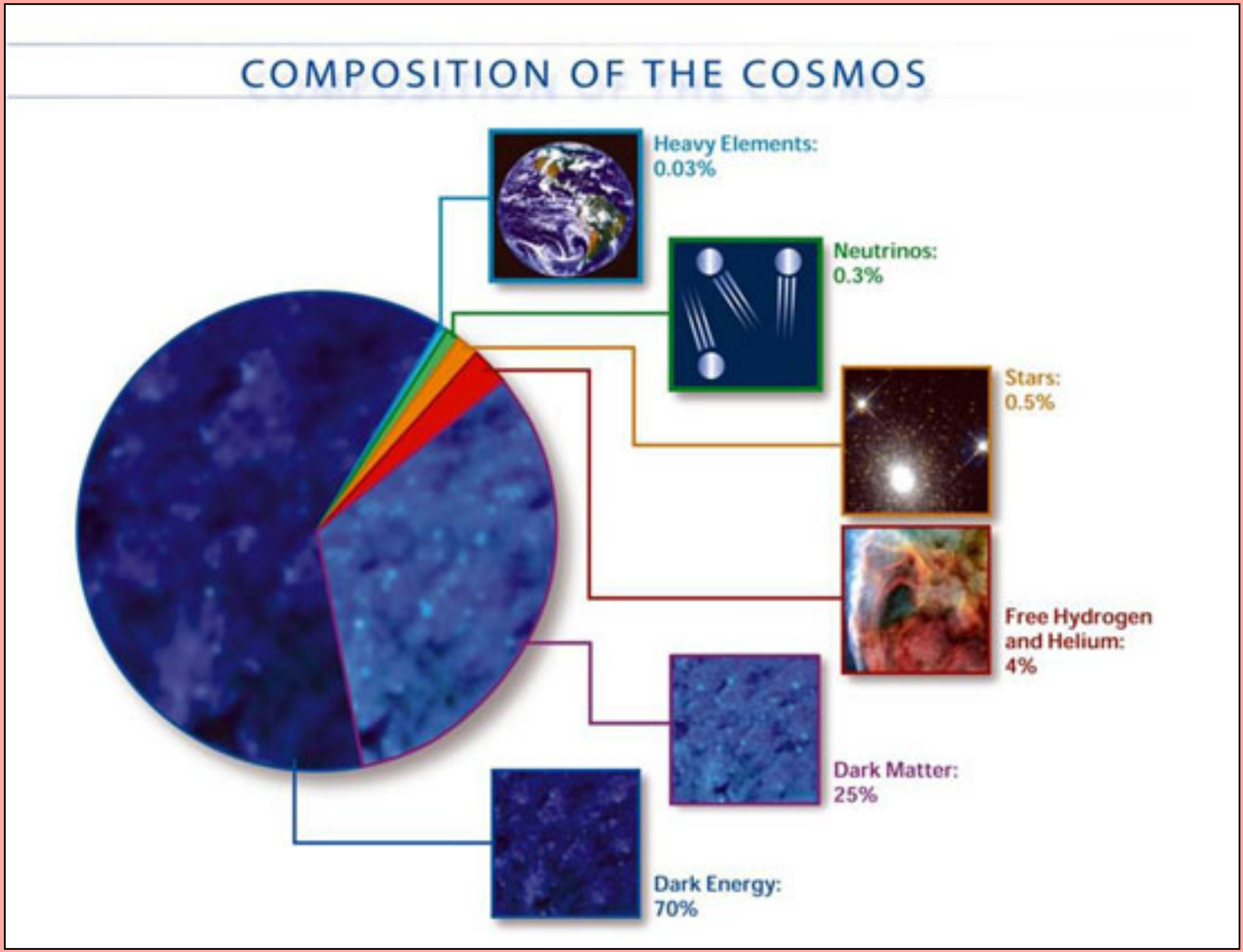
Cosmological Parameters

Cosmological parameter #7: the radiation density

$$\Omega_\gamma \qquad \Omega_\nu$$

$$\boxed{\Omega_\gamma \approx 5 \times 10^{-5}} \qquad \boxed{\Omega_\nu \approx 3.4 \times 10^{-5}}$$

Cosmological Parameters



The Evolution of Energy Density

$$\boxed{\dot{\varepsilon} + 3\frac{\dot{a}}{a}\left(\varepsilon + P\right) = 0}$$

$$\boxed{P = w\varepsilon}$$

$$\dot{\varepsilon} + 3\frac{\dot{a}}{a}\left(1 + w\right)\varepsilon = 0$$

$$\frac{d\varepsilon}{dt} = -\frac{da}{dt}\frac{3}{a}\left(1 + w\right)\varepsilon$$

$$\frac{d\varepsilon}{\varepsilon} = -\frac{da}{a}3\left(1 + w\right)$$

If w is constant with a:

$$\ln\varepsilon = \ln\varepsilon_0 - 3\left(1 + w\right)\ln a$$

$$\boxed{\varepsilon = \varepsilon_0 a^{-3\left(1+w\right)}}$$

The Evolution of Energy Density

$$\boxed{\varepsilon = \varepsilon_0 a^{-3(1+w)}}$$

- Non-relativistic particles (baryons, dark matter)

$$w = 0 \quad \rightarrow \quad \boxed{\varepsilon = \varepsilon_0 a^{-3} = \varepsilon_0 (1+z)^3}$$

- Relativistic particles (photons, neutrinos)

$$w = \frac{1}{3} \quad \rightarrow \quad \boxed{\varepsilon = \varepsilon_0 a^{-4} = \varepsilon_0 (1+z)^4}$$

- Dark energy

$$w = -1 \quad \rightarrow \quad \boxed{\varepsilon = \varepsilon_0}$$

The Friedmann Equation

$$\frac{H(t)^2}{H_0^2} = \frac{H(t)^2}{H_0^2}\Omega(t) + \frac{(1-\Omega_0)}{a(t)^2}$$

$$\left\{ \begin{array}{c} \varepsilon_c = \dfrac{3c^2}{8\pi G} H^2 \\ \dfrac{H^2}{H_0^2}\dfrac{\varepsilon}{\varepsilon_c} = \dfrac{\varepsilon_c}{\varepsilon_{c,0}}\dfrac{\varepsilon}{\varepsilon_c} = \dfrac{\varepsilon}{\varepsilon_{c,0}} \end{array} \right\}$$

$$\boxed{\frac{H^2}{H_0^2} = \frac{\varepsilon}{\varepsilon_{c,0}} + \frac{(1-\Omega_0)}{a^2}}$$

$$\boxed{\varepsilon = \varepsilon_m + \varepsilon_r + \varepsilon_\Lambda = \frac{\varepsilon_{m,0}}{a^3} + \frac{\varepsilon_{r,0}}{a^4} + \varepsilon_{\Lambda,0}}$$

The Friedmann Equation

$$\frac{H^2}{H_0^2} = \frac{1}{\varepsilon_{c,0}}\left(\frac{\varepsilon_{m,0}}{a^3} + \frac{\varepsilon_{r,0}}{a^4} + \varepsilon_{\Lambda,0}\right) + \frac{\left(1-\Omega_0\right)}{a^2}$$

$$\boxed{\frac{H^2}{H_0^2} = \frac{\Omega_{m,0}}{a^3} + \frac{\Omega_{r,0}}{a^4} + \Omega_{\Lambda,0} + \frac{1-\Omega_0}{a^2}}$$

$$\boxed{\Omega_0 = \Omega_{m,0} + \Omega_{r,0} + \Omega_{\Lambda,0}}$$

$$\boxed{1-\Omega_0 = \Omega_{k,0}}$$

$$\boxed{\Omega_{m,0} + \Omega_{r,0} + \Omega_{\Lambda,0} + \Omega_{k,0} = 1}$$

Cosmological Parameters

Cosmological parameter #8: the spatial curvature

$$\Omega_k$$

$$\boxed{\Omega_k = -0.002 \pm 0.004}$$

Cosmological Parameters

Cosmological parameter #9: the equation of state of dark energy

$$w$$

$$\boxed{w \approx -1.04 \pm 0.07}$$

The Friedmann Equation

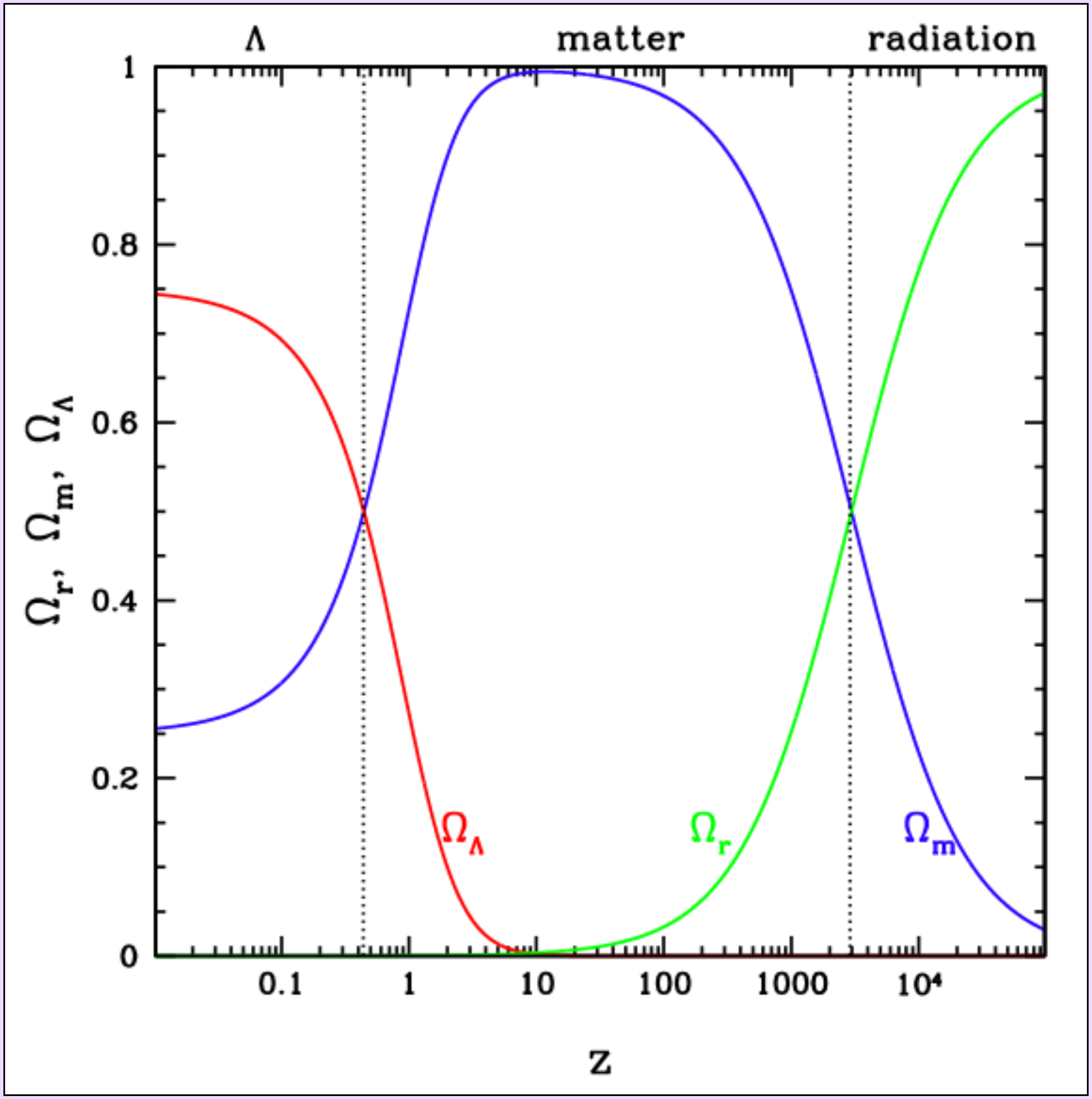
$$\boxed{\frac{H^2}{H_0^2} = \frac{\Omega_{r,0}}{a^4} + \frac{\Omega_{m,0}}{a^3} + \frac{\Omega_{k,0}}{a^2} + \frac{\Omega_{\Lambda,0}}{a^{3(1+w)}}}$$

$$\Omega_m(z) = \frac{\varepsilon_m}{\varepsilon_c} = \frac{\varepsilon_{m,0}}{a^3\varepsilon_c} = \frac{\varepsilon_{m,0}}{a^3\varepsilon_{c,0}}\frac{\varepsilon_{c,0}}{\varepsilon_c} = \frac{\Omega_{m,0}}{a^3}\frac{{H_0}^2}{H^2}$$

$$\Omega_r(z) = \frac{\Omega_{r,0}}{a^4}\frac{{H_0}^2}{H^2}$$

$$\Omega_\Lambda(z) = \frac{\Omega_{\Lambda,0}}{a^{3(1+w)}}\frac{{H_0}^2}{H^2}$$

The Friedmann Equation



The Friedmann Equation

$$\frac{H^2}{H_0^2} = \frac{\Omega_{r,0}}{a^4} + \frac{\Omega_{m,0}}{a^3} + \frac{\Omega_{k,0}}{a^2} + \frac{\Omega_{\Lambda,0}}{a^{3(1+w)}}$$

Solving this equation gives expansion history a(t)
(which also specifies the age of the universe t_0)

Solving the Friedmann Equation

<u>Special case #1: empty universe</u>

$$\boxed{\Omega_{k,0} = 1}$$

$$\boxed{\frac{H^2}{H_0^2} = \cancelto{0}{\frac{\Omega_{r,0}}{a^4}} + \cancelto{0}{\frac{\Omega_{m,0}}{a^3}} + \frac{\Omega_{k,0}}{a^2} + \cancelto{0}{\frac{\Omega_{\Lambda,0}}{a^{3(1+w)}}}}$$

$$\frac{H^2}{H_0^2} = \frac{1}{a^2} \quad \rightarrow H = H_0\frac{1}{a} \quad \rightarrow \dot{a} = H_0 \quad \rightarrow \int_0^a da' = \int_0^t H_0 dt'$$

$$\boxed{a(t) = H_0 t} \qquad \boxed{t_0 = \frac{1}{H_0}}$$

Solving the Friedmann Equation

<u>Special case #2: radiation dominated universe</u>

$$\boxed{\Omega_{r,0} = 1}$$

$$\boxed{\frac{H^2}{H_0^2} = \frac{\Omega_{r,0}}{a^4} + \cancelto{0}{\frac{\Omega_{m,0}}{a^3}} + \cancelto{0}{\frac{\Omega_{k,0}}{a^2}} + \cancelto{0}{\frac{\Omega_{\Lambda,0}}{a^{3(1+w)}}}}$$

$$\frac{H^2}{H_0^2} = \frac{1}{a^4} \rightarrow H = H_0 \frac{1}{a^2} \rightarrow \dot{a} = H_0 \frac{1}{a} \rightarrow \int_0^a a' da' = \int_0^t H_0 dt'$$

$$\boxed{a(t) = \left(2H_0 t\right)^{1/2}} \qquad \boxed{t_0 = \frac{1}{2H_0}}$$

Solving the Friedmann Equation

Special case #3: matter dominated universe

$$\boxed{\Omega_{m,0} = 1}$$

$$\boxed{\frac{H^2}{H_0^2} = \cancelto{0}{\frac{\Omega_{r,0}}{a^4}} + \frac{\Omega_{m,0}}{a^3} + \cancelto{0}{\frac{\Omega_{k,0}}{a^2}} + \cancelto{0}{\frac{\Omega_{\Lambda,0}}{a^{3(1+w)}}}}$$

$$\frac{H^2}{H_0^2} = \frac{1}{a^3} \rightarrow H = H_0 \frac{1}{a^{3/2}} \rightarrow \dot{a} = H_0 \frac{1}{\sqrt{a}} \rightarrow \int_0^a \sqrt{a'}\,da' = \int_0^t H_0\,dt'$$

$$\boxed{a(t) = \left(\frac{3}{2} H_0 t\right)^{2/3}}$$

$$\boxed{t_0 = \frac{2}{3H_0}}$$

Solving the Friedmann Equation

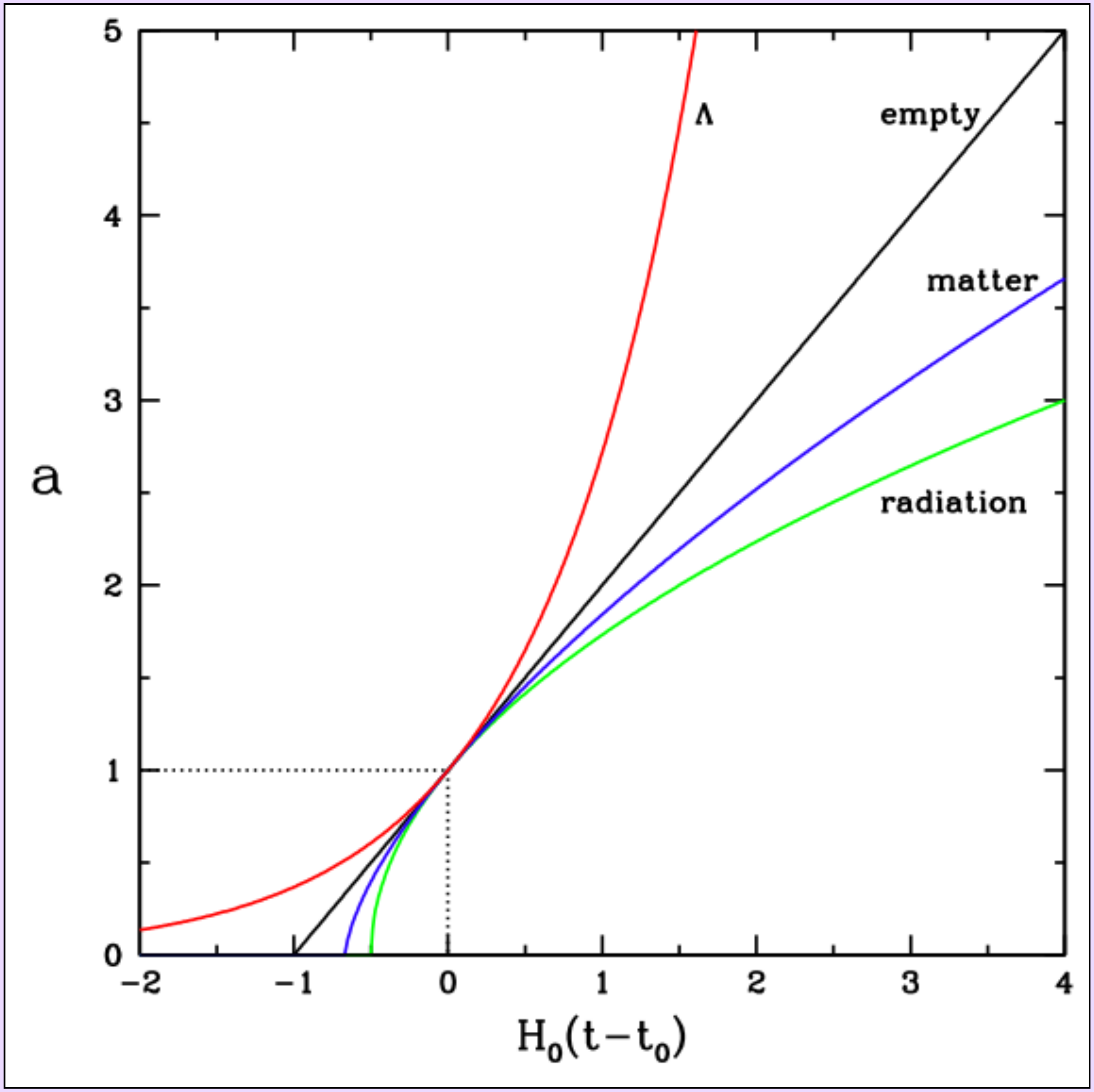
Special case #4: dark energy dominated universe $\boxed{\Omega_{\Lambda,0} = 1}$

$$\boxed{\frac{H^2}{H_0^2} = \cancelto{0}{\frac{\Omega_{r,0}}{a^4}} + \cancelto{0}{\frac{\Omega_{m,0}}{a^3}} + \cancelto{0}{\frac{\Omega_{k,0}}{a^2}} + \Omega_{\Lambda,0}}$$

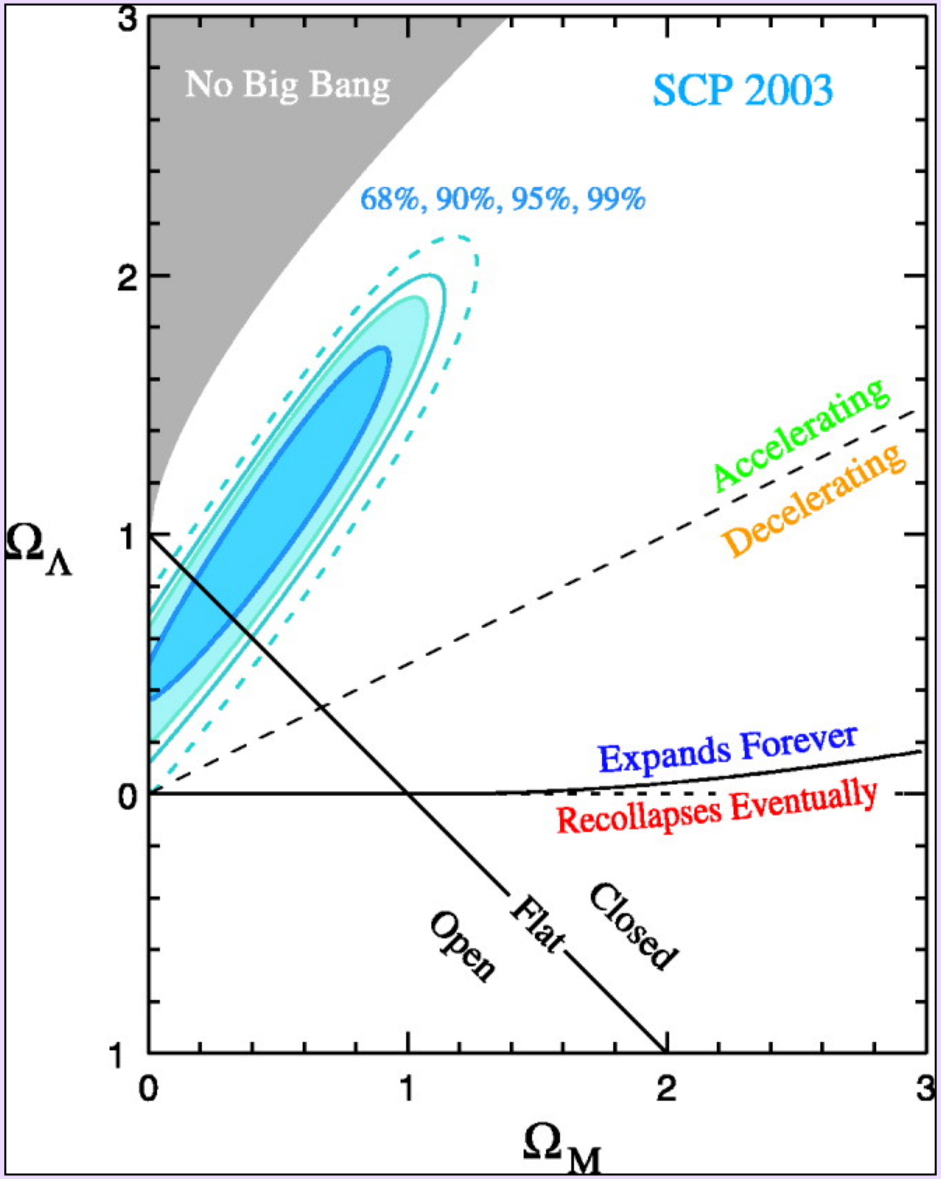
$$\frac{H^2}{H_0^2} = 1 \quad \rightarrow H = H_0 \quad \rightarrow \dot{a} = H_0 a \quad \rightarrow \int_a^1 \frac{da'}{a'} = \int_t^{t_0} H_0 dt'$$

$$\boxed{a(t) = e^{H_0 (t - t_0)}} \qquad \boxed{t_0 = \infty}$$

Solving the Friedmann Equation



Solving the Friedmann Equation



Knop et al. (2003)

Distance Measures in Cosmology

Hubble time: Time it took the universe to reach its present size if expansion rate has always been the same.

$$t_H \equiv \frac{1}{H_0} = 9.78h^{-1}\mathrm{Gyr}$$

Hubble distance: Distance light travels in a Hubble time.

$$D_H \equiv \frac{c}{H_0} = 3000h^{-1}\mathrm{Mpc}$$

Proper distance: Distance measured in rulers between two points.

$$D_P$$

Distance Measures in Cosmology

- Proper distance between point at *a* and *a+da:*

$$dD_P = c \times \frac{da}{\dot{a}}$$

$$= c\frac{da}{a\left(\frac{\dot{a}}{a}\right)} = c\frac{da}{aH} = \frac{c}{H_0}\frac{da}{a}\left(\frac{H}{H_0}\right)^{-1}$$

$$= D_H \frac{dz}{(1+z)E(z)}$$

- Integrating: $\boxed{D_P = D_H \int_0^z \frac{dz'}{(1+z')E(z')}}$

The Friedmann Equation

$$\boxed{\frac{H^2}{H_0^2} = \cancelto{0}{\frac{\Omega_{r,0}}{a^4}} + \frac{\Omega_{m,0}}{a^3} + \frac{\Omega_{k,0}}{a^2} + \frac{\Omega_{\Lambda,0}}{a^{3(1+w)}}}$$

$$\boxed{\frac{H}{H_0} = \sqrt{\Omega_{m,0}\left(1+z\right)^3 + \Omega_{k,0}\left(1+z\right)^2 + \Omega_{\Lambda,0}\left(1+z\right)^{3(1+w)}} = E\left(z\right)}$$

Distance Measures in Cosmology

<u>Comoving distance</u>: Distance between points that remains constant if both points move with the hubble flow.

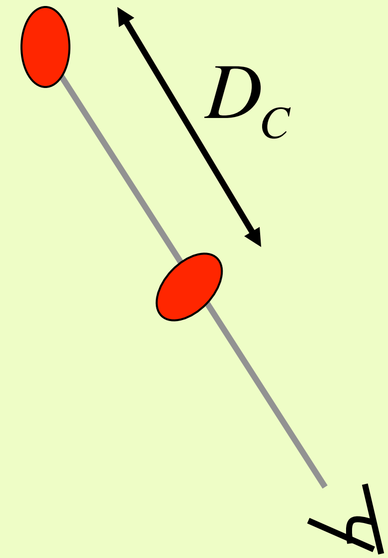
$$D_C \equiv \frac{D_P}{a} = D_P(1+z)$$

$$\boxed{D_C = D_H \int_0^z \frac{dz'}{E(z')}}$$

Distance Measures in Cosmology

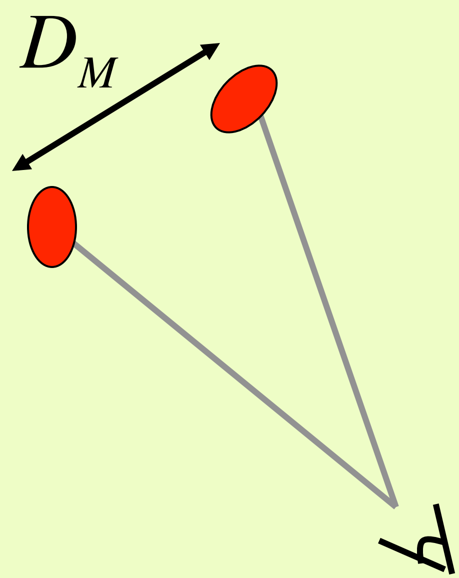
<u>Comoving distance</u> (line-of-sight)

$$D_C = D_H \int_0^z \frac{dz'}{E(z')}$$



<u>Comoving distance</u> (transverse)

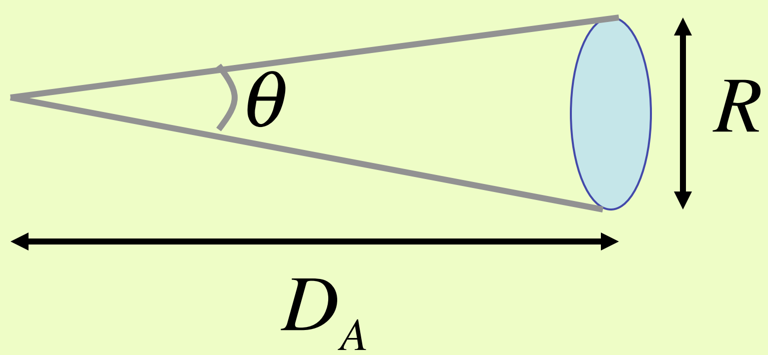
$$D_M = \begin{cases} D_H \dfrac{1}{\sqrt{\Omega_k}} \sinh\left(\sqrt{\Omega_k}\, D_C / D_H\right) & \text{for } \Omega_k > 0 \\ D_C & \text{for } \Omega_k = 0 \\ D_H \dfrac{1}{\sqrt{|\Omega_k|}} \sin\left(\sqrt{\Omega_k}\, D_C / D_H\right) & \text{for } \Omega_k < 0 \end{cases}$$



Distance Measures in Cosmology

<u>Angular diameter distance</u>: Ratio of object's physical size to angular size.

$$\boxed{D_A = \frac{D_M}{(1+z)}}$$



$$D_A = \frac{R}{\theta}$$

Distance Measures in Cosmology

Luminosity distance: Distance that defines the relationship between luminosity and flux.

$$D_L = \sqrt{\frac{L}{4\pi F}}$$

$$\boxed{D_L = (1+z)\, D_M = (1+z)^2\, D_A}$$

Distance Measures in Cosmology

Comoving volume: Volume in which densities of non-evolving objects are constant with redshift.

$$dV_C = D_H \frac{(1+z)^2 D_A^2}{E(z)} d\Omega dz$$

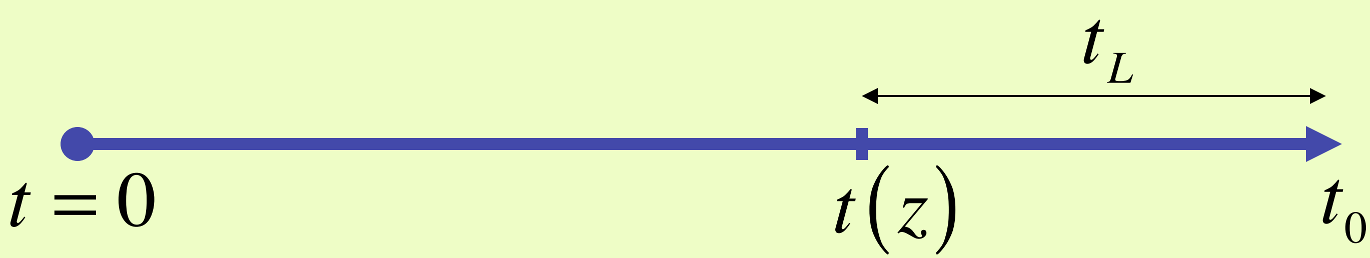
$$\boxed{V_C = \frac{4\pi}{3} D_M^3 \quad \text{for} \;\; \Omega_k = 0}$$

Distance Measures in Cosmology

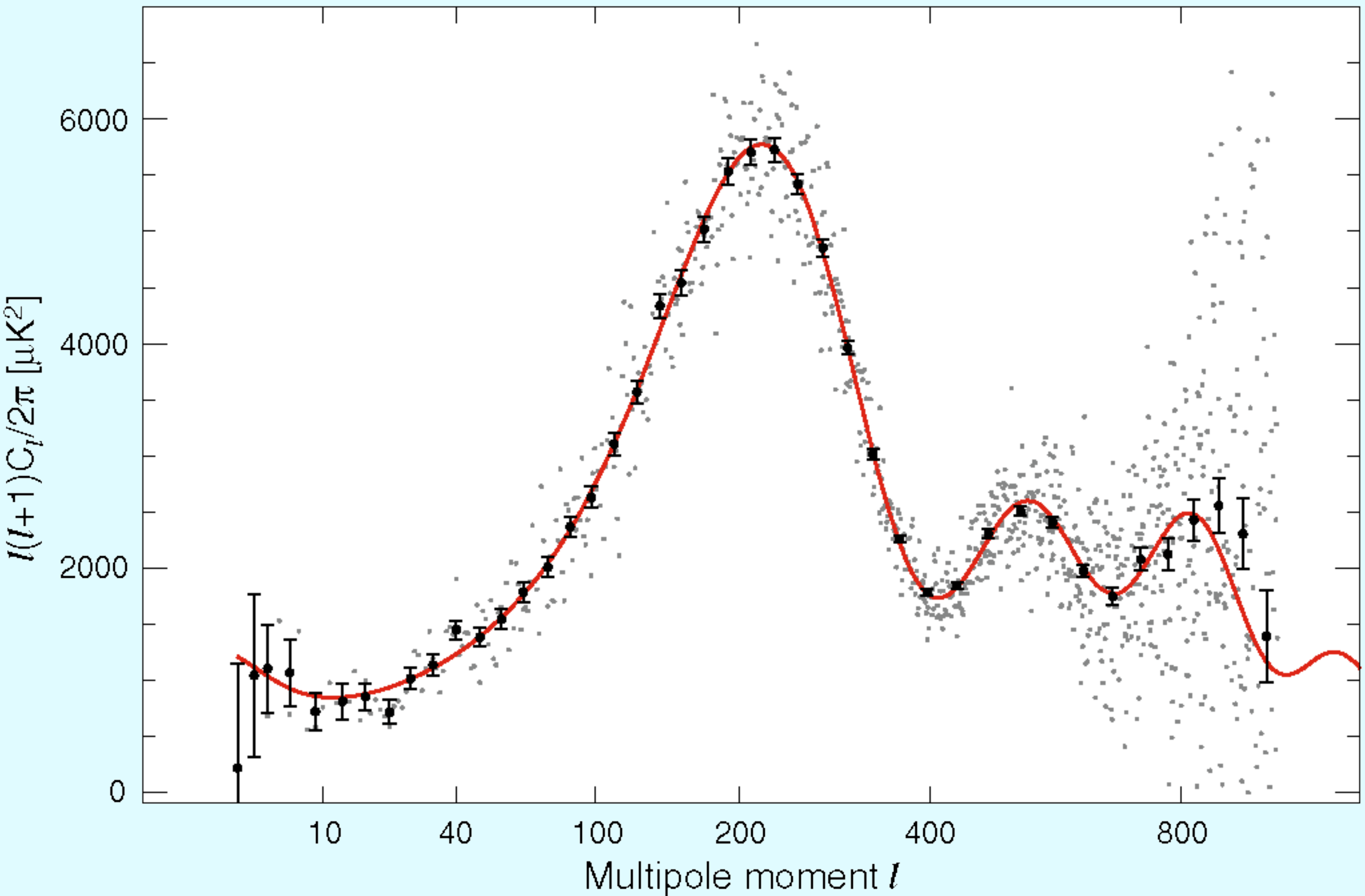
Lookback time: Difference between age of universe now and age of universe at the time photons were emitted from object.

$$\boxed{t_L = t_H \int_0^z \frac{dz'}{(1+z')E(z')}}$$

Age of the universe at redshift z: $t_0 - t_L(z)$

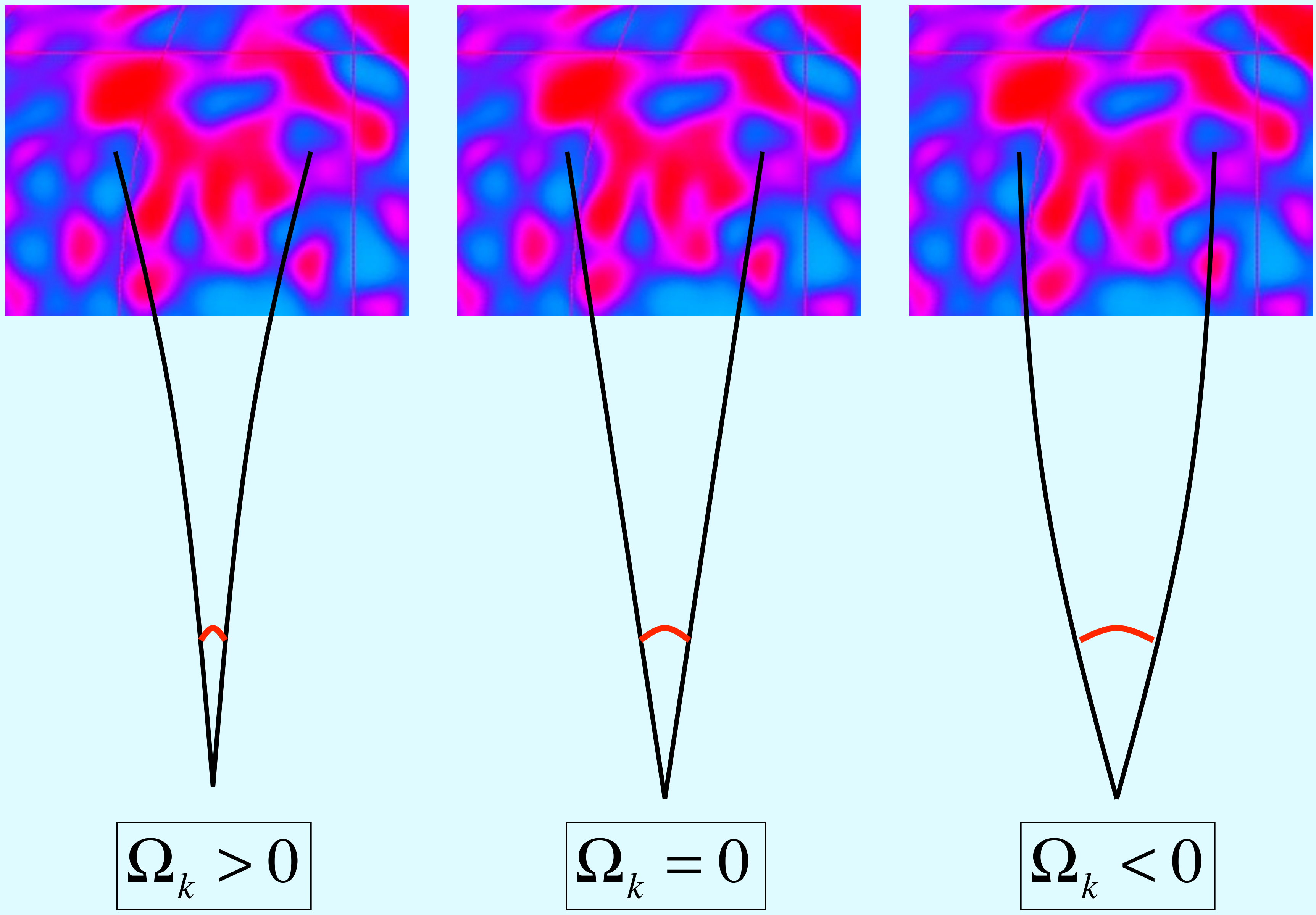


Constraining Cosmological Parameters



Dunkley et al. (2009)

Constraining Cosmological Parameters



Constraining Cosmological Parameters

Measurement

Physics

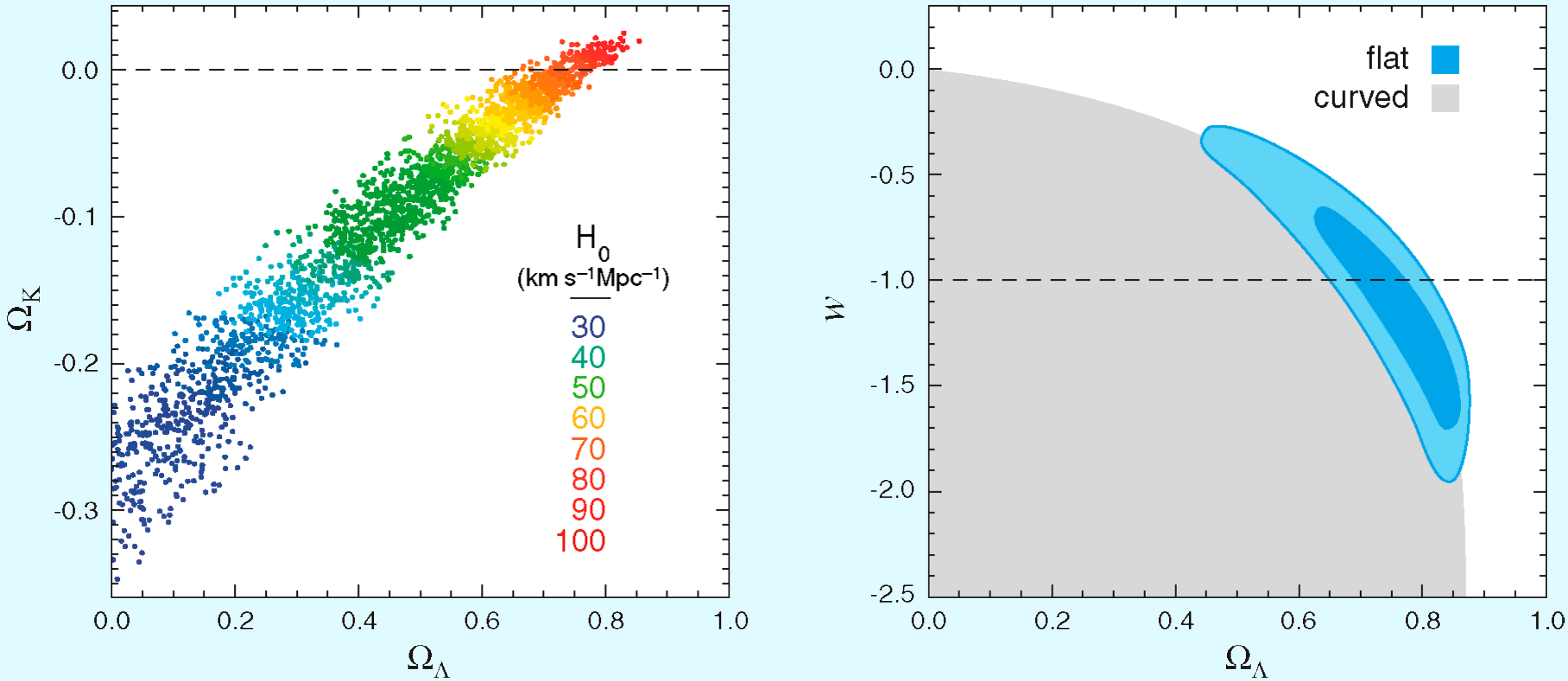
$$D_A = \frac{\lambda}{\theta} = \frac{l}{2\pi}\lambda$$

Cosmological parameters

$$D_A = \frac{c}{H_0(1+z)}\int_0^z \frac{dz'}{E(z')}$$

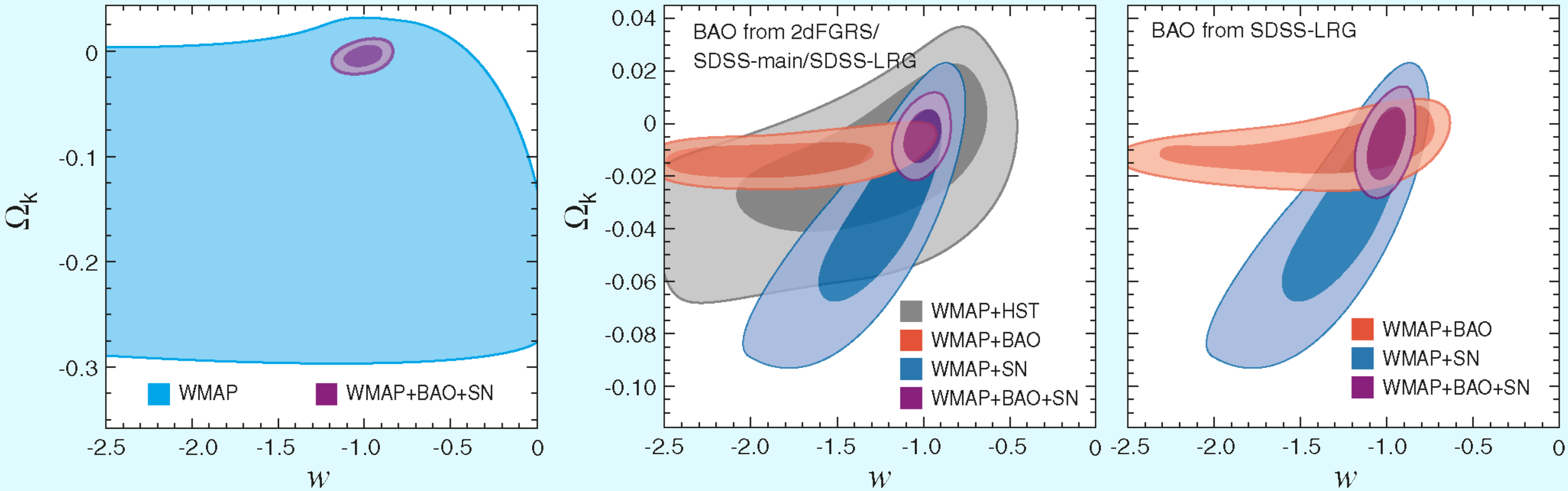
$$E(z) = \sqrt{\Omega_{m,0}(1+z)^3 + \Omega_{k,0}(1+z)^2 + \Omega_{\Lambda,0}(1+z)^{3(1+w)}}$$

Constraining Cosmological Parameters



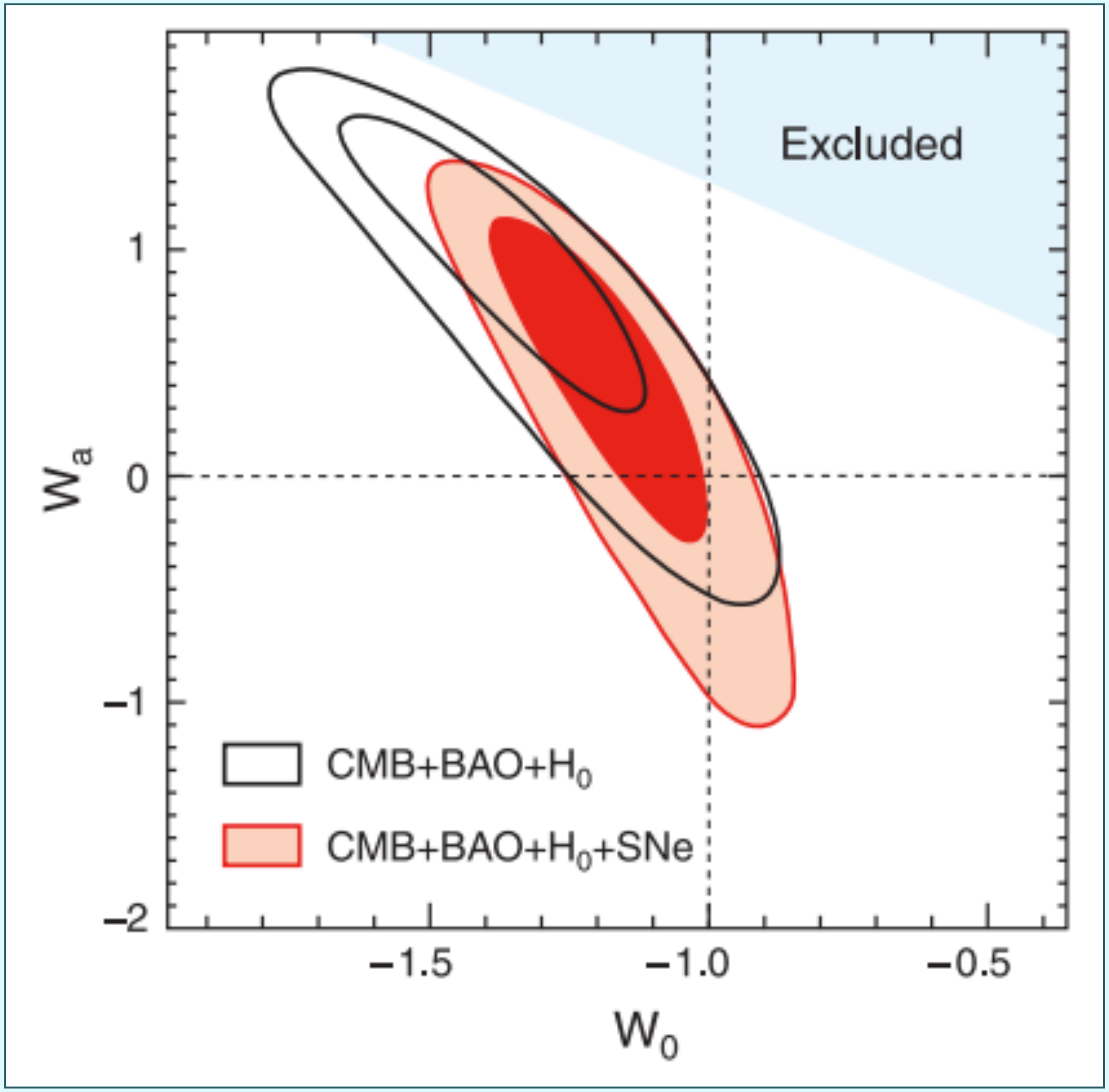
Dunkley et al. (2009)

Constraining Cosmological Parameters



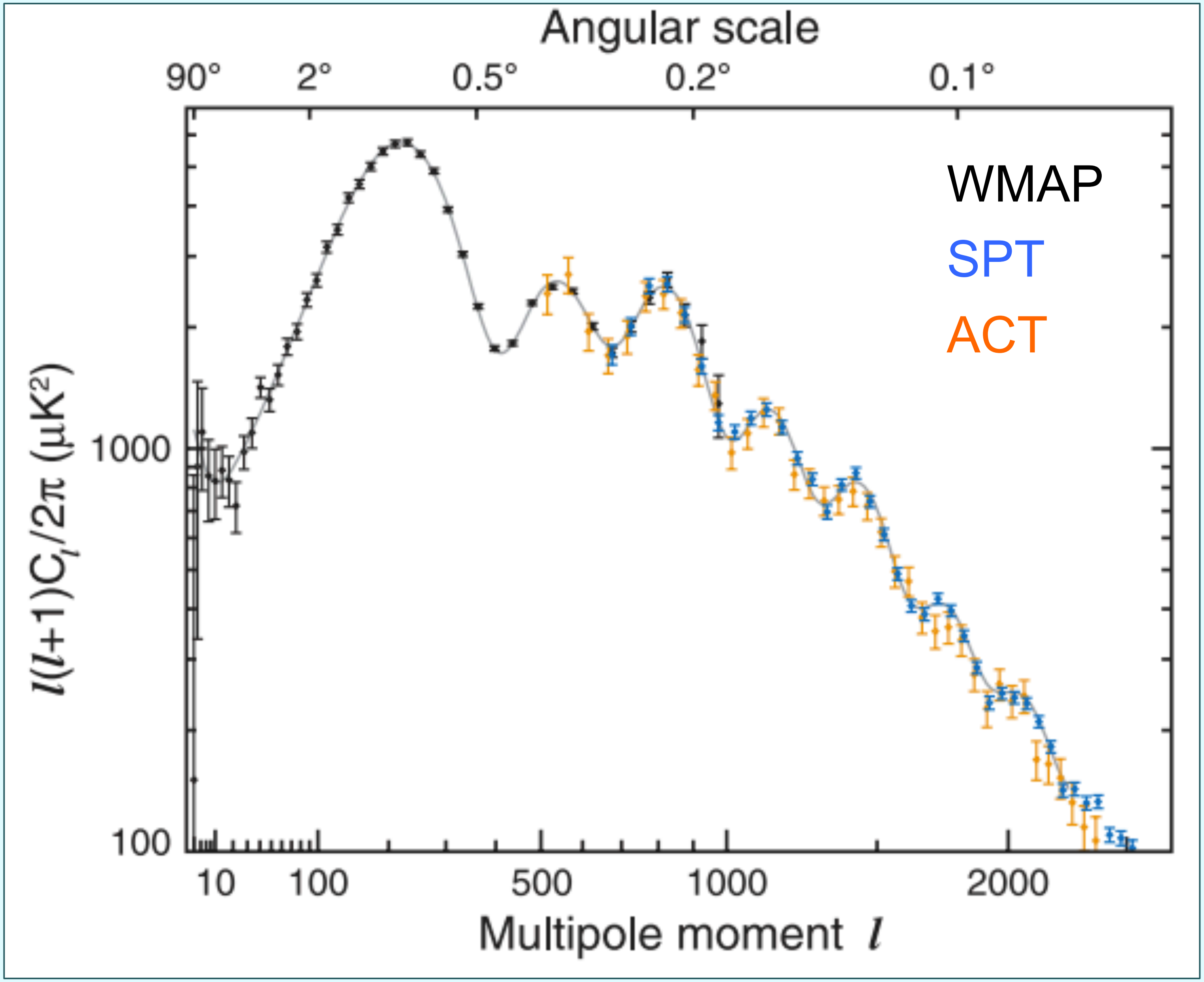
Komatsu et al. (2009)

Constraining Cosmological Parameters



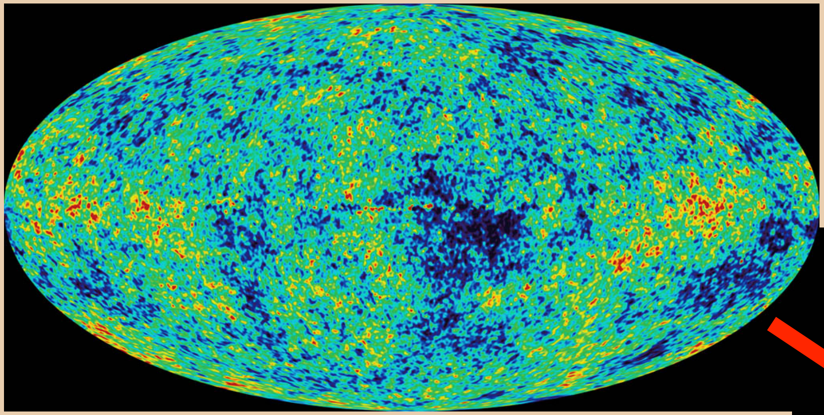
Hinshaw et al. (2013)

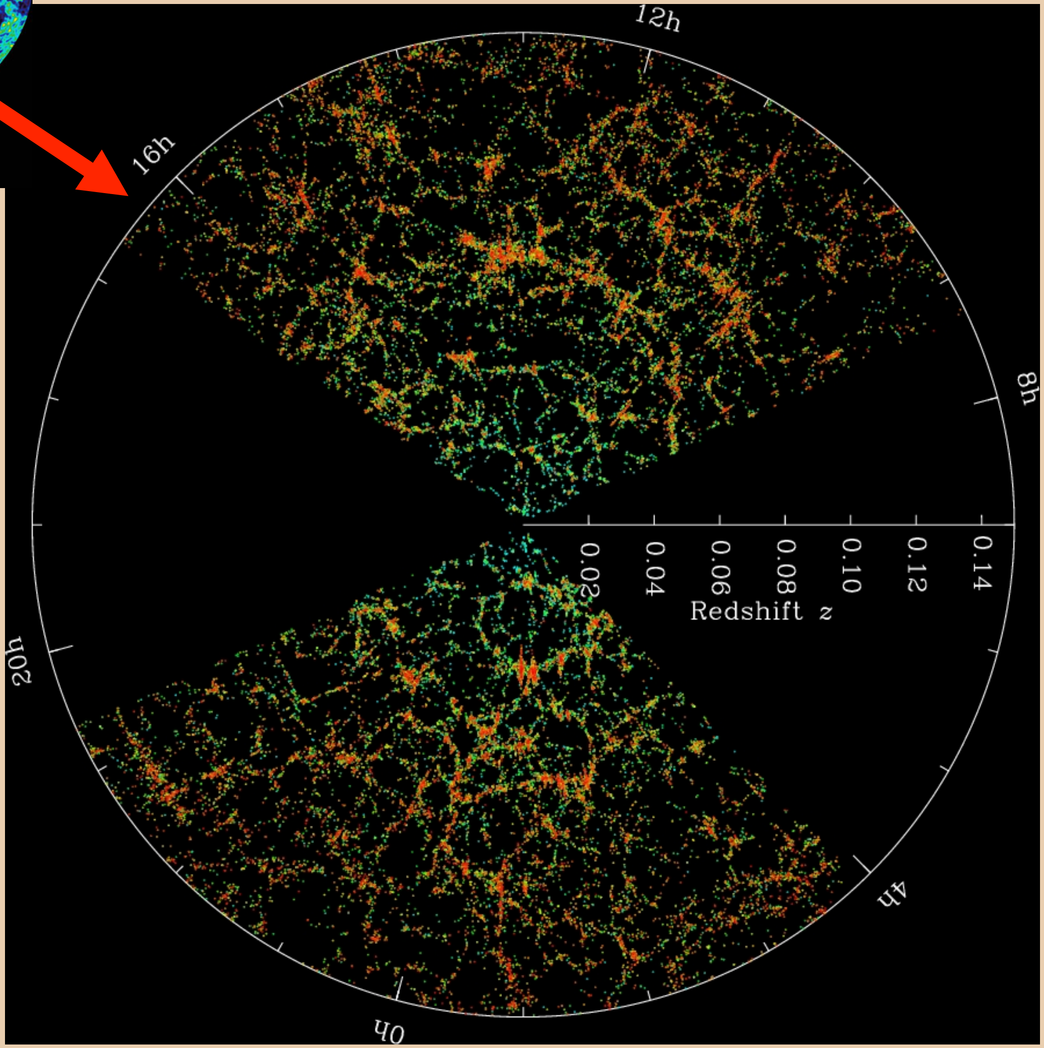
Constraining Cosmological Parameters



Hinshaw et al. (2013)

The Growth of Structure





The Growth of Structure

$$\boxed{\delta \equiv \frac{\rho - \bar{\rho}}{\bar{\rho}}}$$

CMB: $\delta \sim 10^{-4}$

Superclusters: $\delta \sim 10$

Clusters: $\delta \sim 10^{2}$

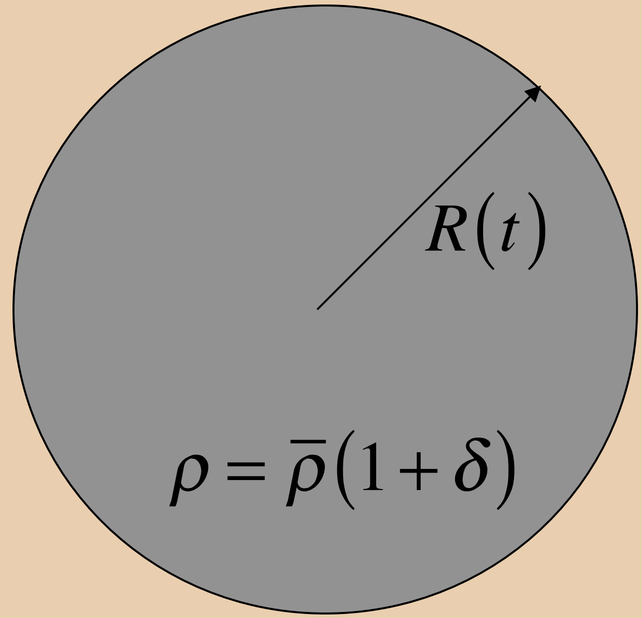
Galaxies: $\delta \sim 10^{4}$

Stars: $\delta \sim 10^{29}$

People: $\delta \sim 10^{30}$

Stellar black hole: $\delta \sim 10^{45}$

Assume small spherical overdensity in a *static* universe.



$$\ddot{R} = -\frac{G(\Delta M)}{R^2}$$

$$= -\frac{G}{R^2}\left(\frac{4\pi}{3}R^3\bar{\rho}\delta\right)$$

$$\boxed{\frac{\ddot{R}}{R} = -\frac{4\pi G\bar{\rho}}{3}\delta(t)}$$

Two unknowns: $R(t)$ and $\delta(t)$

The Growth of Structure

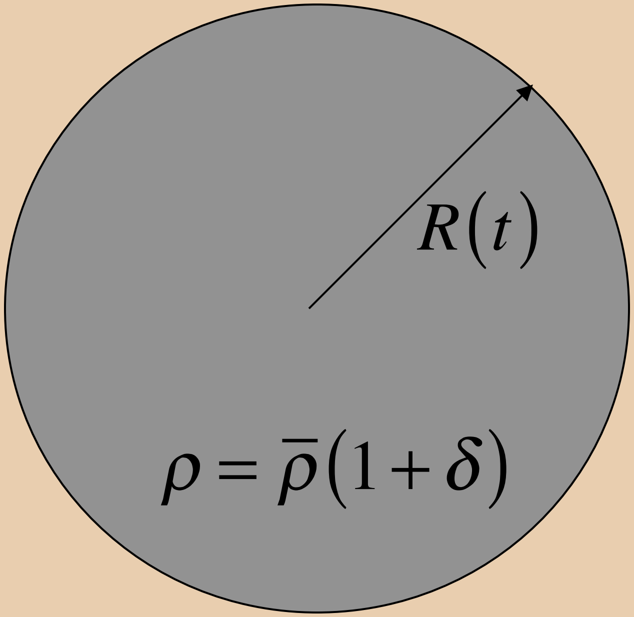
<u>Conservation of mass</u>

$$M = \frac{4\pi}{3} R(t)^3 \, \bar{\rho} \left[1 + \delta(t)\right]$$

$$R(t) = R_0 \left[1 + \delta(t)\right]^{-1/3}$$

$$R_0 = \left(\frac{3M}{4\pi\bar{\rho}}\right)^{1/3} = \text{const}$$

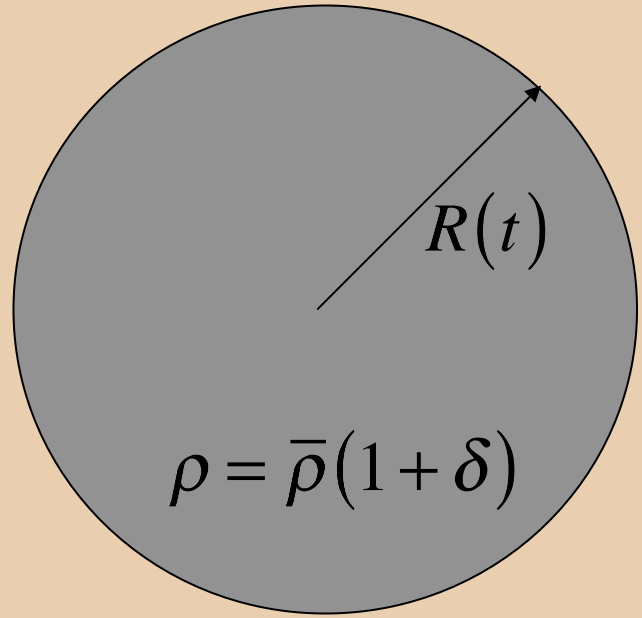
$$\delta \ll 1 \quad \rightarrow \quad R(t) \approx R_0 \left[1 - \frac{1}{3}\delta(t)\right]$$



The Growth of Structure

$$\ddot{R} \approx -\frac{1}{3} R_0 \ddot{\delta} \approx -\frac{1}{3} R \ddot{\delta}$$

$$\boxed{\frac{\ddot{R}}{R} \approx -\frac{1}{3} \ddot{\delta}}$$

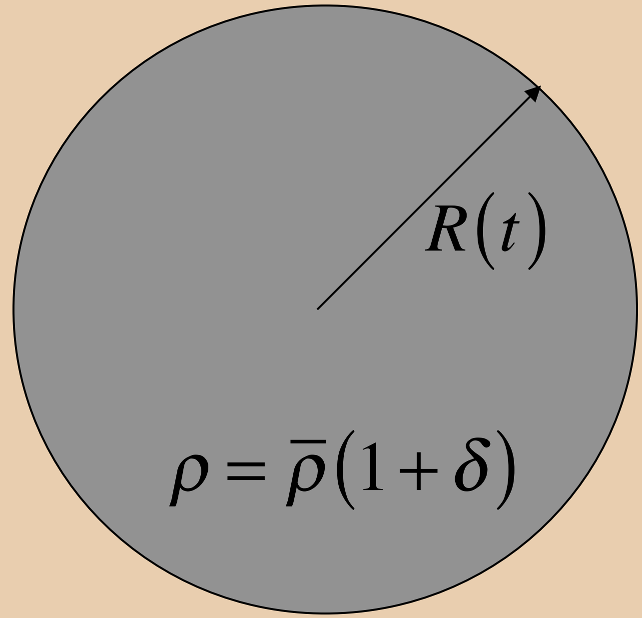


Combine with:

$$\boxed{\frac{\ddot{R}}{R} = -\frac{4\pi G \bar{\rho}}{3} \delta(t)}$$

$$\boxed{\ddot{\delta} = 4\pi G \bar{\rho} \delta}$$

The Growth of Structure



$$\boxed{\ddot{\delta} = 4\pi G\bar{\rho}\delta}$$

General solution:

$$\boxed{\delta(t) = A_1 e^{t/t_{dyn}} + A_2 e^{-t/t_{dyn}}}$$

where

$$\boxed{t_{dyn} = \left(4\pi G\bar{\rho}\right)^{-1/2}}$$

After a few dynamical times, only the growing mode is significant

The Growth of Structure

$$\boxed{\delta(t) \sim e^{t}}$$

In reality, density perturbations do not grow this fast in the universe because:

- there is some pressure support

- the universe is not static, but expanding

$$t_{dyn} \sim \left(G\bar{\rho}\right)^{-1/2} = \left(c^2/G\bar{\varepsilon}\right)^{1/2}$$

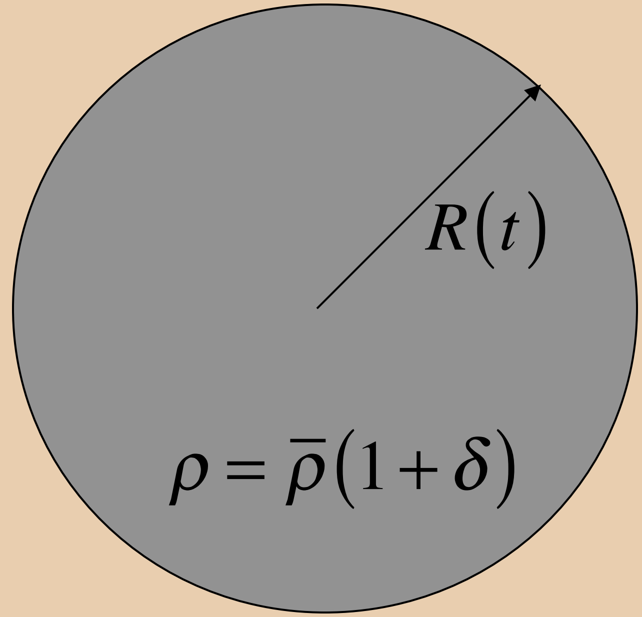
$$\bar{\varepsilon} = \frac{3c^2}{8\pi G}H^2 \rightarrow H^{-1} \sim \left(c^2/G\bar{\varepsilon}\right)^{1/2}$$

The Growth of Structure

$$\ddot{R} = -\frac{GM}{R^2} = -\frac{G}{R^2}\left(\frac{4\pi}{3}\rho R^3\right)$$

$$= -\frac{4\pi}{3}G\bar{\rho}R - \frac{4\pi}{3}G\left(\bar{\rho}\delta\right)R$$

$$\frac{\ddot{R}}{R} = -\frac{4\pi}{3}G\bar{\rho} - \frac{4\pi}{3}G\bar{\rho}\delta$$

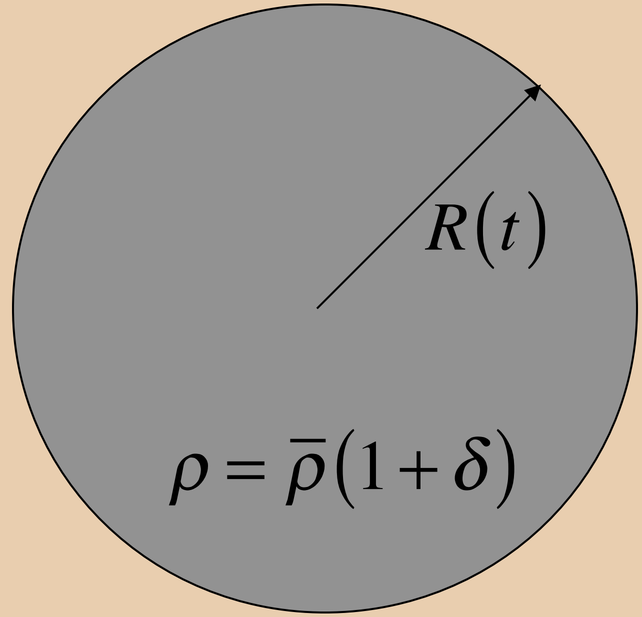


Mass conservation:

$$M = \frac{4\pi}{3}R(t)^3\,\bar{\rho}(t)\left[1+\delta(t)\right] = \text{const}$$

The Growth of Structure

$$R(t) \propto \bar{\rho}(t)^{-1/3}\left[1+\delta(t)\right]^{-1/3}$$



$$\bar{\rho} \propto a^{-3}$$

$$R(t) \propto a(t)\left[1+\delta(t)\right]^{-1/3}$$

$$\approx a(t)\left[1-\frac{1}{3}\delta(t)\right]$$

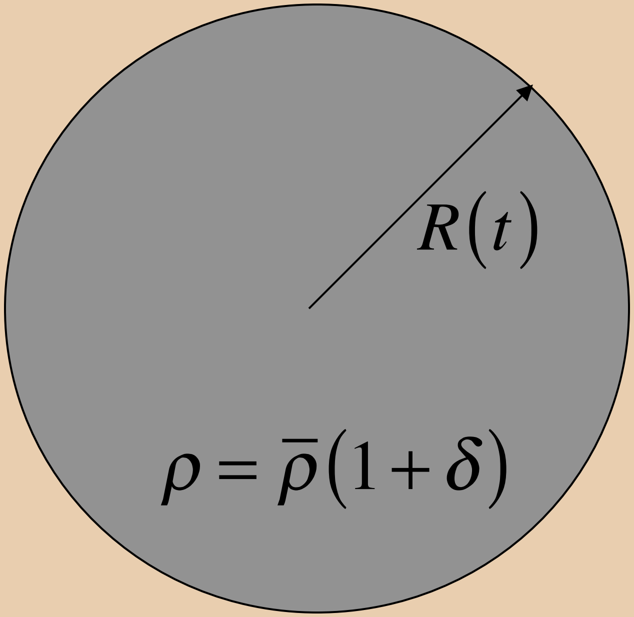
An overdense region will grow slightly less rapidly than the scale factor.

$$\frac{\ddot{R}}{R} = \frac{\ddot{a}}{a} - \frac{1}{3}\ddot{\delta} - \frac{2}{3}\frac{\dot{a}}{a}\dot{\delta} \qquad = -\frac{4\pi}{3}G\bar{\rho} - \frac{4\pi}{3}G\bar{\rho}\delta$$

The Growth of Structure

With δ~0, this reduces to:

$$\frac{\ddot{a}}{a} = -\frac{4\pi}{3} G\bar{\rho}$$



Subtract this from previous equation:

$$-\frac{1}{3}\ddot{\delta} - \frac{2}{3}\frac{\dot{a}}{a}\dot{\delta} = -\frac{4\pi}{3} G\bar{\rho}\delta$$

$$\boxed{\ddot{\delta} + 2H\dot{\delta} = 4\pi G\bar{\rho}\delta}$$

Has extra term that acts to slow collapse in an expanding universe.

The Growth of Structure

$$\left(\frac{\partial \rho}{\partial t}\right)_r + \rho \nabla_r \cdot \vec{u} = 0$$

Continuity (mass)

$$\left(\frac{\partial \vec{u}}{\partial t}\right)_r + \left(\vec{u} \cdot \nabla_r\right)\vec{u} = -\nabla_r \Phi$$

Euler (momentum)

$$\nabla_r^2 \Phi = 4\pi G \rho$$

Poisson (gravity)

Transform to comoving coordinates

$$\vec{r} = a\vec{x}$$

$$\vec{u} = \dot{\vec{r}} = \dot{a}\vec{x} + \vec{v}$$

The Growth of Structure

- Add a small perturbation

$$\rho(\vec{x},t) = \bar{\rho}(t) + \delta(\vec{x},t)\,\bar{\rho}(t)$$

- Linear approximation: keep first order terms in *δ* and *v*
- Rearrange equations to get rid of *v*
- Subtract off equation for unperturbed case to get *δ*

$$\boxed{\ddot{\delta} + 2H\dot{\delta} = 4\pi G\bar{\rho}\delta}$$

The Growth of Structure

$$\ddot{\delta} + 2H\dot{\delta} = \frac{4\pi G}{c^2}\bar{\varepsilon}_m \delta$$

δ is the density of *matter* only

$$\delta = \frac{\varepsilon_m - \bar{\varepsilon}_m}{\bar{\varepsilon}_m}$$

$$\Omega_m = \frac{\bar{\varepsilon}_m}{\varepsilon_c} = \frac{8\pi G \bar{\varepsilon}_m}{3c^2 H^2}$$

$$\boxed{\ddot{\delta} + 2H\dot{\delta} - \frac{3}{2}H^2 \Omega_m \delta = 0}$$

The solution to this equation depends on Ω_m

The Growth of Structure

- Radiation-dominated phase in early universe $\Omega_m \ll 1$

$$H = \frac{1}{2t}$$

$$\boxed{\ddot{\delta} + \frac{1}{t}\dot{\delta} = 0}$$

Solution: $\boxed{\delta(t) \approx B_1 + B_2 \ln t}$

Perturbations grow at a logarithmic rate.

The Growth of Structure

- Lambda-dominated phase in late universe $\Omega_m \ll 1$

$$H = H_\Lambda = \text{const}$$

$$\boxed{\ddot{\delta} + 2H_\Lambda \dot{\delta} = 0}$$

Solution: $\boxed{\delta(t) \approx C_1 + C_2 e^{-2H_\Lambda t}}$

Perturbations reach a constant amplitude.

The Growth of Structure

- Matter-dominated phase in recent universe $\Omega_m \approx 1$

$$H = \frac{2}{3t}$$

$$\boxed{\ddot{\delta} + \frac{4}{3t}\dot{\delta} - \frac{2}{3t^2}\delta = 0}$$

Guess: $\delta(t) \approx Dt^n \quad \rightarrow \quad \dot{\delta} = nDt^{n-1} \qquad \ddot{\delta} = n(n-1)Dt^{n-2}$

$$n(n-1)Dt^{n-2} + \frac{4}{3t}nDt^{n-1} - \frac{2}{3t^2}Dt^n = 0$$

$$n(n-1)Dt^{n-2} + \frac{4}{3}nDt^{n-2} - \frac{2}{3}Dt^{n-2} = 0$$

$$n = \begin{cases} -1 \\ 2/3 \end{cases}$$

$$n(n-1) + \frac{4}{3}n - \frac{2}{3} = n^2 + \frac{1}{3}n - \frac{2}{3} = 0$$

The Growth of Structure

- Matter-dominated phase in recent universe $\Omega_m \approx 1$

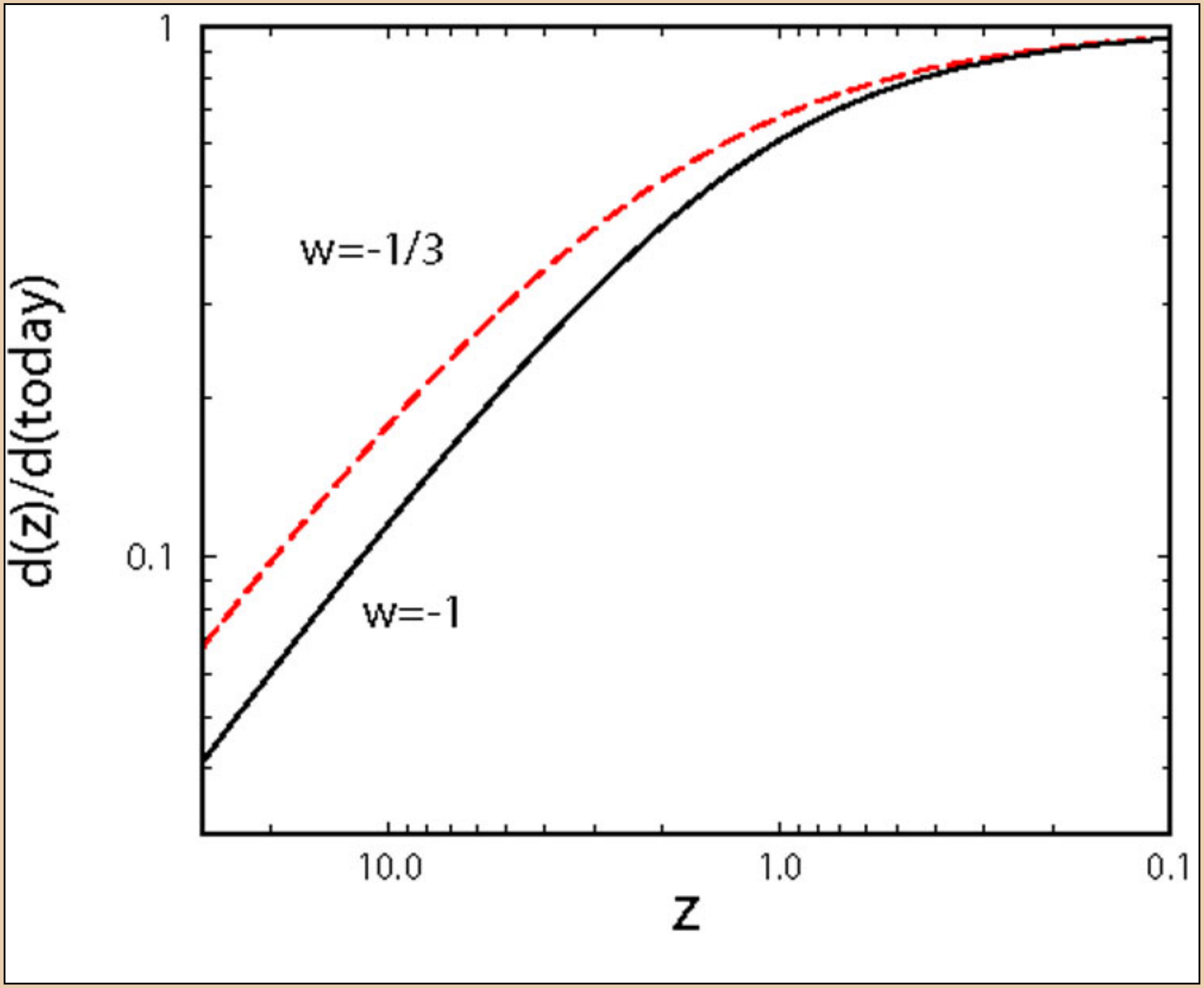
$$H = \frac{2}{3t}$$

$$\boxed{\ddot{\delta} + \frac{4}{3t}\dot{\delta} - \frac{2}{3t^2}\delta = 0}$$

Solution: $\boxed{\delta(t) \approx D_1 t^{2/3} + D_2 t^{-1}}$

When growing mode dominates, $\delta \propto t^{2/3} \propto a \propto \dfrac{1}{1+z}$

The Growth of Structure



The Growth of Structure

$$\boxed{\ddot{\delta} + 2H\dot{\delta} - \frac{3}{2}H^2\Omega_m\delta = 0}$$

None of the terms or derivatives depend on location $\vec{x}$

The solution may thus be written as:

$$\boxed{\delta(\vec{x},t) = D(t)\tilde{\delta}(\vec{x})}$$

$D(t)$ is called the "*growth factor*" and it satisfies the above differential equation. It is also normalized to be equal to unity at t = today.

The Growth of Structure

Since the growth function is normalized to be unity today, $\tilde{\delta}(\vec{x})$ must be the density at *t* = today assuming linear theory. It is the linearly extrapolated density fluctuation.

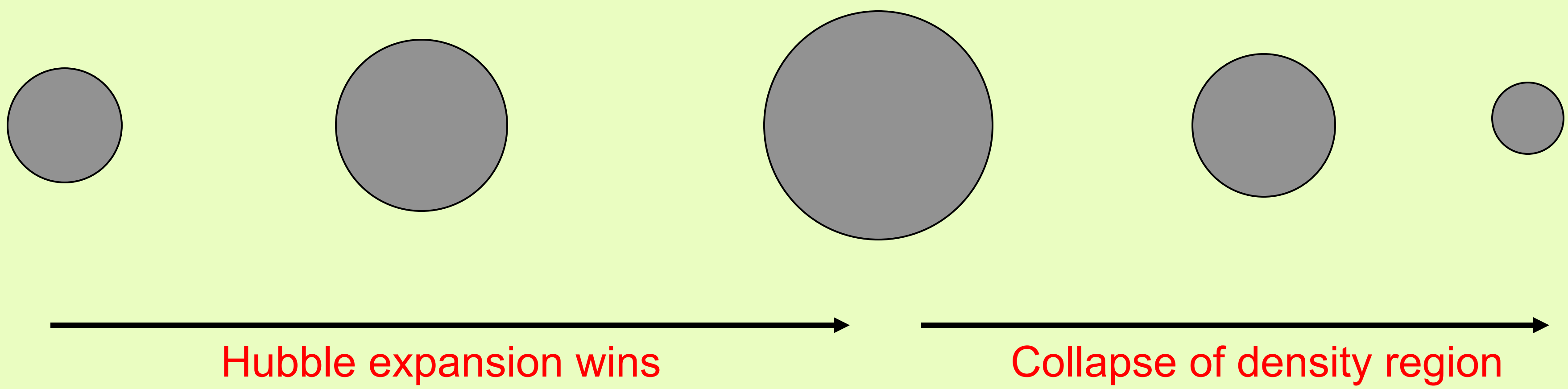
e.g., in a matter dominated universe: $D(t) = \left(\dfrac{t}{t_0}\right)^{2/3}$

The Spherical Collapse model

Reminder: these solutions are for linear theory only!

Once δ grows to ~1, they do not apply.

We need different solutions to describe the collapse of density fluctuations.



The Spherical Collapse model

The Friedmann equation applies to a small density perturbation, in addition to the whole universe.

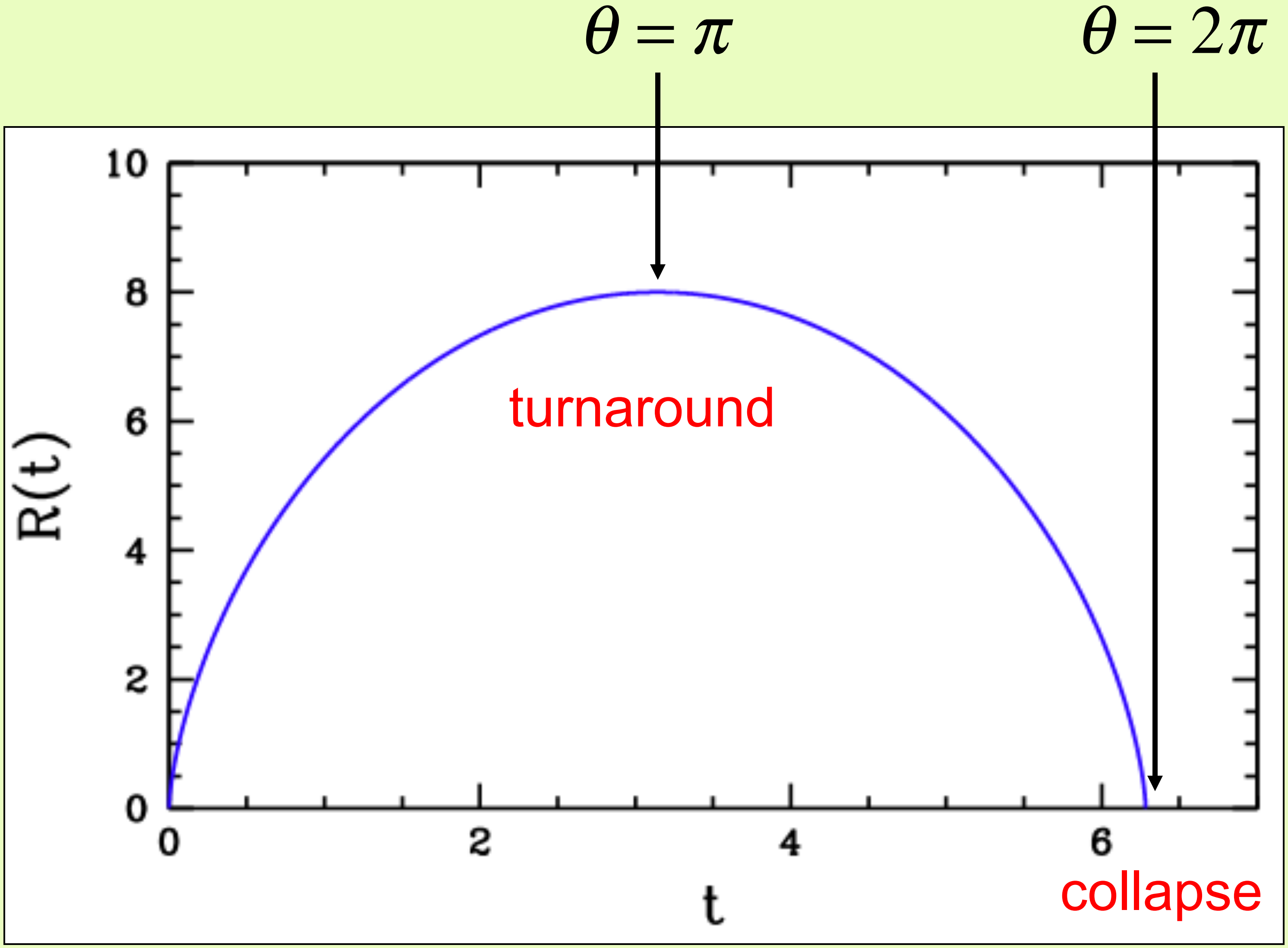
In a matter-dominated universe, $\ddot{R} = -\frac{GM}{R^2}$ has a parametric solution:

$$R = A\left[1 - \cos(\theta)\right]$$
$$t = B\left[\theta - \sin(\theta)\right]$$

(the "cycloid" solution)

Where, $A^3 = GMB^3$

The Spherical Collapse model



The Spherical Collapse model

Expand and only keep low order terms:

$$R = A\left[1-\cos(\theta)\right]$$

$$t = B\left[\theta-\sin(\theta)\right]$$

$$\cos(\theta) \approx 1-\frac{\theta^2}{2}+\frac{\theta^4}{24}-\ldots$$

$$\sin(\theta) \approx \theta-\frac{\theta^3}{6}+\ldots$$

$$R \approx A\left[\frac{\theta^2}{2}-\frac{\theta^4}{24}\right] \quad = \frac{A}{2}\theta^2\left[1-\frac{\theta^2}{12}\right]$$

$$t \approx B\left[\frac{\theta^3}{6}\right] \qquad \rightarrow \; \theta \approx \left(\frac{6t}{B}\right)^{1/3}$$

$$\boxed{\rightarrow \; \mathrm{R} \approx \frac{A}{2}\left(\frac{6t}{B}\right)^{2/3}\left[1-\frac{1}{12}\left(\frac{6t}{B}\right)^{2/3}\right]}$$

The Spherical Collapse model

$$R(t) \approx \frac{A}{2}\left(\frac{6t}{B}\right)^{2/3}\left[1 - \frac{1}{12}\left(\frac{6t}{B}\right)^{2/3}\right]$$

Compare this to our previous linear theory result:

$$R(t) \approx a(t)\left[1 - \frac{1}{3}\delta(t)\right]$$

where:

$$a(t) = \left(\frac{3}{2}H_0 t\right)^{2/3}$$ and: $$\delta(t) \propto t^{2/3}$$

The cycloid solution at small t agrees with linear theory.

The Spherical Collapse model

Turnaround

The sphere breaks away from general expansion and reaches a maximum radius at $\theta=\pi$. At this point, linear theory predicts that the density contrast is $\delta_{lin}=1.06$.

Collapse

The sphere collapses to a singularity at $\theta=2\pi$. This occurs when $\delta_{lin}=1.69$.

Virialization

Complete collapse never occurs in practice because the kinetic energy of collapse is converted into random motions. When the sphere has collapsed to half its maximum size, its kinetic energy is $K=-0.5U$, where U is the potential energy. This is the condition for equilibrium according to the virial theorem. This occurs at $\theta=3\pi/2$ when the density contrast is $\delta_{lin}=1.58$.

The Spherical Collapse model

If virialization occurs at 3π/2: $$1+\delta_{\text{vir}} \equiv \Delta_{\text{vir}} = \frac{\rho}{\bar{\rho}} \approx 147$$

If virialization occurs at 2π: $$1+\delta_{\text{vir}} \equiv \Delta_{\text{vir}} = \frac{\rho}{\bar{\rho}} \approx 178$$

More generally: $$\Delta_{\text{vir}} \approx 178\,\Omega_m^{-0.7}$$

Even more generally (for flat matter + dark energy models):

$$\Delta_{\text{vir}} \approx \left[18\pi^2 + 82(\Omega_m - 1) - 39(\Omega_m - 1)^2\right]\Omega_m^{-1}$$

Bryan & Norman (1998)

Large Scale Velocity Field

According to linear theory, the peculiar velocity is:

$$\vec{v}(\vec{x}) = \frac{f(\Omega_m)}{4\pi} \int \delta_m(\vec{x}') \frac{(\vec{x}' - \vec{x})}{|\vec{x}' - \vec{x}|^3} d^3x'$$

where:

$$f(\Omega_m) \approx \Omega_m^{0.6}$$

The differential form of this equation is:

$$\vec{\nabla} \cdot \vec{v}(\vec{x}) = -f(\Omega_m)\delta_m(\vec{x})$$

Large Scale Velocity Field

We can measure the radial peculiar velocity of a galaxy by measuring its redshift and a redshift-independent distance.

$$v_r = cz - H_0 d$$

By comparing the observed velocity field to the observed density field, we can constrain Ω. There are two main approaches that have been used:

- velocity-velocity comparison
- density-density comparison

Large Scale Velocity Field

Velocity-Velocity

- Measure the density field from a galaxy redshift survey, smoothing on some (small) scale.
- Use linear theory to predict the full 3D velocity field.
- Predict radial velocities for all the galaxies.
- Compare these predictions to the actual measured galaxy velocities.
- The slope of the predicted vs. observed relation gives us $f(\Omega_m)$.

Large Scale Velocity Field

Density-Density

- Measure the radial velocity field from a galaxy redshift survey, smoothing on some (large) scale.

- Integrate this radially to get the potential field and compute the gradient of the potential field to get the full 3D velocity field. Then use linear theory to predict the density field.

- Compare this prediction to the actual measured galaxy density field.

- The slope of the predicted vs. observed relation gives us $f(\Omega_m)$.

Density-Density

$$\Phi(\vec{r}) = -\int_0^r v_r(r', \theta, \varphi)\, dr'$$

$$\vec{v}(\vec{r}) = -\nabla_r \Phi(\vec{r})$$

$$\delta_m(\vec{r}) = -f(\Omega_m)^{-1} \nabla_r \cdot \vec{v}(\vec{r})$$

Large Scale Velocity Field

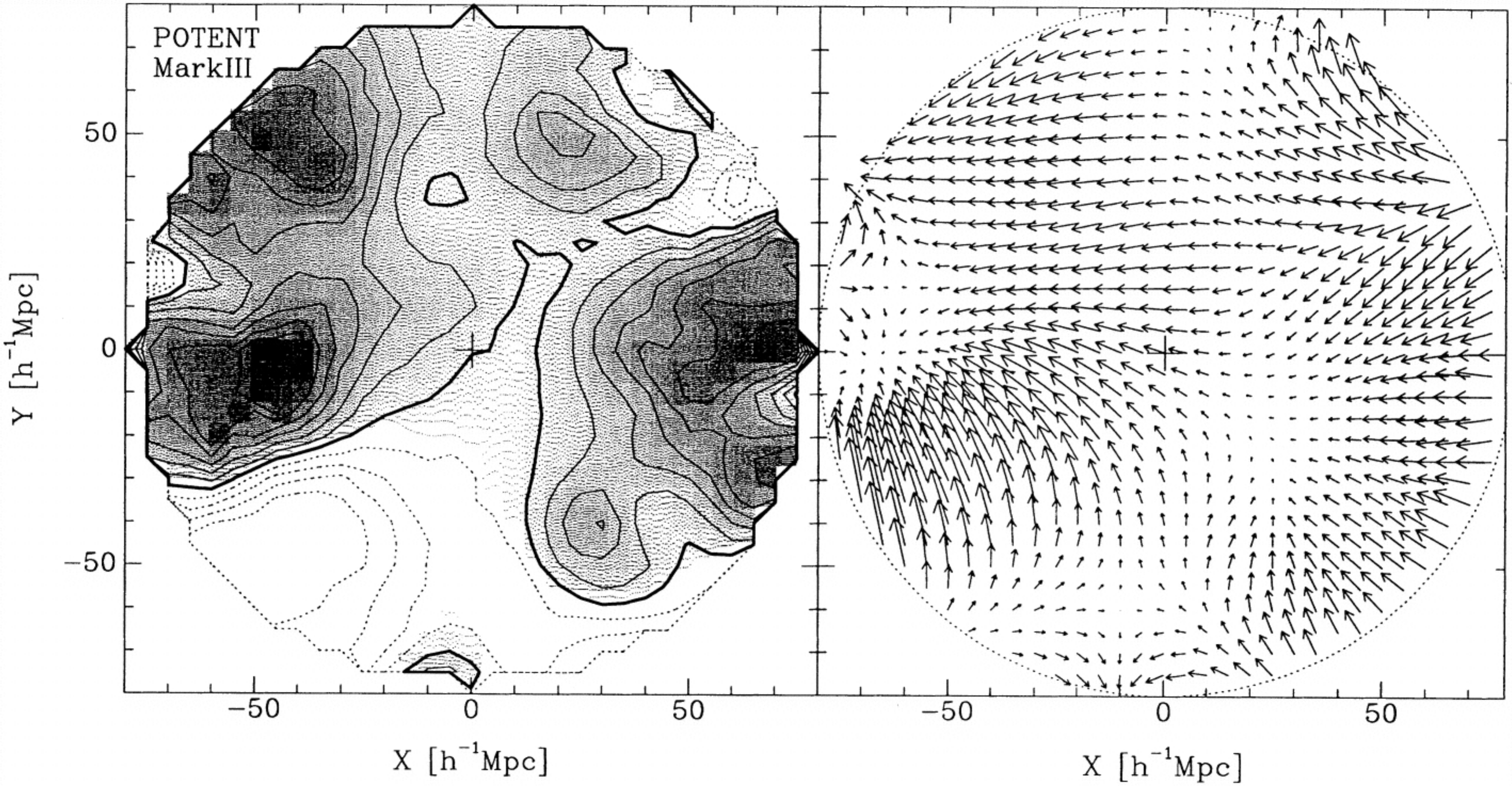
We only actually measure the *galaxy* density field, which on large scales is related to the mass density via a linear bias factor:

$$\delta_g = b\delta_m$$

So these methods actually constrain the quantity:

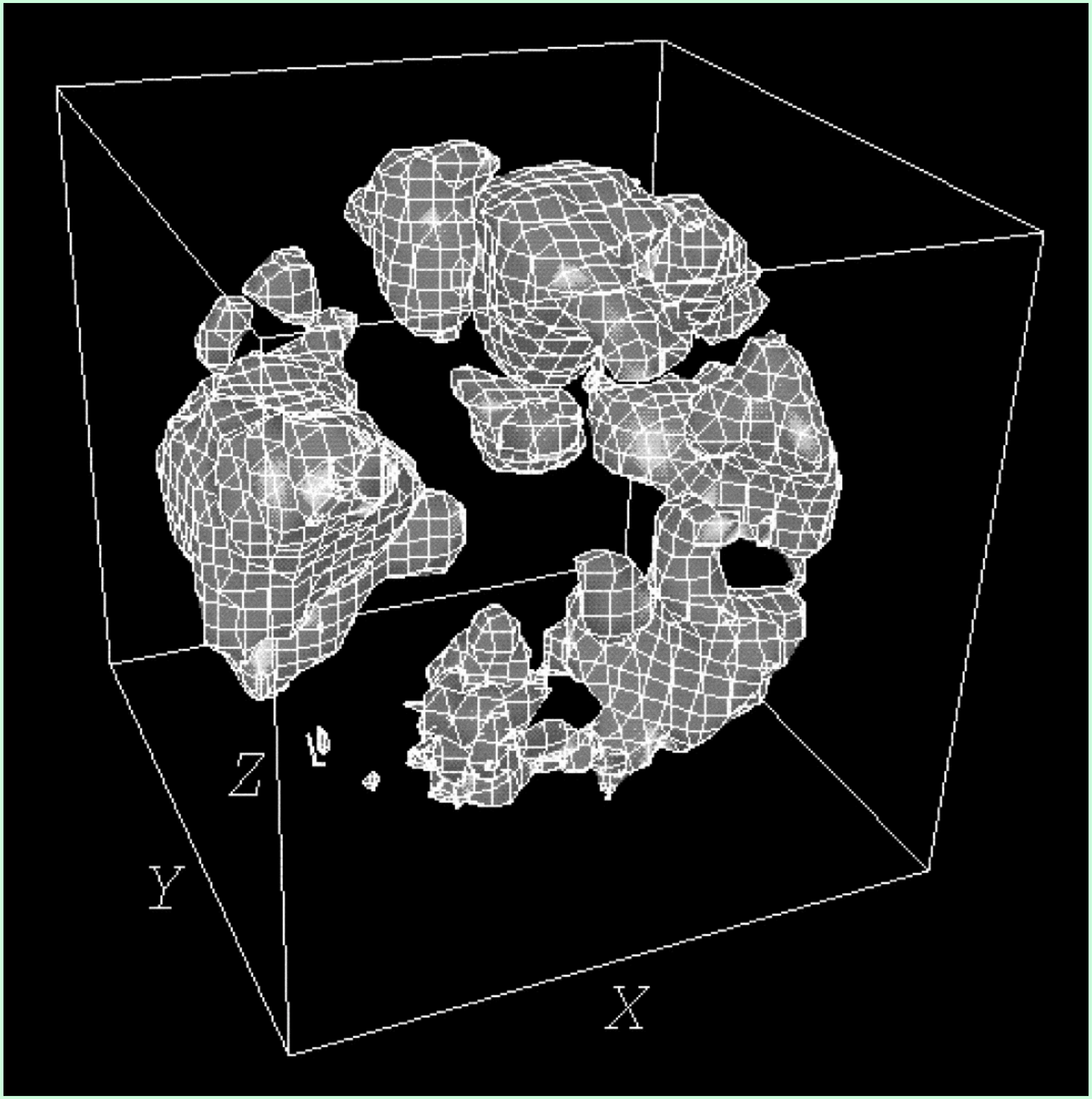
$$\beta = \frac{f\left(\Omega_m\right)}{b} \approx \frac{\Omega_m^{0.6}}{b}$$

Large Scale Velocity Field



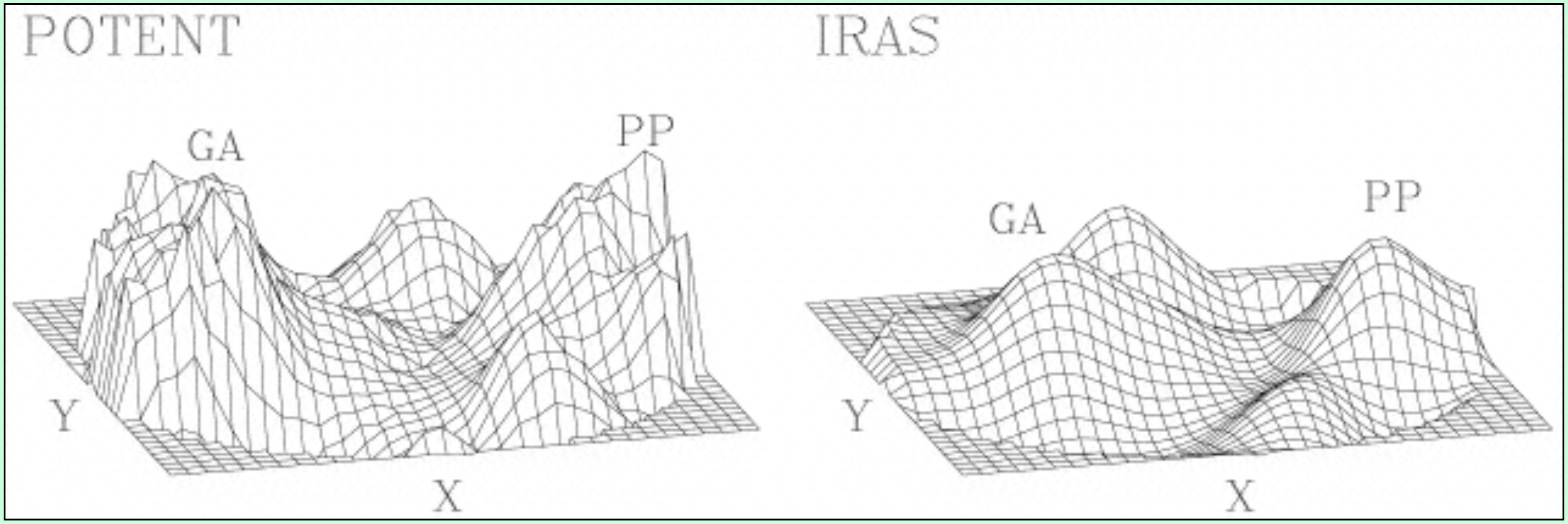
Dekel et al. (1999)

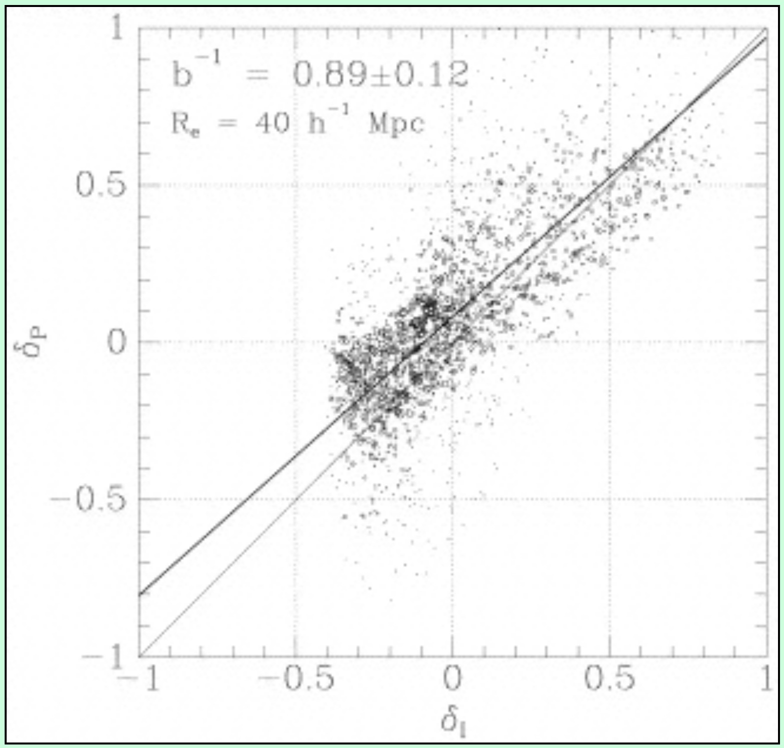
Large Scale Velocity Field



Dekel et al. (1999)

Large Scale Velocity Field





Sigad et al. (1998)

Large Scale Velocity Field

Many systematic errors!

For example, homogeneous and inhomogeneous Malmquist bias, which is caused by anisotropic scattering of galaxy positions due to large distance errors.

N-body Simulations

THE ASTROPHYSICAL JOURNAL

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VOLUME 94 | NOVEMBER 1941 | NUMBER 3

ON THE CLUSTERING TENDENCIES AMONG THE NEBULAE

II. A STUDY OF ENCOUNTERS BETWEEN LABORATORY MODELS OF STELLAR SYSTEMS BY A NEW INTEGRATION PROCEDURE

ERIK HOLMBERG

ABSTRACT

In a previous paper[1] the writer discussed the possibility of explaining the observed clustering effects among extragalactic nebulae as a result of captures. The present investigation deals with the important problem of whether the loss of energy resulting from the tidal disturbances at a close encounter between two nebulae is large enough to effect a capture. The tidal deformations of two models of stellar systems, passing each other at a small distance, are studied by reconstructing, piece by piece, the orbits described by the individual mass elements. The difficulty of integrating the total gravitational force acting upon a certain element at a certain point of time is solved by replacing gravitation by light. The mass elements are represented by light-bulbs, the candle power being proportional to mass, and the total light is measured by a photocell (Fig. 1). The nebulae are assumed to have a flattened shape, and each is represented by 37 light-bulbs. It is found that the tidal deformations cause an increase in the attraction between the two objects, the increase reaching its maximum value when the nebulae are separating, i.e., after the passage. The resulting loss of energy (Fig. 6) is comparatively large and may, in favorable cases, effect a capture. The spiral arms developing during the encounter (Figs. 4) represent an interesting by-product of the investigation. The direction of the arms depends on the direction of rotation of the nebulae with respect to the direction of their space motions.

N-body Simulations

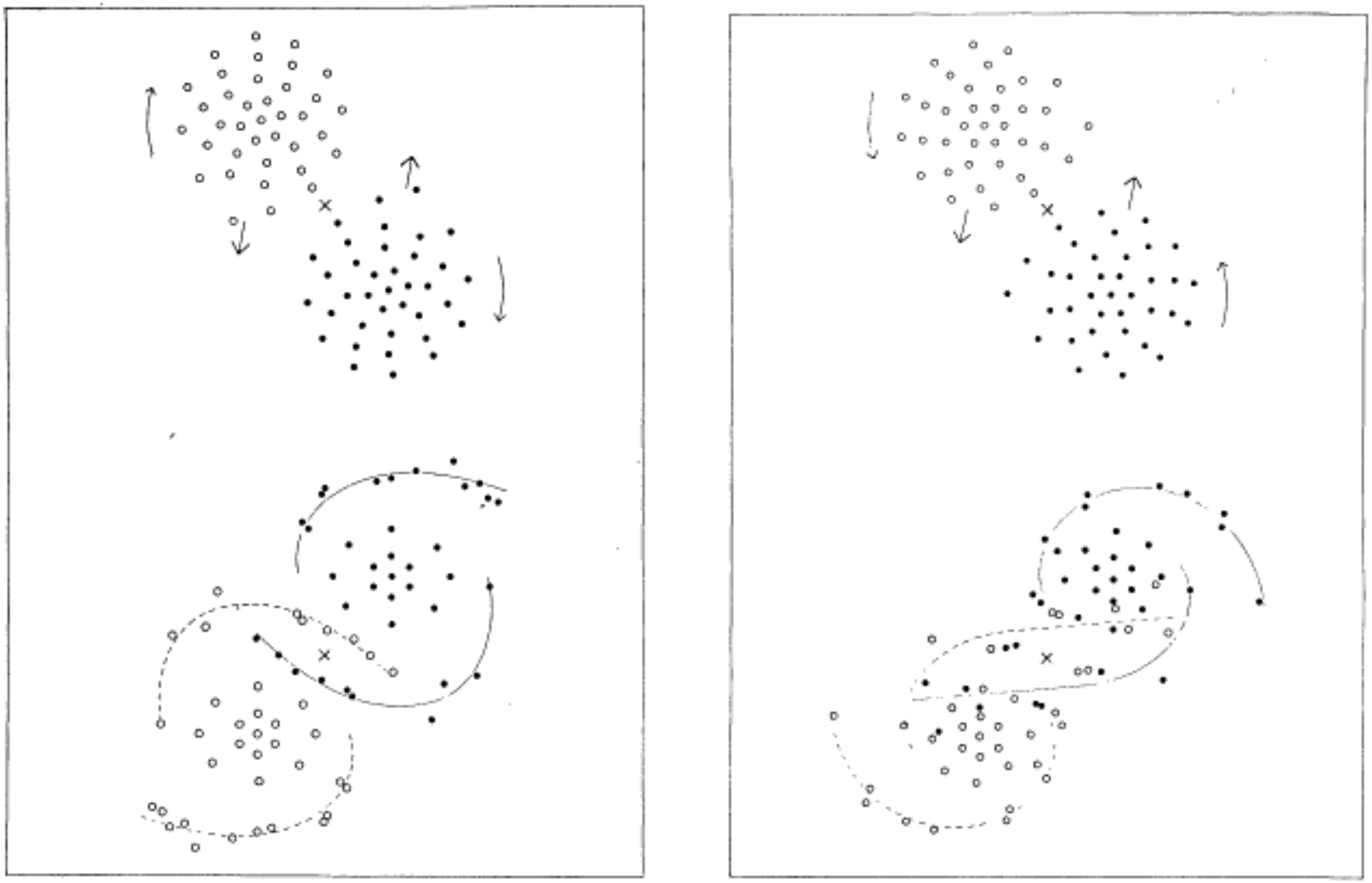


FIG. 4a FIG. 4b

FIG. 4a.—Tidal deformations corresponding to parabolic motions, clockwise rotations, and a distance of closest approach equal to the diameters of the nebulae. The spiral arms point in the direction of the rotation.

FIG. 4b.—Same as above, with the exception of counterclockwise rotations. The spiral arms point in the direction opposite to the rotation.

N-body Simulations

<u>Initial conditions:</u>

- What kind of Dark Matter?
- How much Dark Matter?
- Initial density fluctuations P(k)

↓

GRAVITY

↓

Final distribution of dark matter.

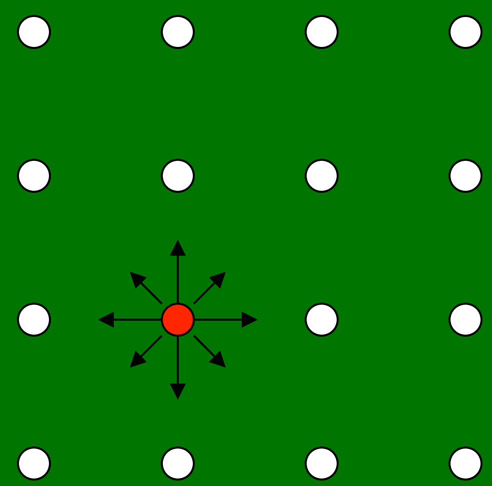
N-body Simulations

• Start with a grid of particles representing dark matter.

• Give them initial smooth density fluctuations.

• Compute the force of gravity on every particle from every other particle in a series of time steps.

A very computationally expensive technique! Only made possible recently with fast computers.



N-body Simulations: Initial Conditions

Cosmological model

$$h,\Omega_m,\Omega_b,\Omega_v,\sigma_8,n_s \;\rightarrow\; P(k)$$

Phases for modes

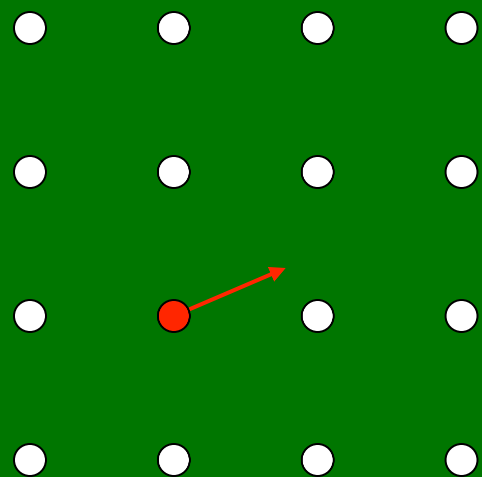
- Gaussian random phase δ(r)
- Non-Gaussian?

Evolve to starting redshift

- z_{init}=30-200
- Zel'dovich approximation
- 2LPT

N-body Simulations: Initial Conditions

Assign initial positions and velocities using Zel'dovich approximation



$$\vec{x} = \vec{q} + D(t)\vec{\psi}(\vec{q})$$

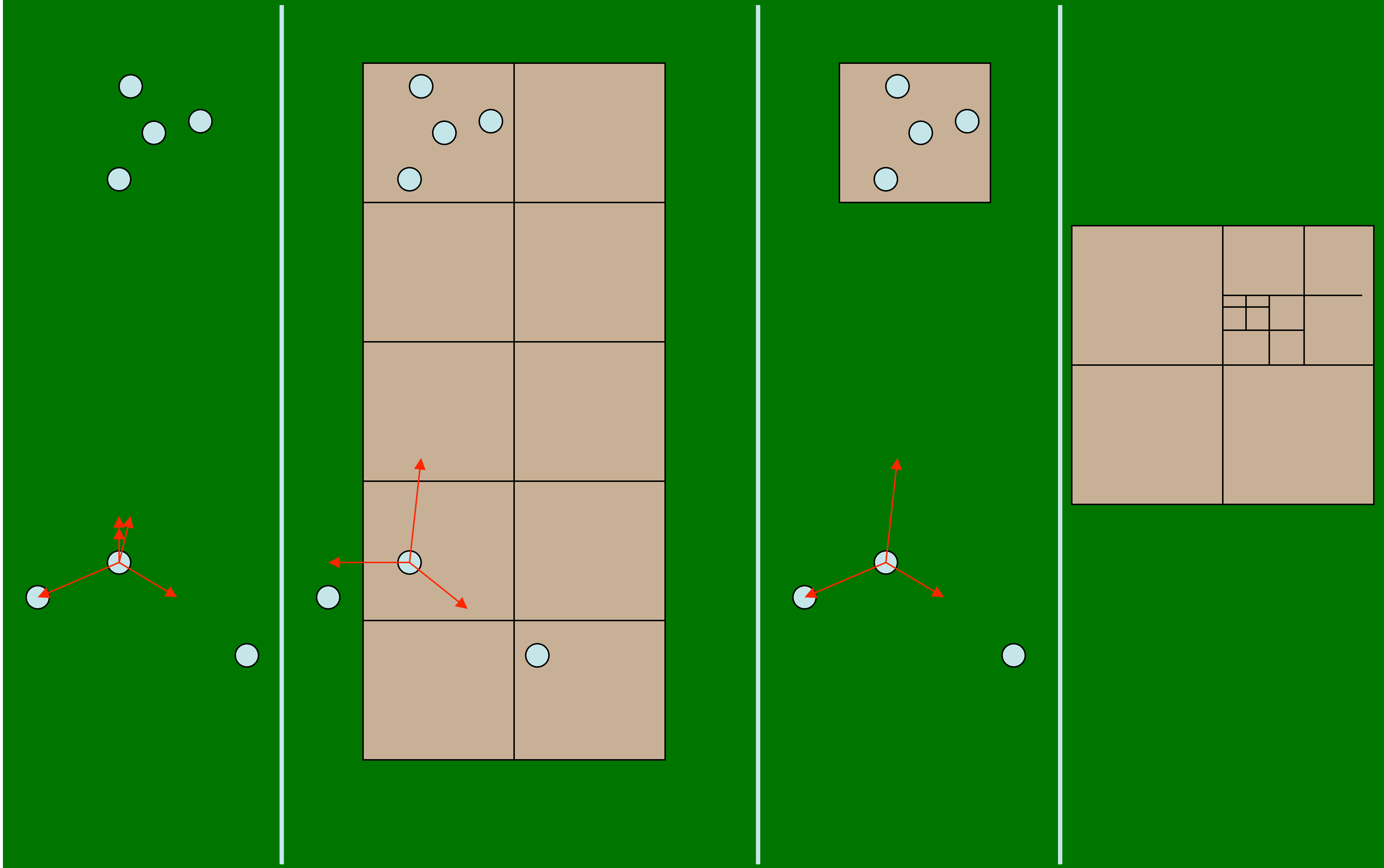
$$\vec{v} = a\frac{dD}{dt}\vec{\psi}(\vec{q})$$

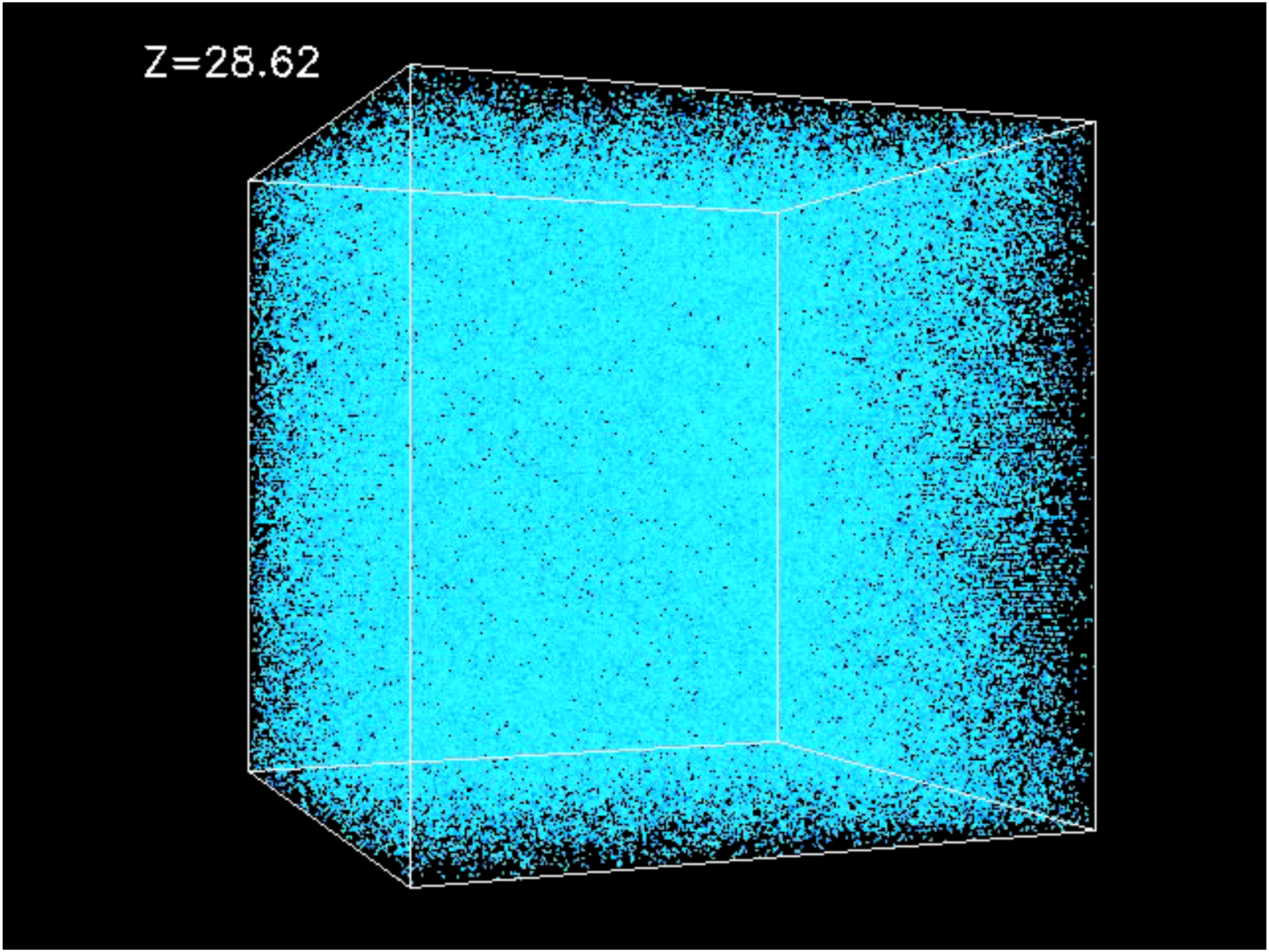
$$\vec{\nabla} \cdot \vec{\psi} = -\frac{\delta(\vec{q})}{D(t)}$$

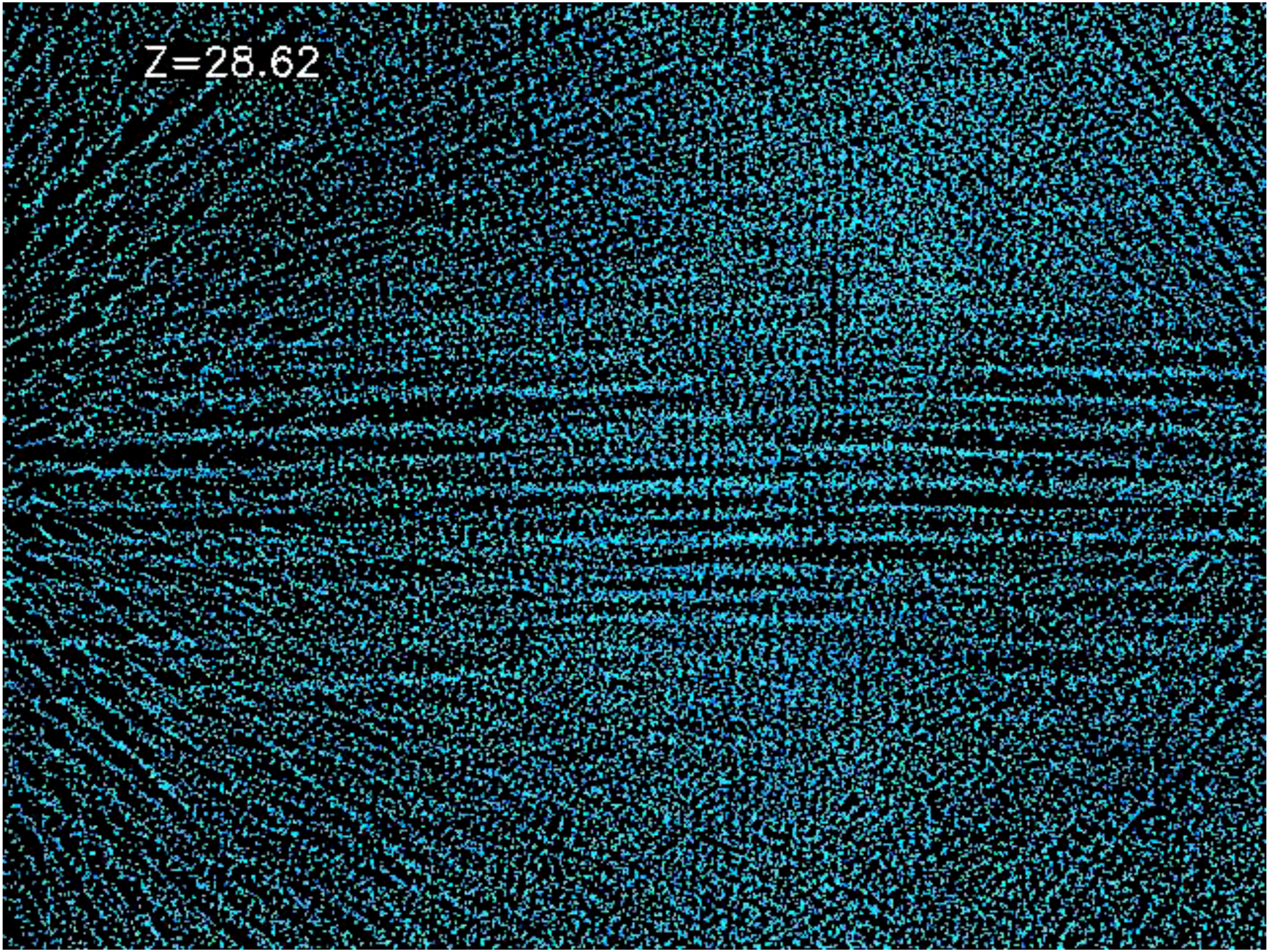
N-body Simulations: Force calculations

• Direct particle-particle	(N^2)
• Particle-Mesh (PM)	$(N_g \log N_g)$
• Particle-particle particle-mesh (P^3M)	$(N^2 \ / \ N_g \log N_g)$
• Tree	$(N \log N)$
• Tree-PM	$(N \log N \ / \ N_g \log N_g)$
• Adaptive mesh refinement (AMR)	
• Adaptive refinement tree (ART)	
• Moving mesh (AREPO)	

N-body Simulations: Force calculations

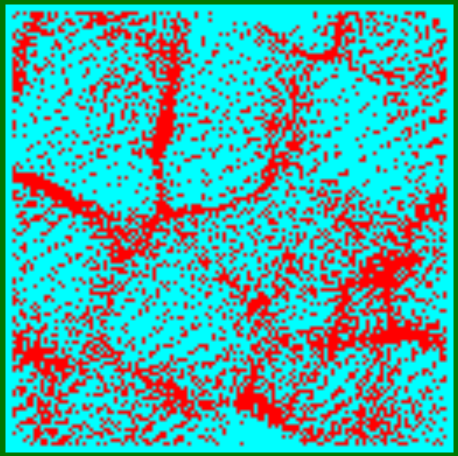



Z=28.62

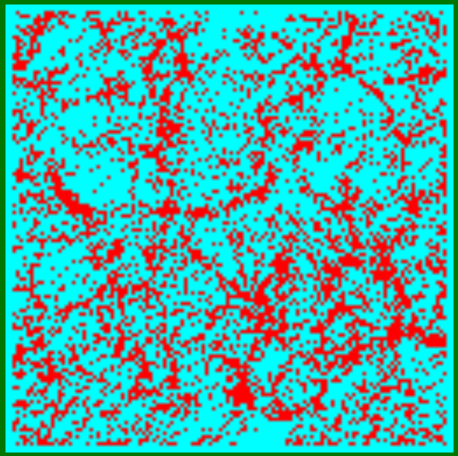

Z=28.62

We can construct cosmological models.

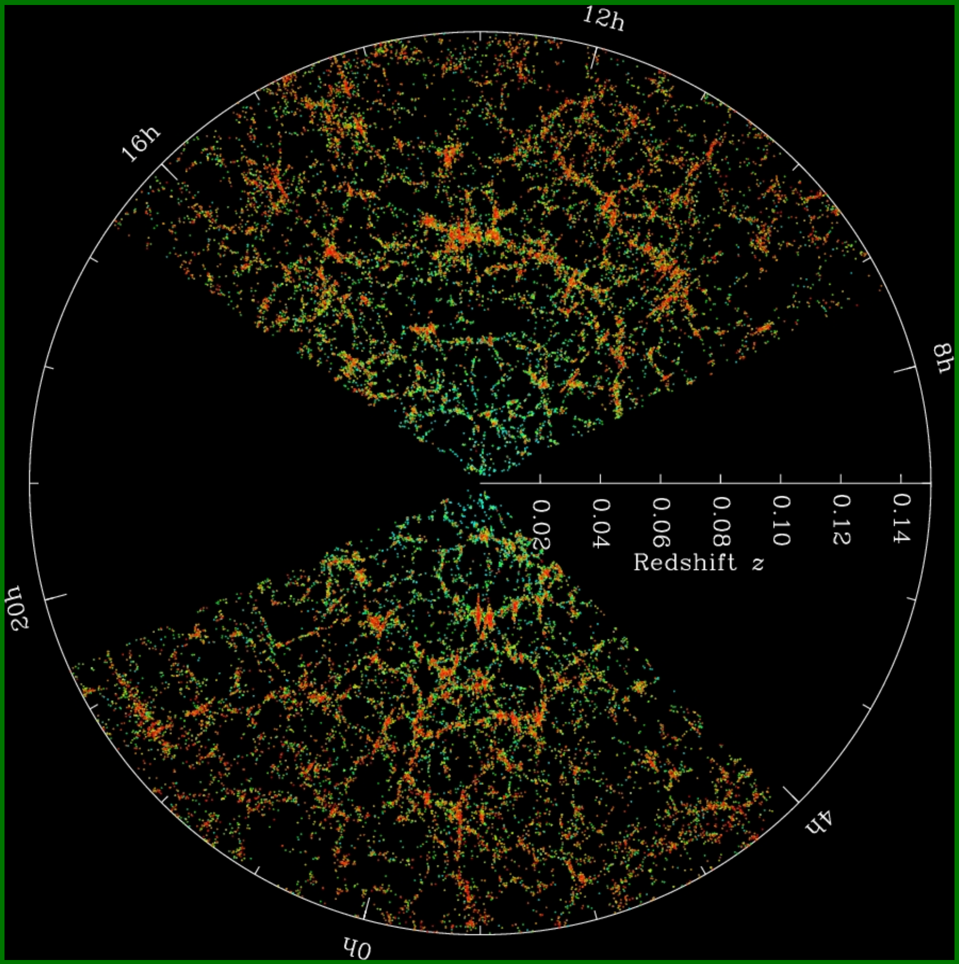
e.g., dark matter models



massive neutrinos

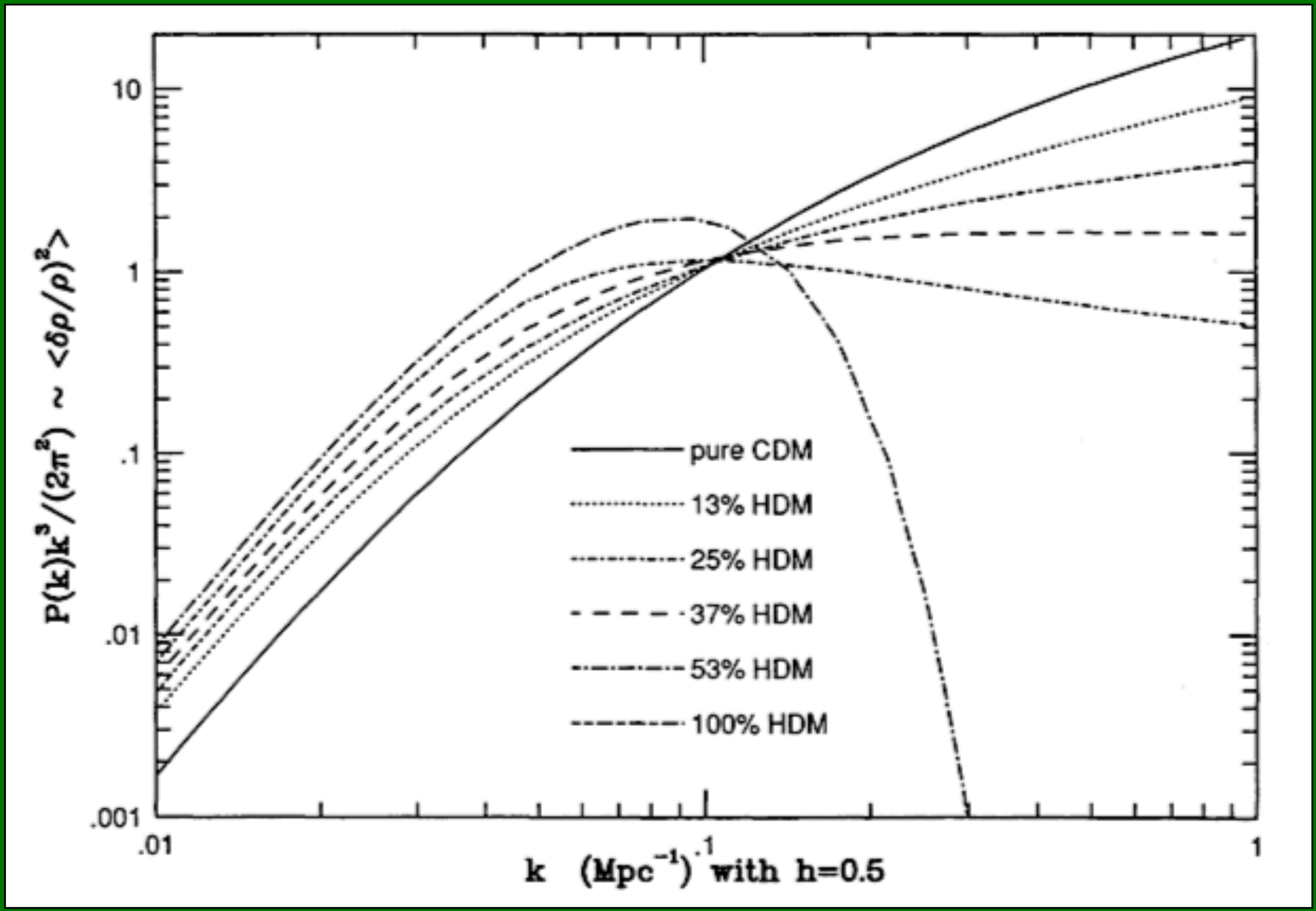


“cold” particles



SDSS observations

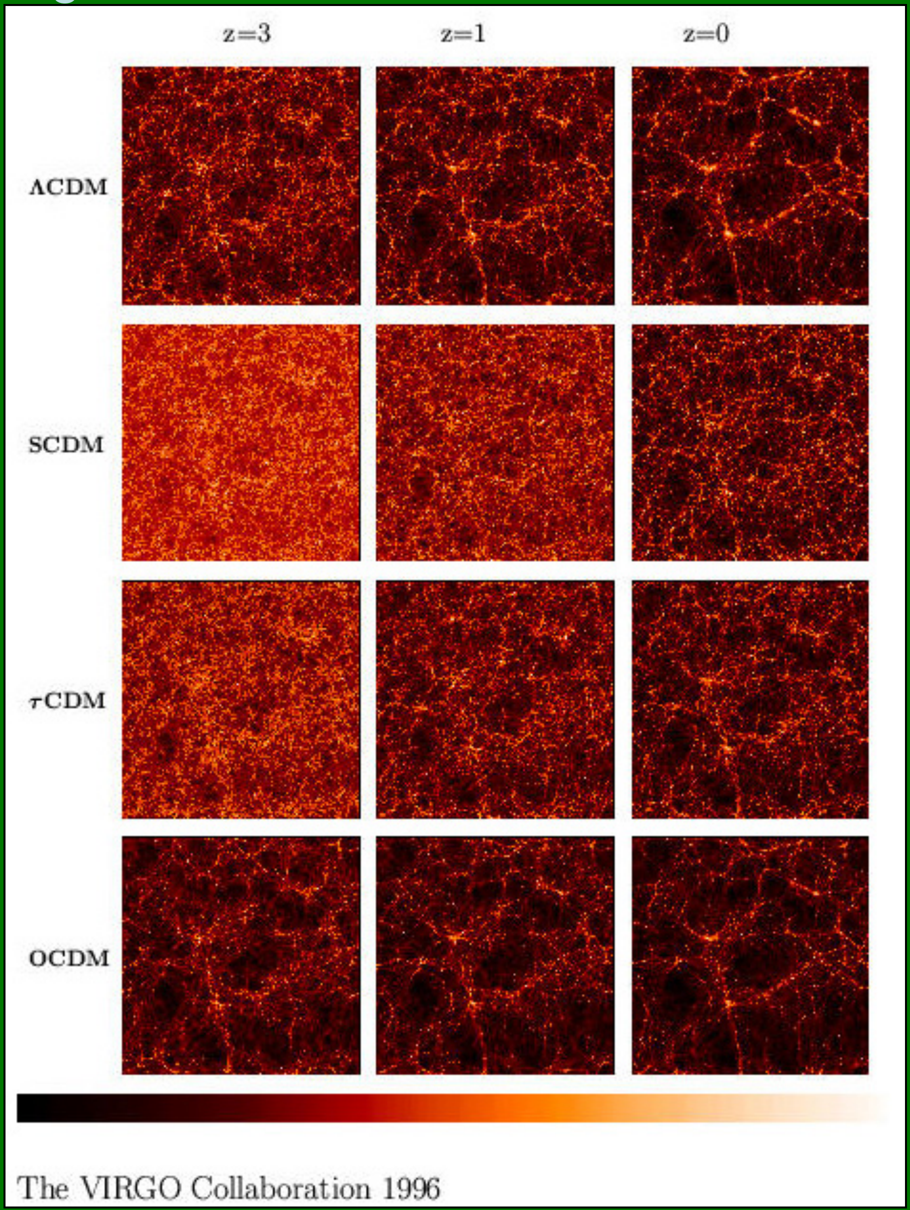
We can constrain cosmological models.



van Dalen & Schaefer (1992)

We can constrain cosmological models.

Fraction of present age: 1/10 1/2 1



Flat universe with dark energy

$\Omega_m = 0.3 \quad \Omega_\Lambda = 0.7$

Flat universe with high DM

$\Omega_m = 1$

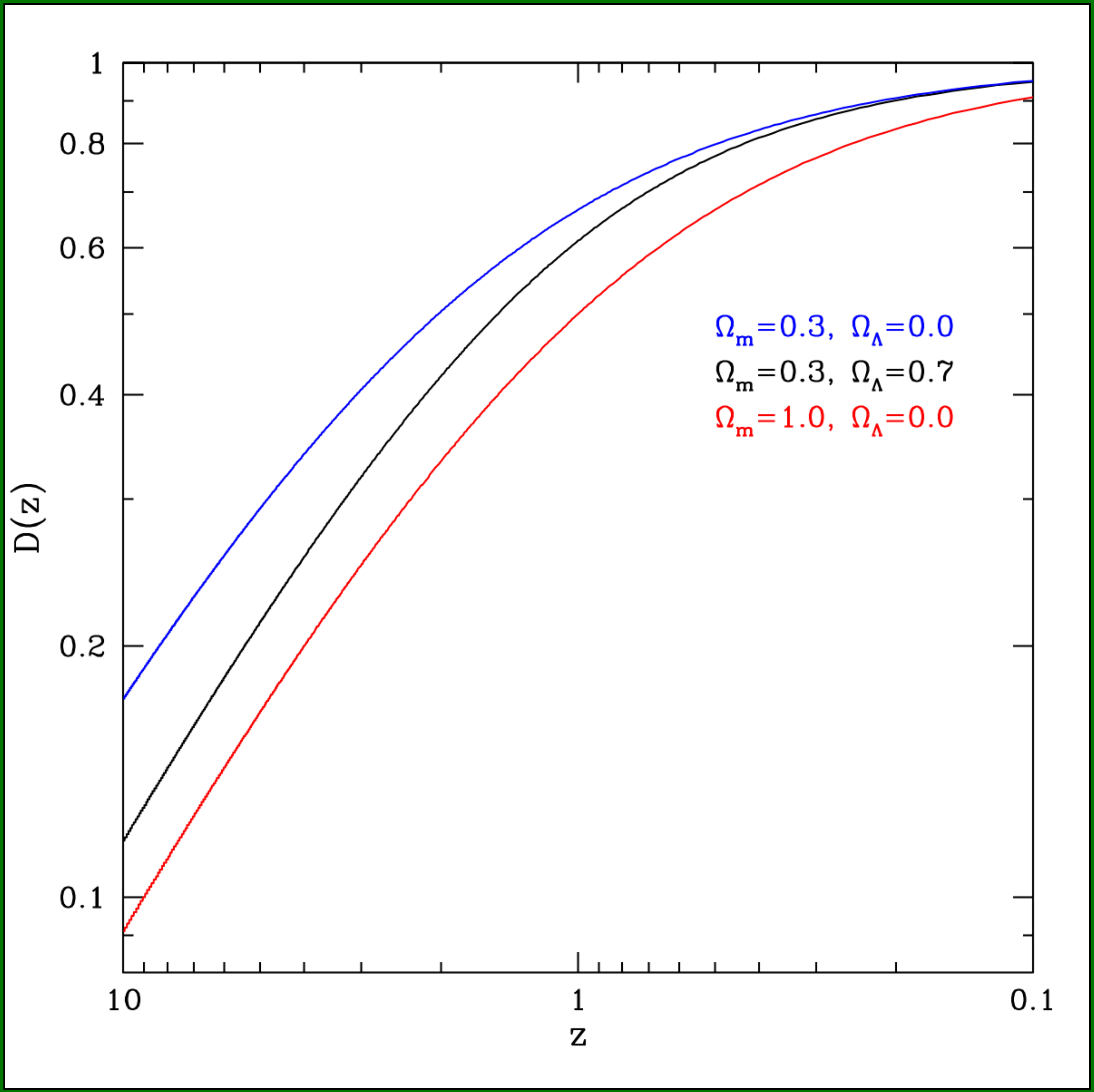
Different initial P(k)

$\Omega_m = 1$

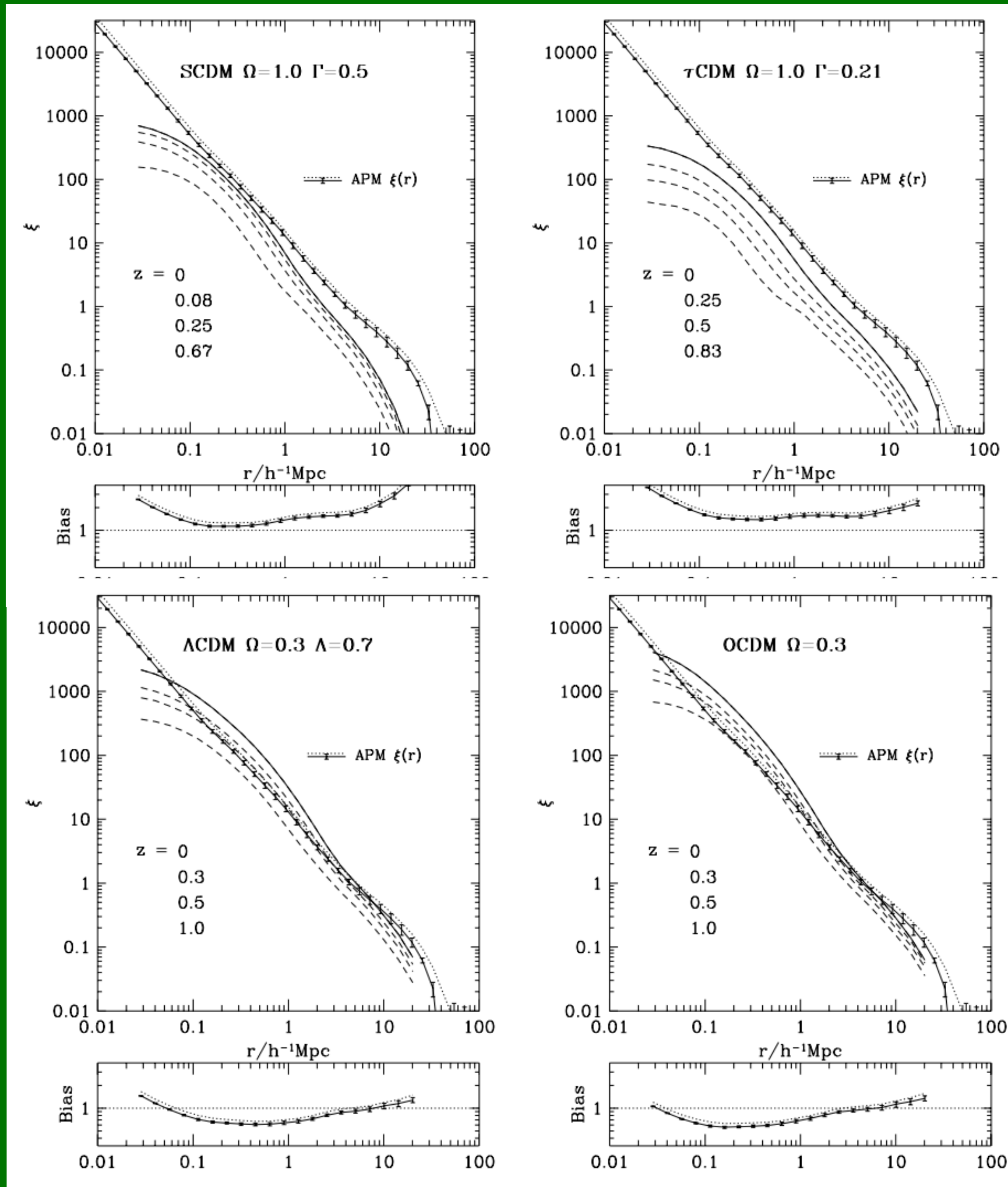
Open universe with low DM

$\Omega_m = 0.3$

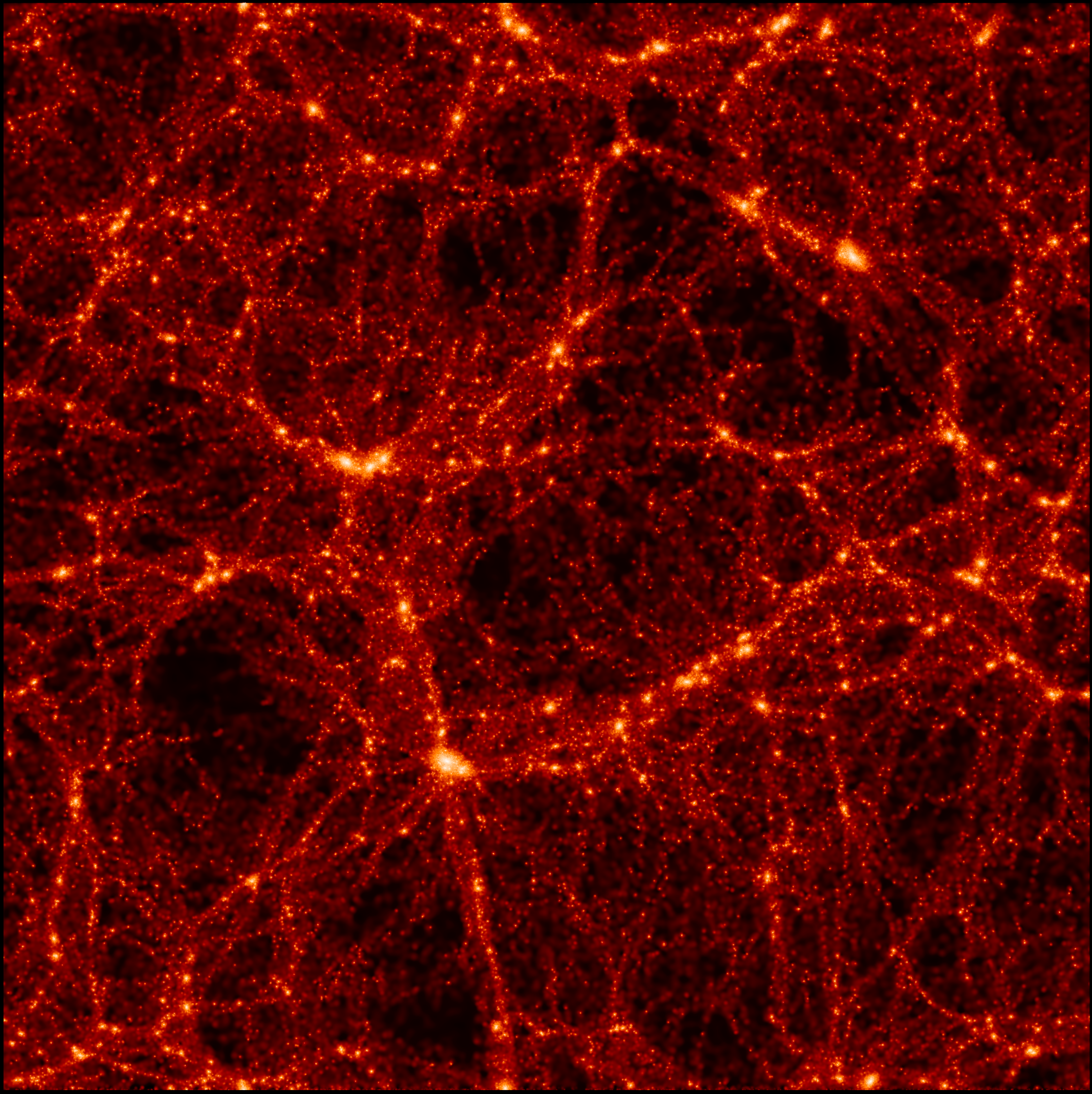
We can constrain cosmological models.



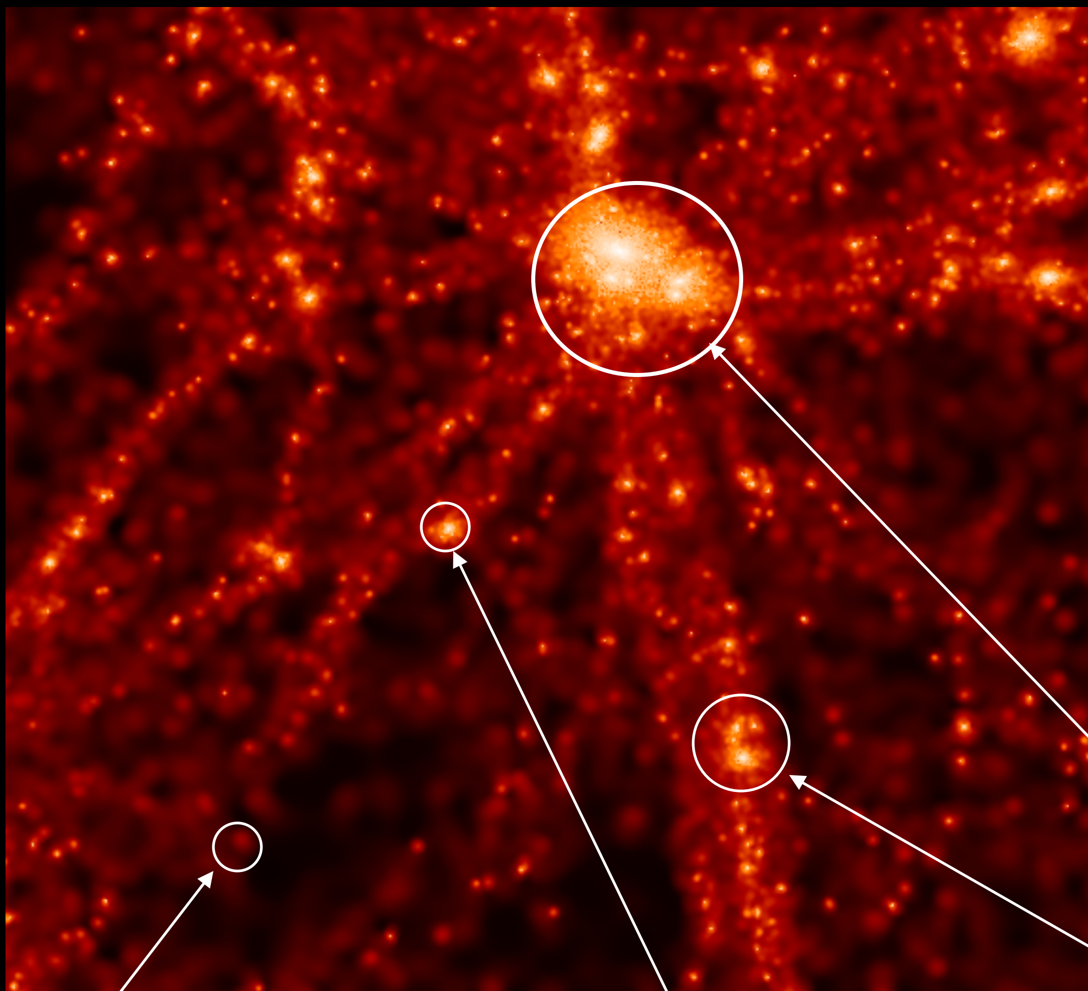
We can constrain cosmological models.



Jenkins et al. (1998)


Virgo Collaboration

What is a dark matter halo?



Dark Matter collapses under its own self-gravity into "virialized" regions, or "halos".

Halos are typically defined as regions with density of ~200 times the mean density, but there are several halo-finding algorithms.

Halos come in different sizes, masses, and shapes.

High mass halo (the kind that would host a galaxy cluster)

Intermediate mass halo (the kind that would host a galaxy group)

Very low mass halo (the kind that would host no galaxy at all)

Low mass halo (the kind that would host a single galaxy)

What is a dark matter halo?

- Friends-of-Friends (FoF)

  linking length *b*

- Spherical Overdensity (SO)

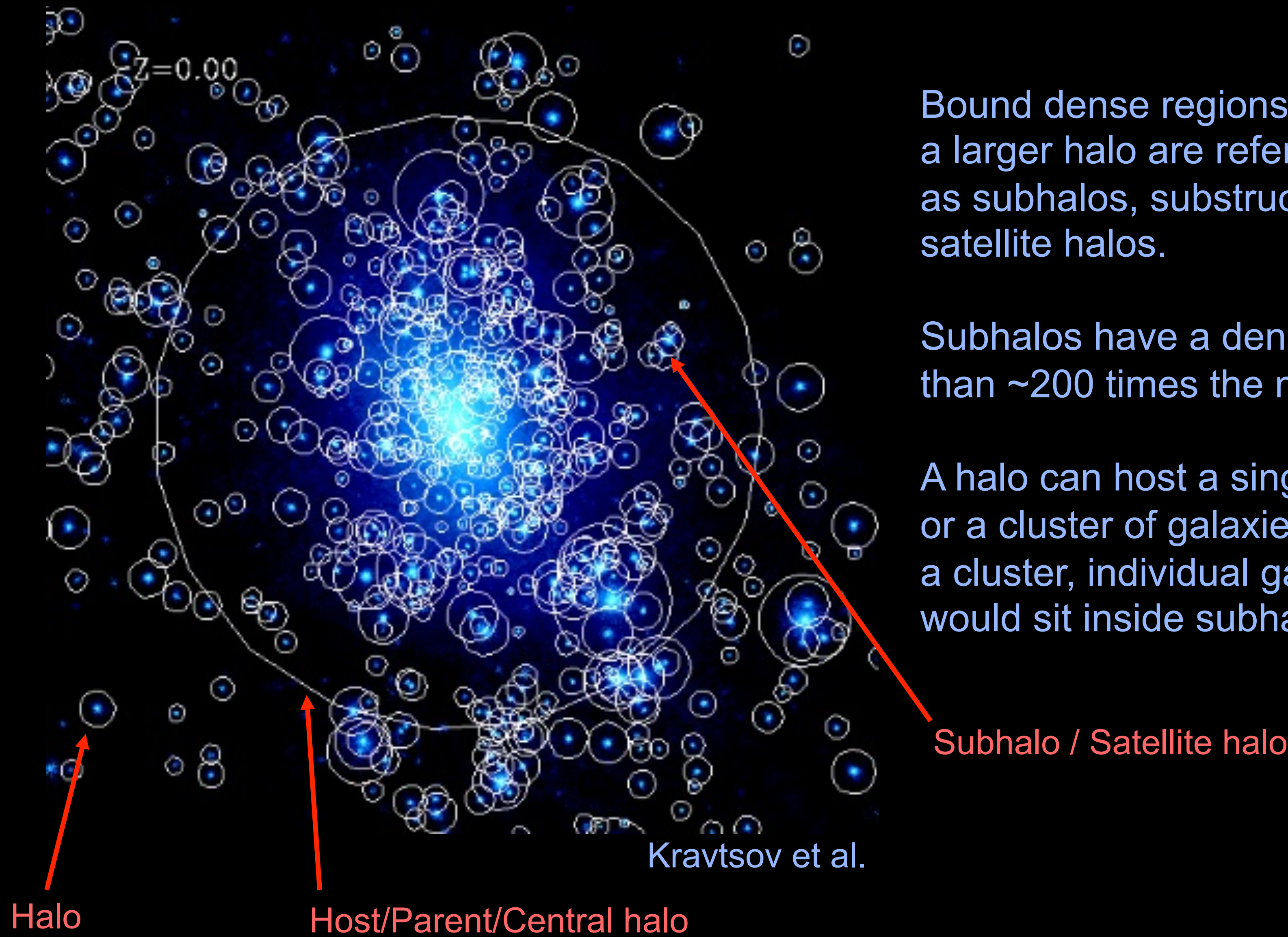
  choice of center, density threshold Δ_{vir}

- Density Maxima (DENMAX, BDM)

  choice of center, density threshold Δ_{vir} , criteria for unbinding

- Other (e.g., Voronoi tesselation)

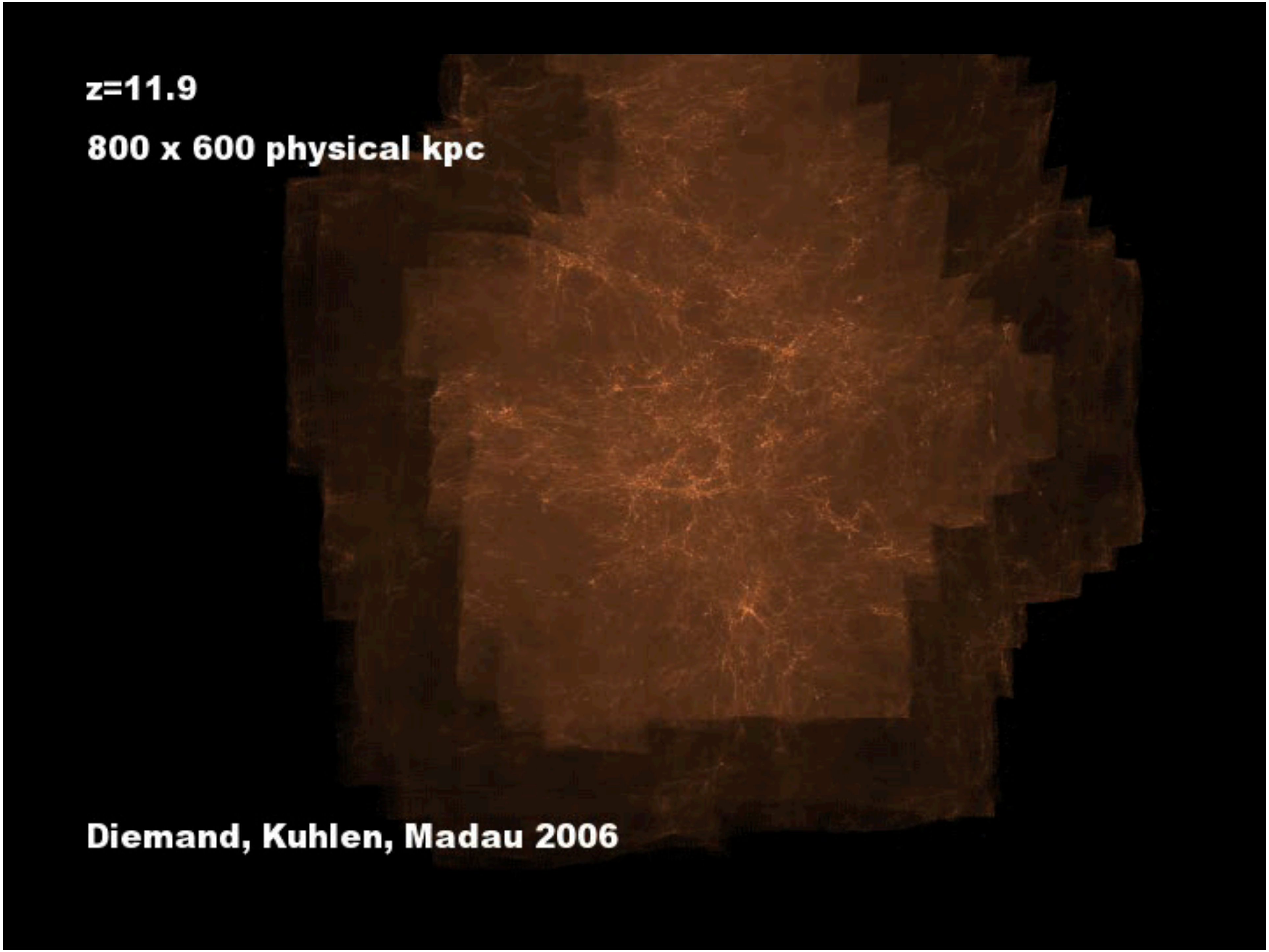
What is a dark matter halo?



Bound dense regions within a larger halo are referred to as subhalos, substructure, or satellite halos.

Subhalos have a density higher than ~200 times the mean.

A halo can host a single galaxy, or a cluster of galaxies. Within a cluster, individual galaxies would sit inside subhalos.

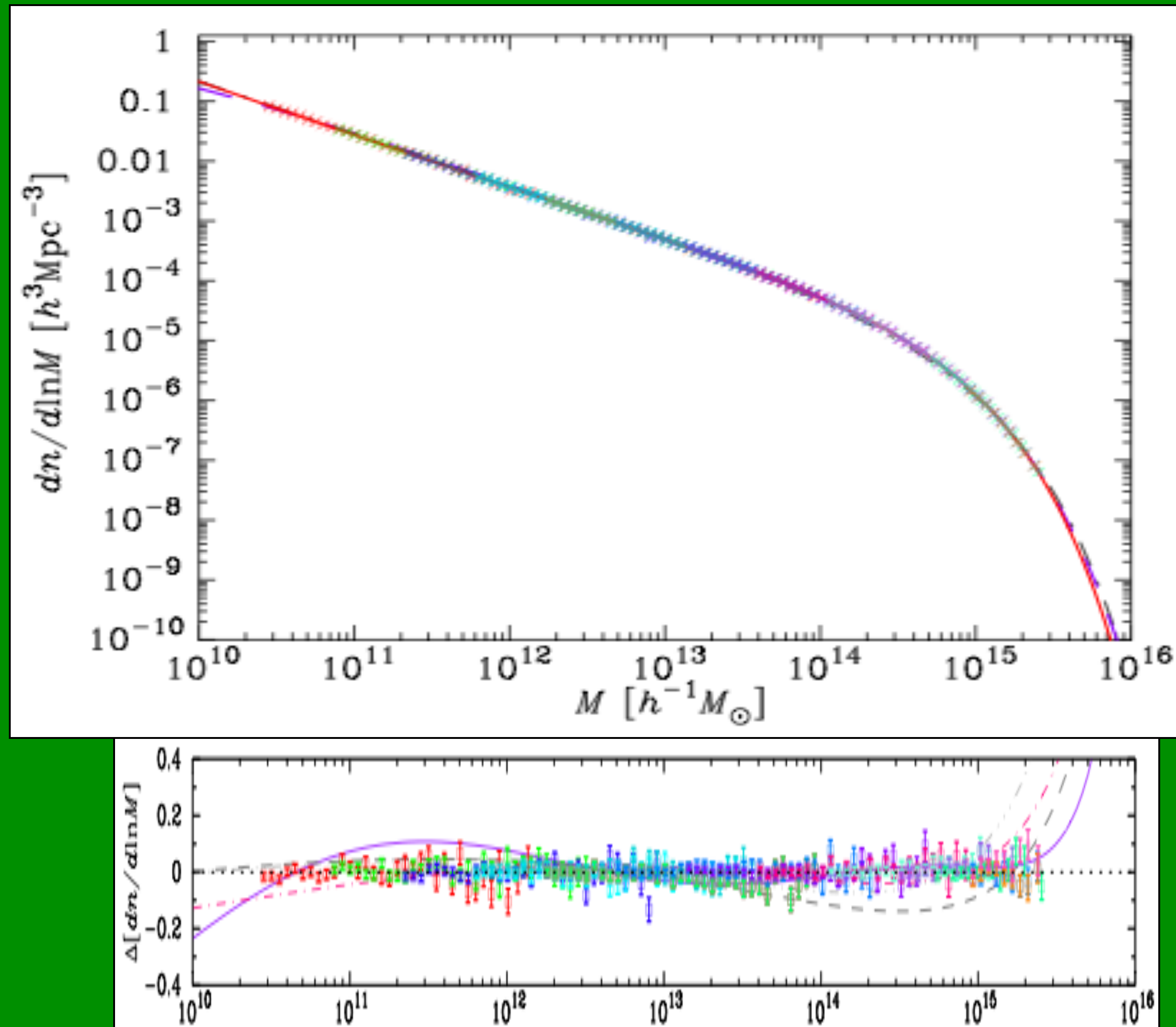

z=11.9
800 x 600 physical kpc
Diemand, Kuhlen, Madau 2006


1600 kpc
via lactea II
Diemand, Kuhlen, Madau, Zemp, Moore, Potter, Stadel, 2008

Halo properties: Abundance (mass function)

We now know the z=0 mass function to ~5% for reasonable choices of cosmological parameters. (*For one N-body code and one halo-finder*)

There may be larger uncertainties for higher redshifts or more exotic cosmological models.



M

Warren et al. (2006)

Halo properties: Abundance (mass function)

Press-Schechter (1974) theory

Halos collapse from regions in the primordial density field that exceed a threshold density. One halo forming does not influence the likelihood of other halos forming nearby.

The halo mass function thus depends on:	
• The distribution of initial densities	$P(k)$
• The linear growth of fluctuations	$D(z)$
• The density threshold for collapse	δ_{crit}

Halo properties: Abundance (mass function)

Consider a spherical region of mass *M*. This region corresponds to a scale:

$$R = \left(\frac{3M}{4\pi\bar{\rho}} \right)^{1/3}$$

The density field smoothed on this scale has a variance of:

$$\sigma_R^2 = \int P(k) \tilde{W}_R(k)^2 d^3k \quad \rightarrow \quad \sigma(M)$$

The probability of the density having a value between δ and δ+dδ is:

$$P(\delta \mid M) d\delta = \frac{1}{\sqrt{2\pi}\sigma(M)} \exp\left[-\frac{\delta^2}{2\sigma(M)^2} \right] d\delta$$

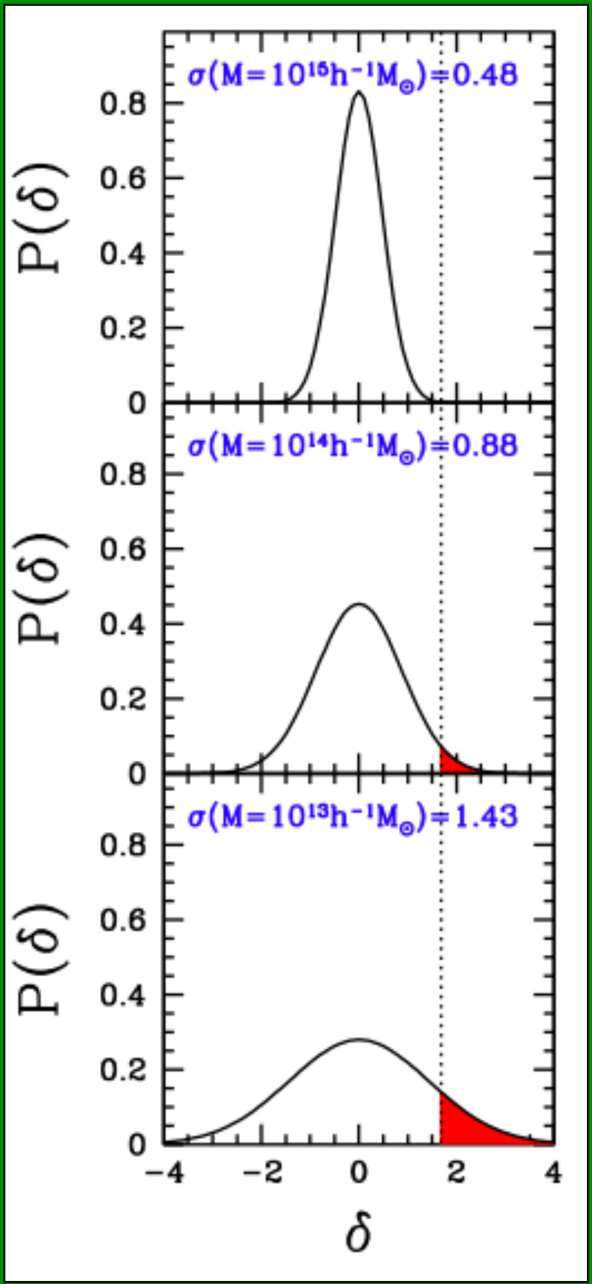
Halo properties: Abundance (mass function)

The fractional volume in this smoothed density field with $\delta > \delta_c$ is:

$$F(>M) = \int_{\delta_c}^{\infty} P(\delta \mid M)\, d\delta$$

The fractional volume corresponding to masses in the range *M* to *M*+*dM* is:

$$\frac{dF(>M)}{dM}\, dM$$



Halo properties: Abundance (mass function)

The volume of a region that will make a single halo of mass *M* is:

$$\frac{M}{\bar{\rho}}$$

The number of halos of mass in the range *M* to *M*+*dM* is:

$$\frac{\text{fraction of volume} \times V_{\text{tot}}}{\text{volume of 1 halo}} = \frac{\bar{\rho}}{M}\frac{dF\left(>M\right)}{dM}dM \times V_{\text{tot}}$$

The number density of halos of mass in the range *M* to *M*+*dM* is:

$$\frac{dn}{dM}dM = \frac{\bar{\rho}}{M}\frac{dF\left(>M\right)}{dM}dM$$

Halo properties: Abundance (mass function)

$$\frac{dn}{dM} = \sqrt{\frac{2}{\pi}} \frac{\bar{\rho}}{M^2} \frac{\delta_c}{\sigma} \frac{d \ln \sigma}{d \ln M} \exp\left[-\frac{\delta_c^{\ 2}}{2\sigma^2} \right]$$

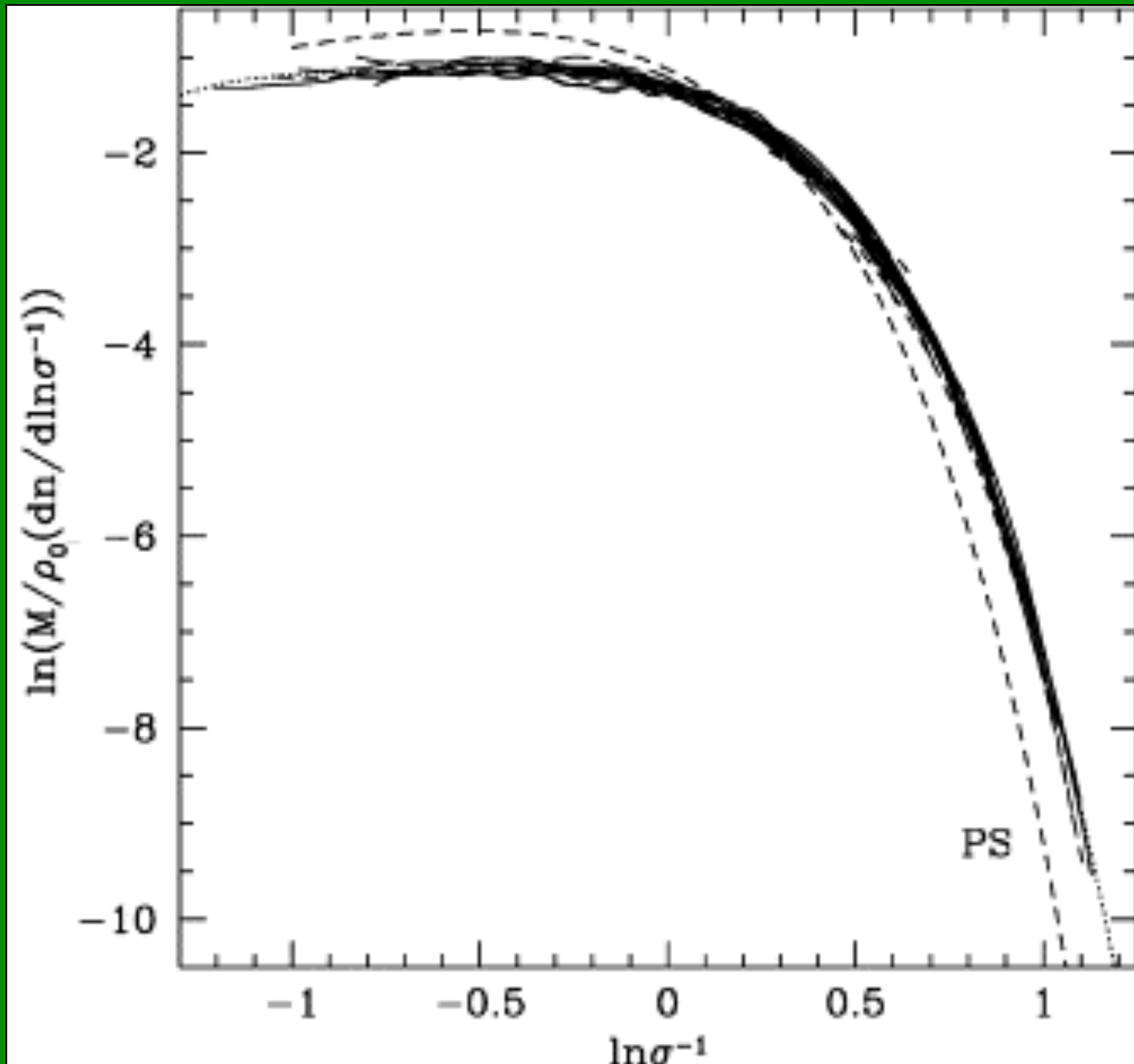
Spherical collapse model

Power spectrum

Linear theory growth rate $\sigma(M,z) = \sigma(M, z=0) D(z)$

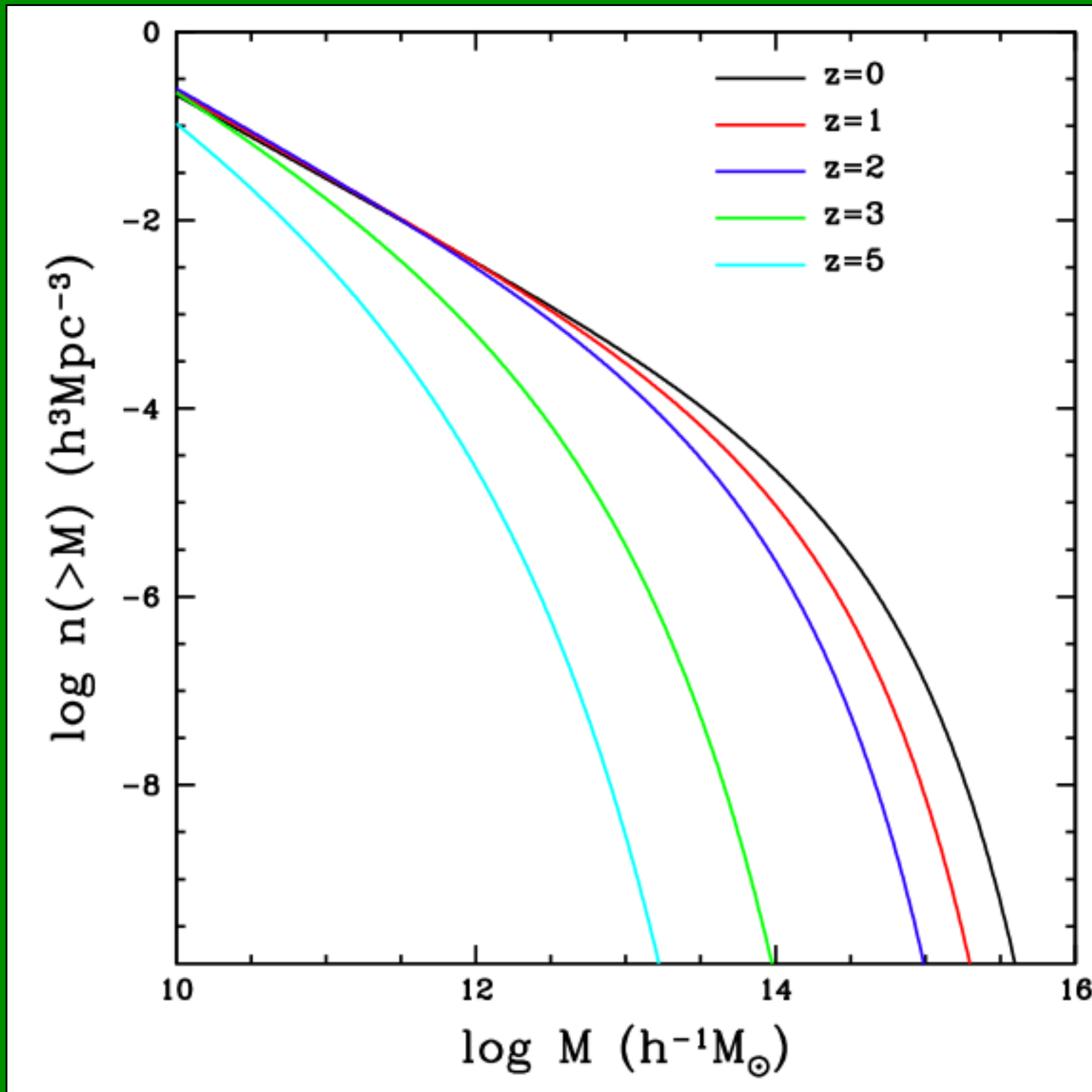
There are numerous improvements to the Press-Schechter mass function

Halo properties: Abundance (mass function)

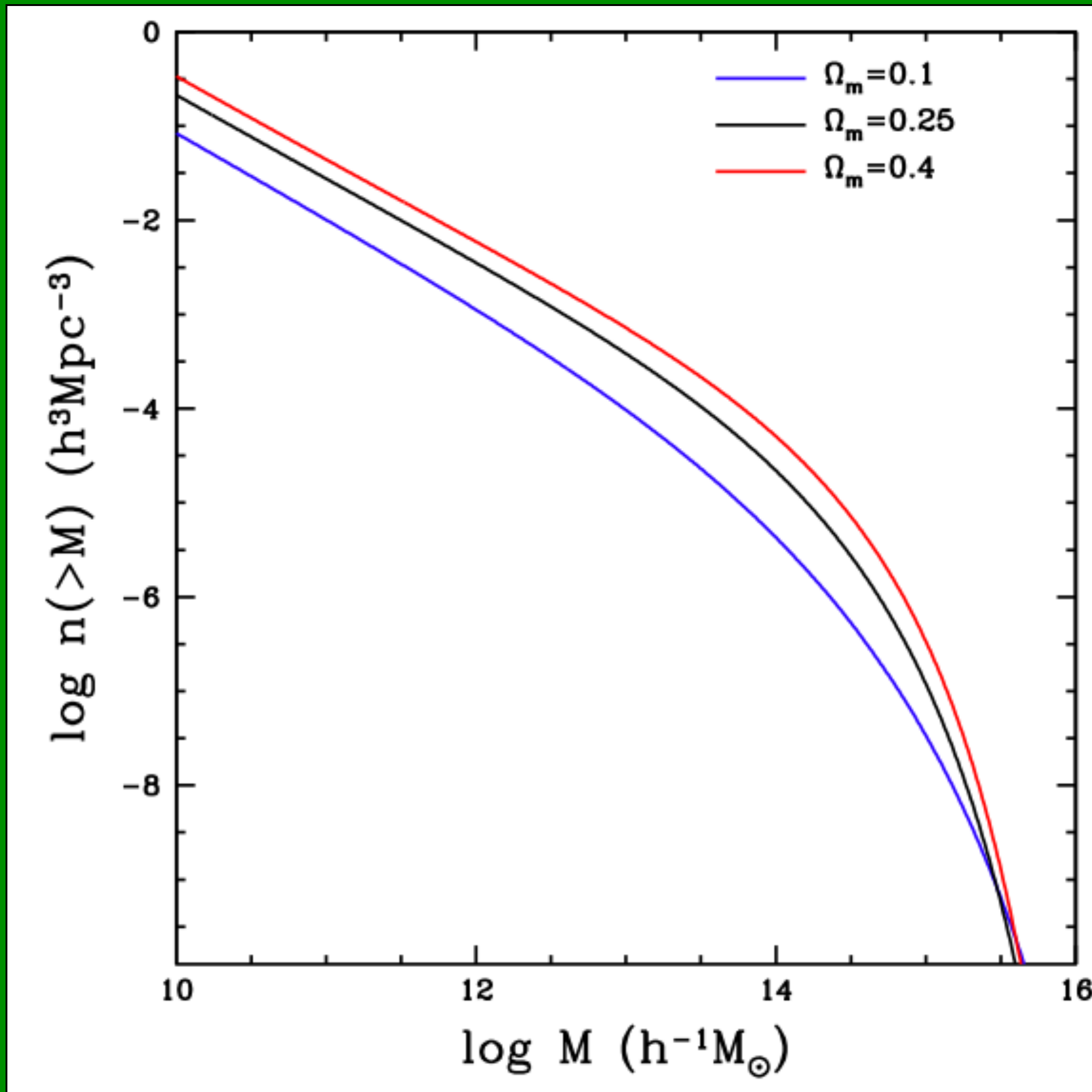


Cooray & Sheth (2002)

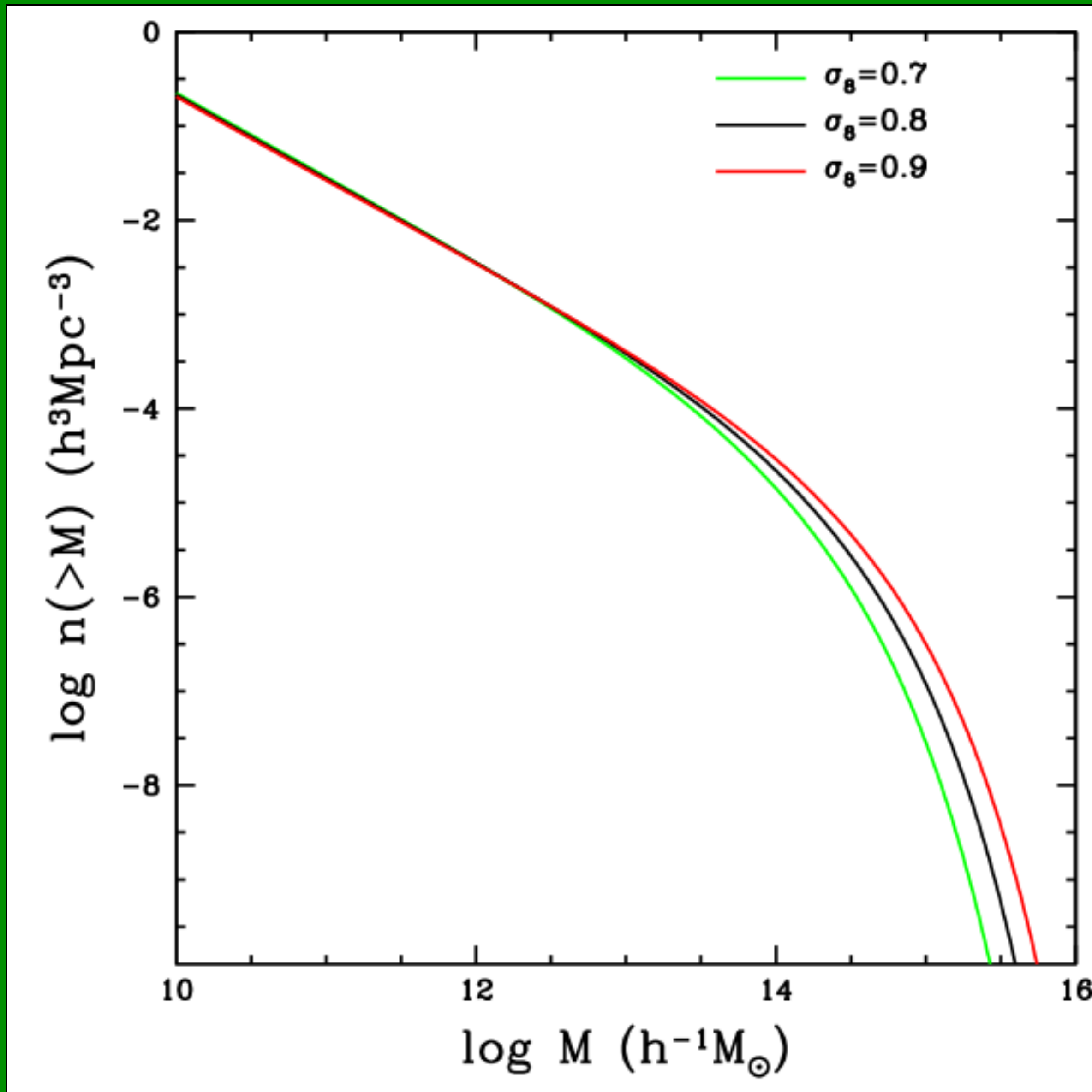
Halo properties: Abundance (mass function)



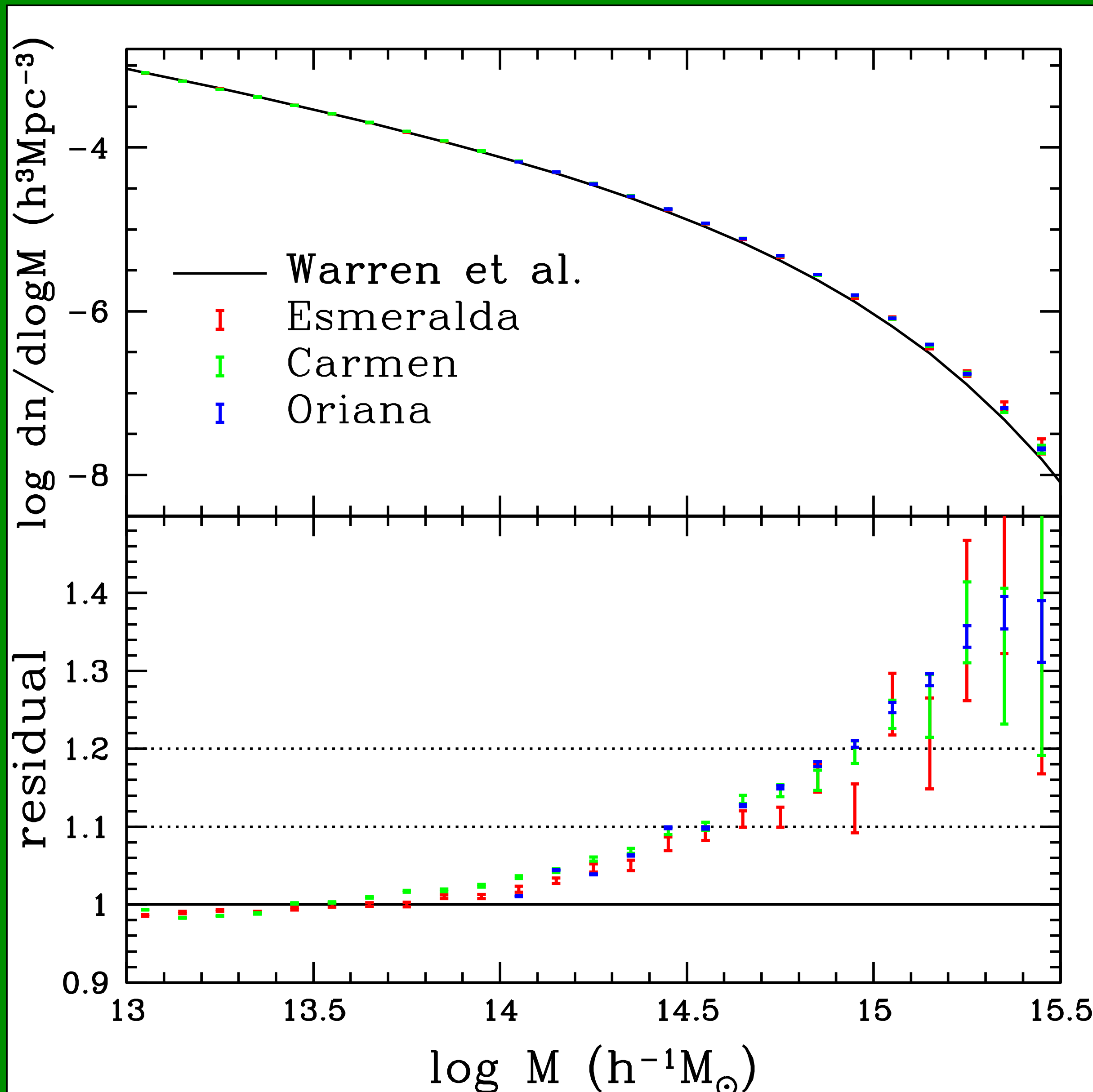
Halo properties: Abundance (mass function)



Halo properties: Abundance (mass function)

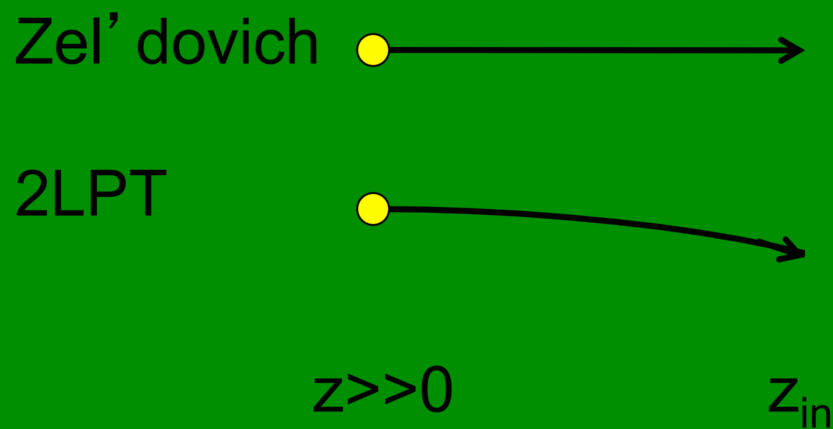


Halo properties: Abundance (mass function)

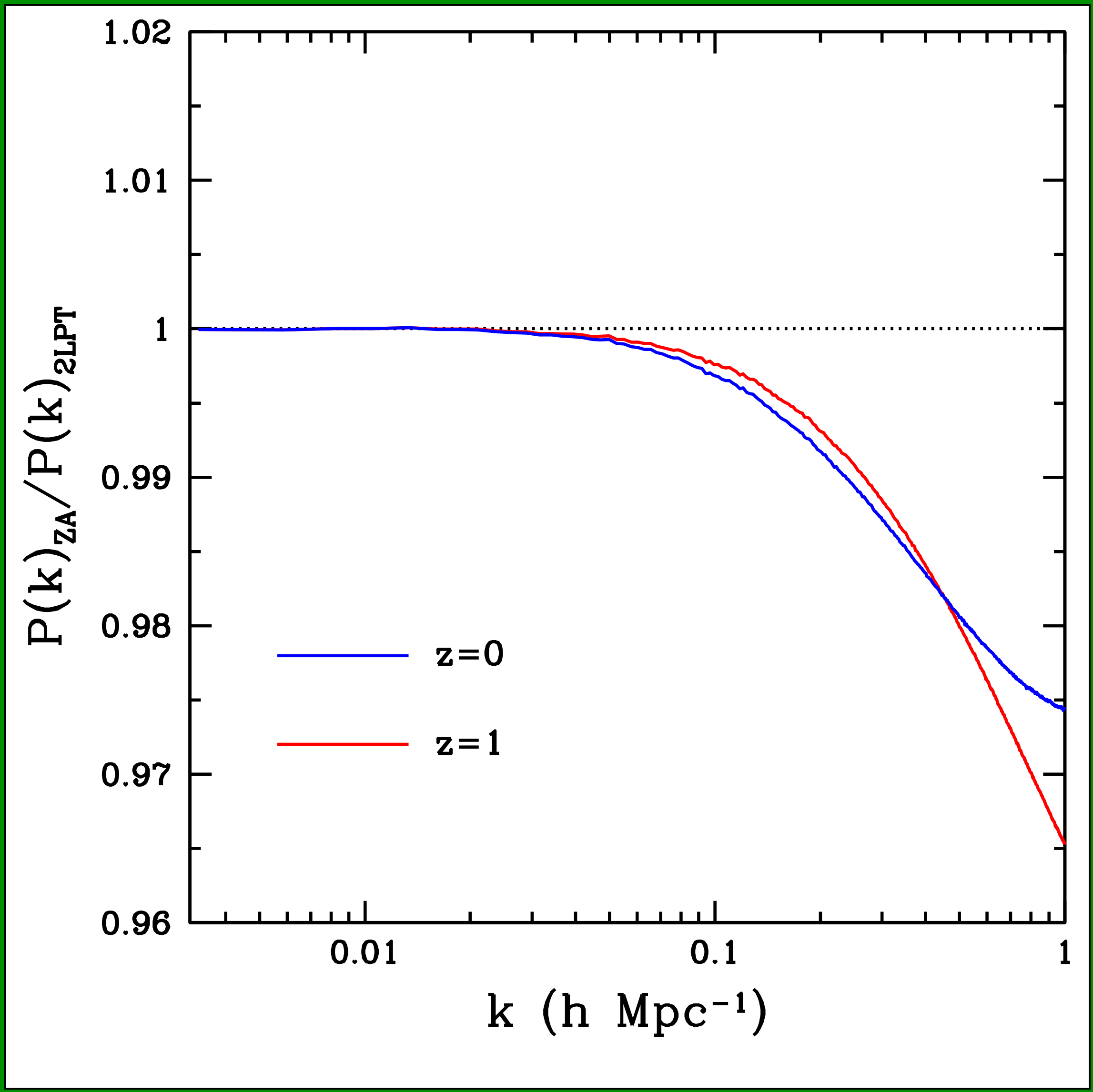


LasDamas

Halo properties: Abundance (mass function)

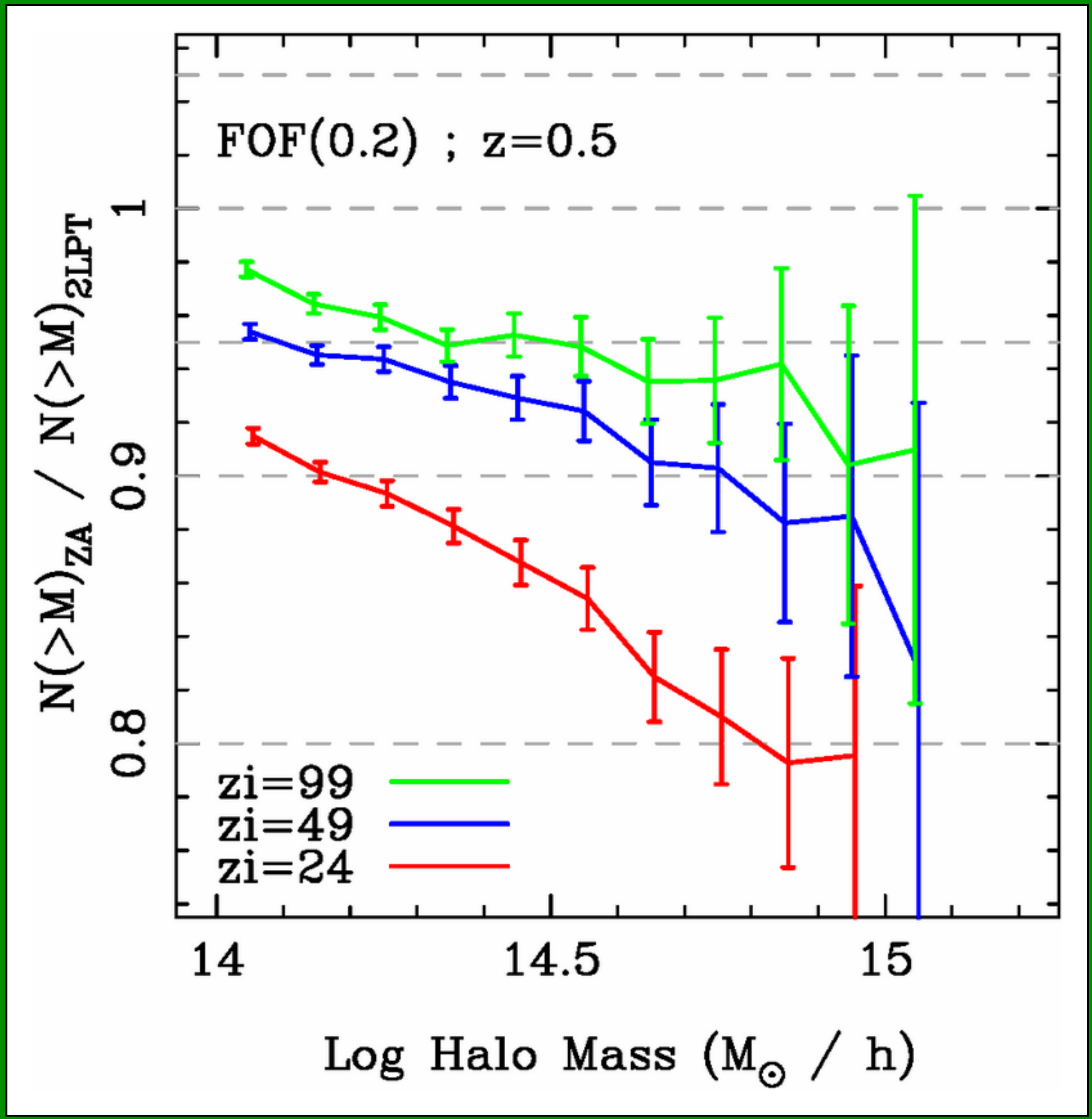


Halo properties: Abundance (mass function)



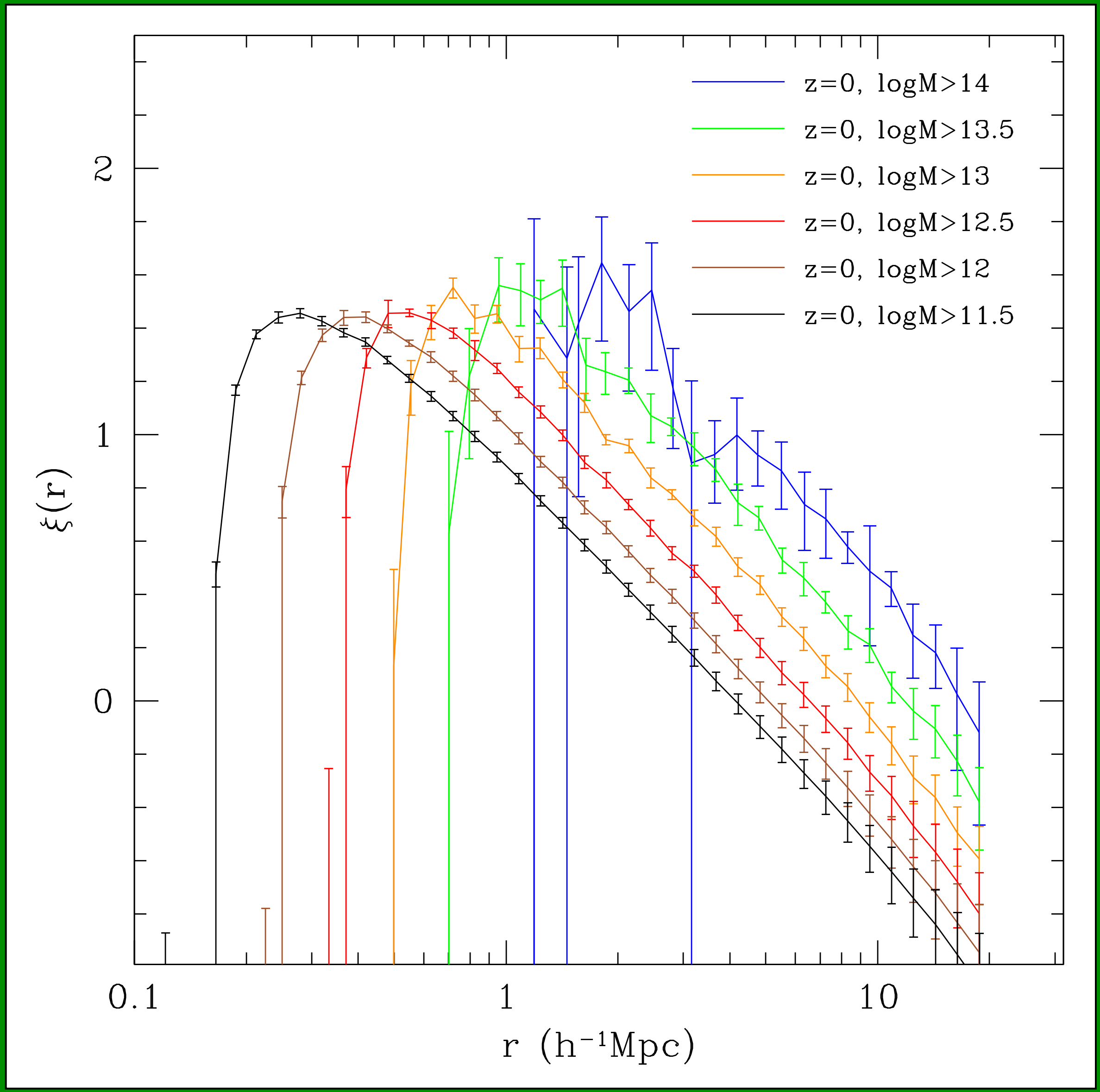
LasDamas

Halo properties: Abundance (mass function)



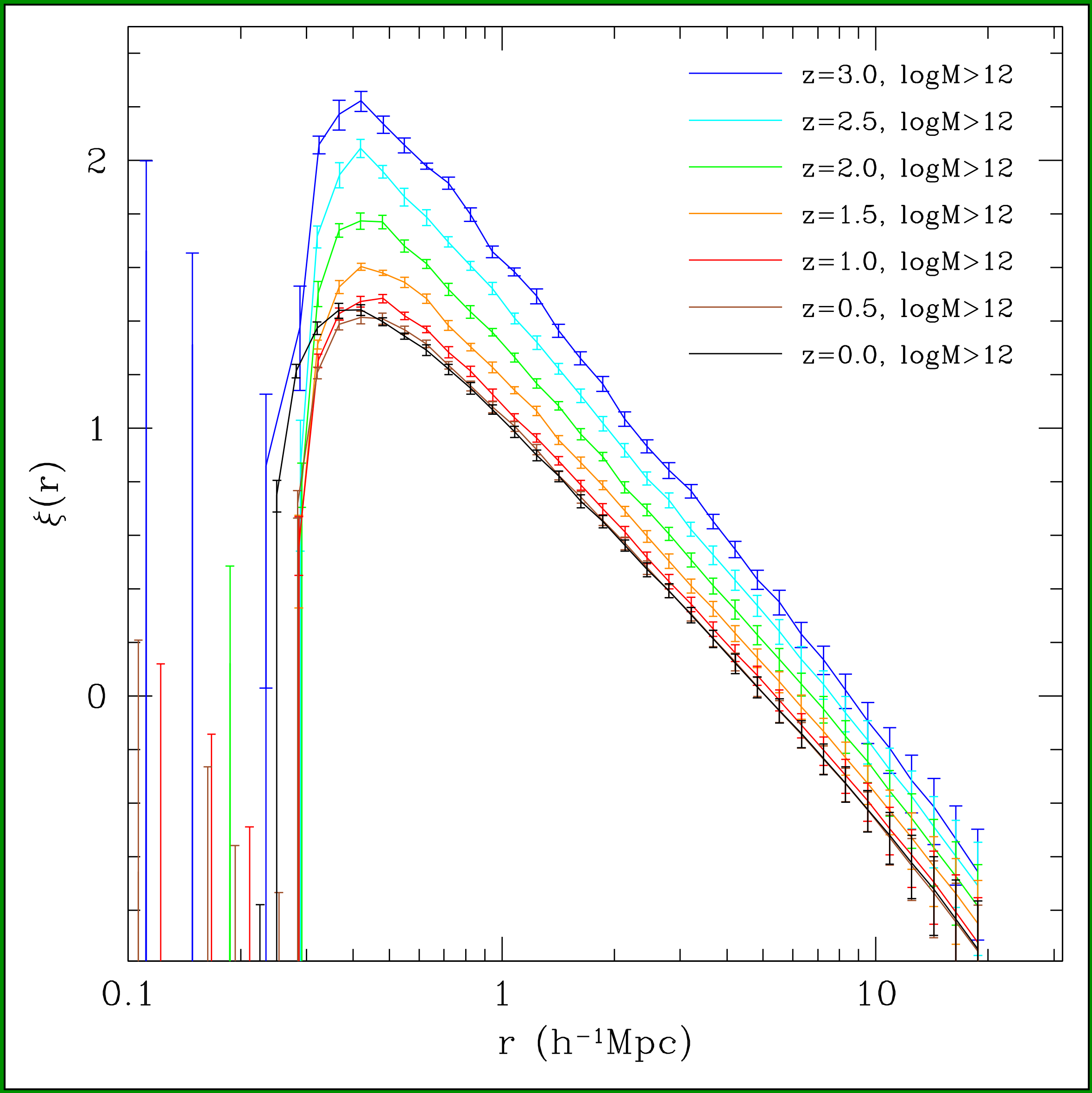
LasDamas

Halo properties: Clustering (halo bias)



LasDamas

Halo properties: Clustering (halo bias)

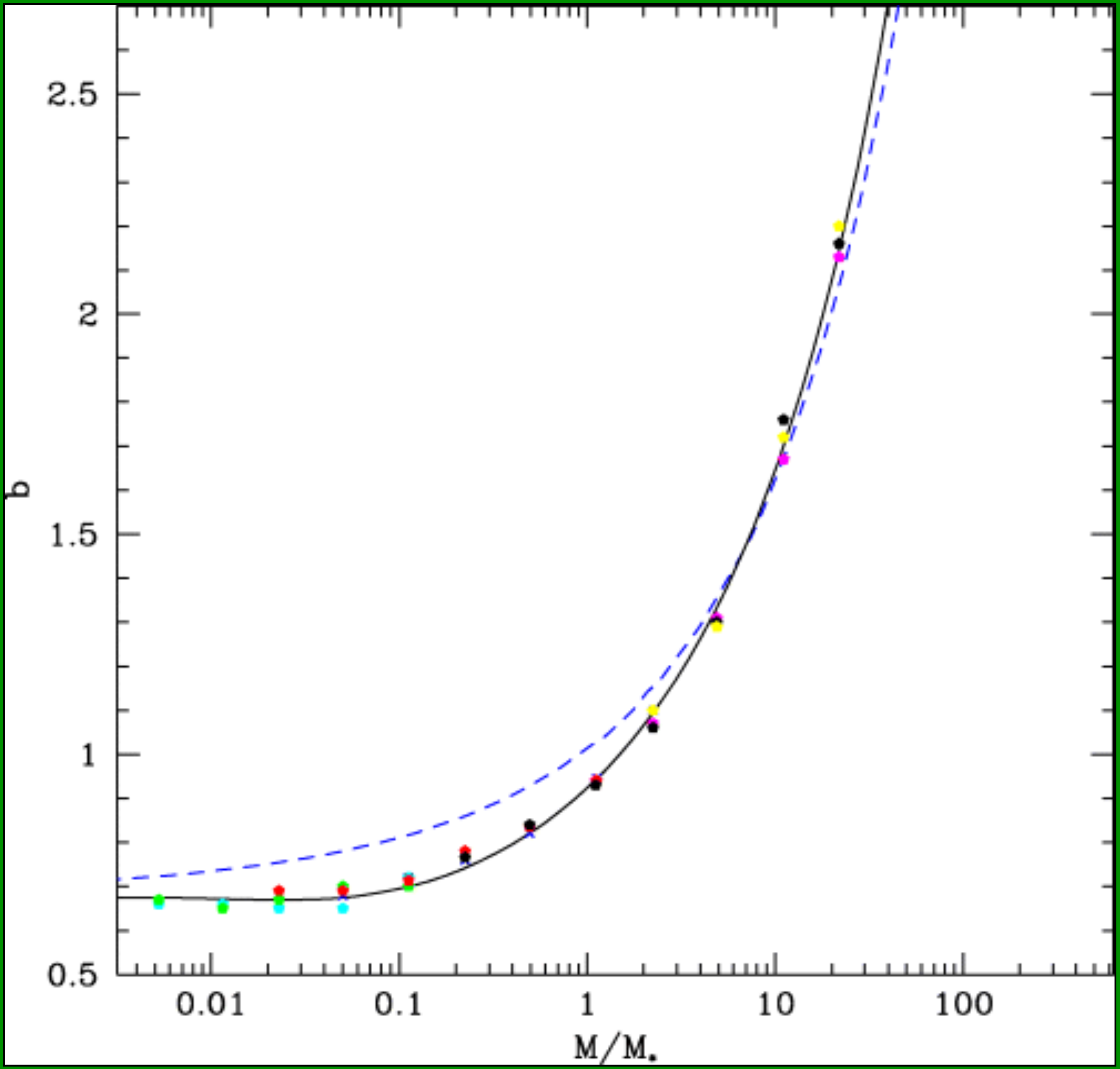


LasDamas

Halo properties: Clustering (halo bias)

Good to ~3% for standard cosmology.

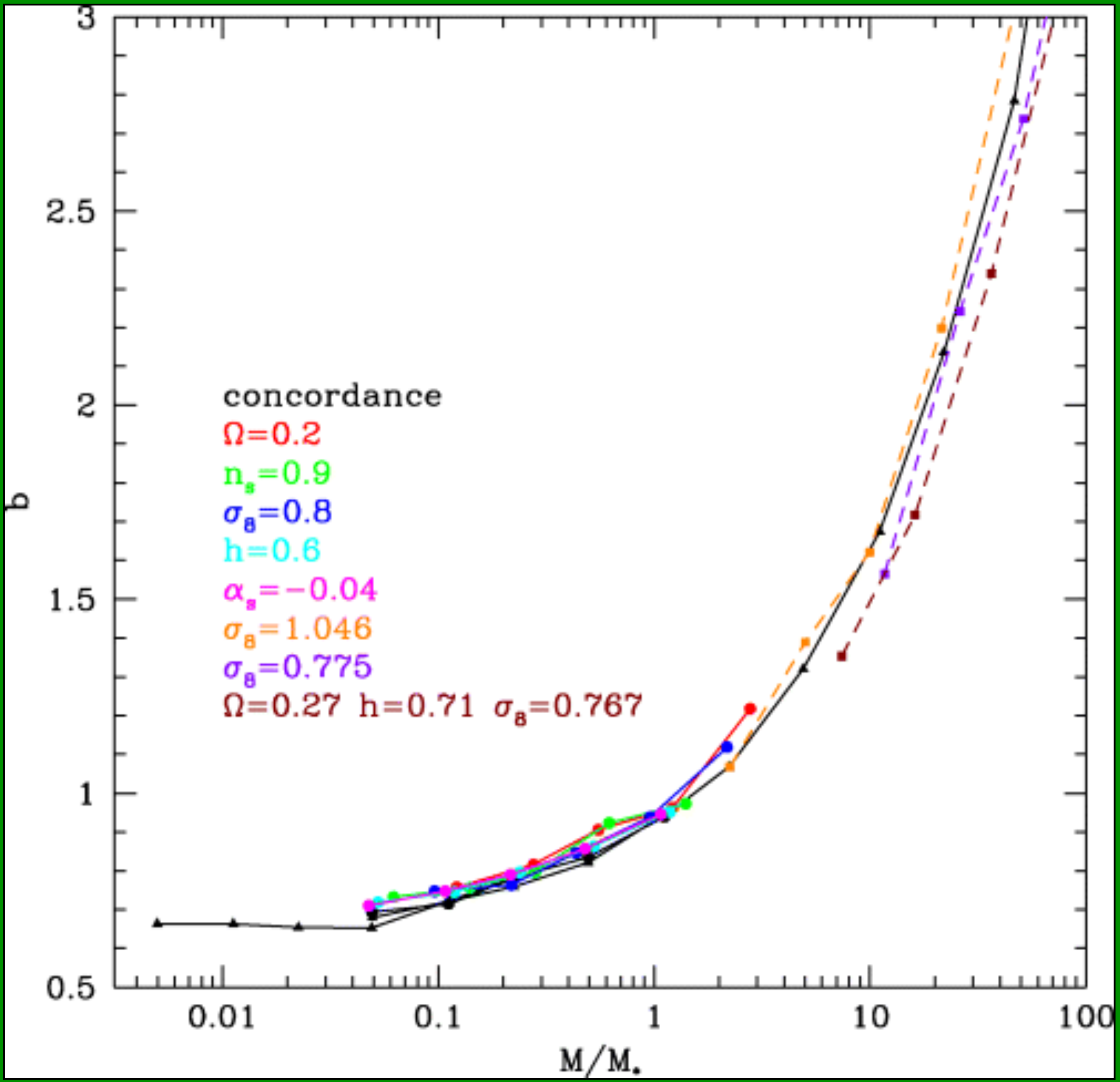
$$b_h = \sqrt{\frac{P_{hh}(k<0.1)}{P_{mm}(k<0.1)}}$$



Seljak & Warren (2005)

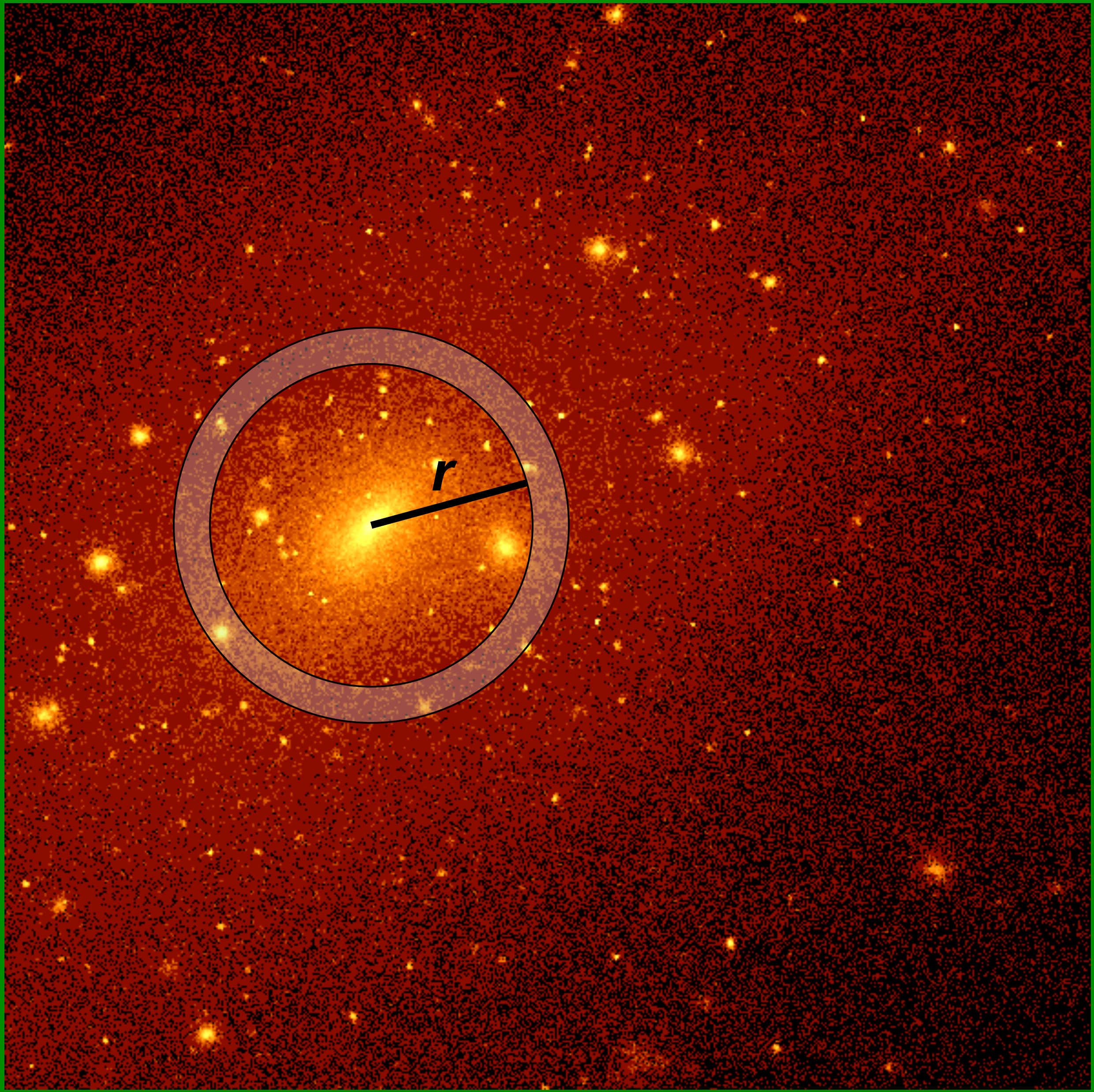
Halo properties: Clustering (halo bias)

Good to ~10% across different cosmologies.

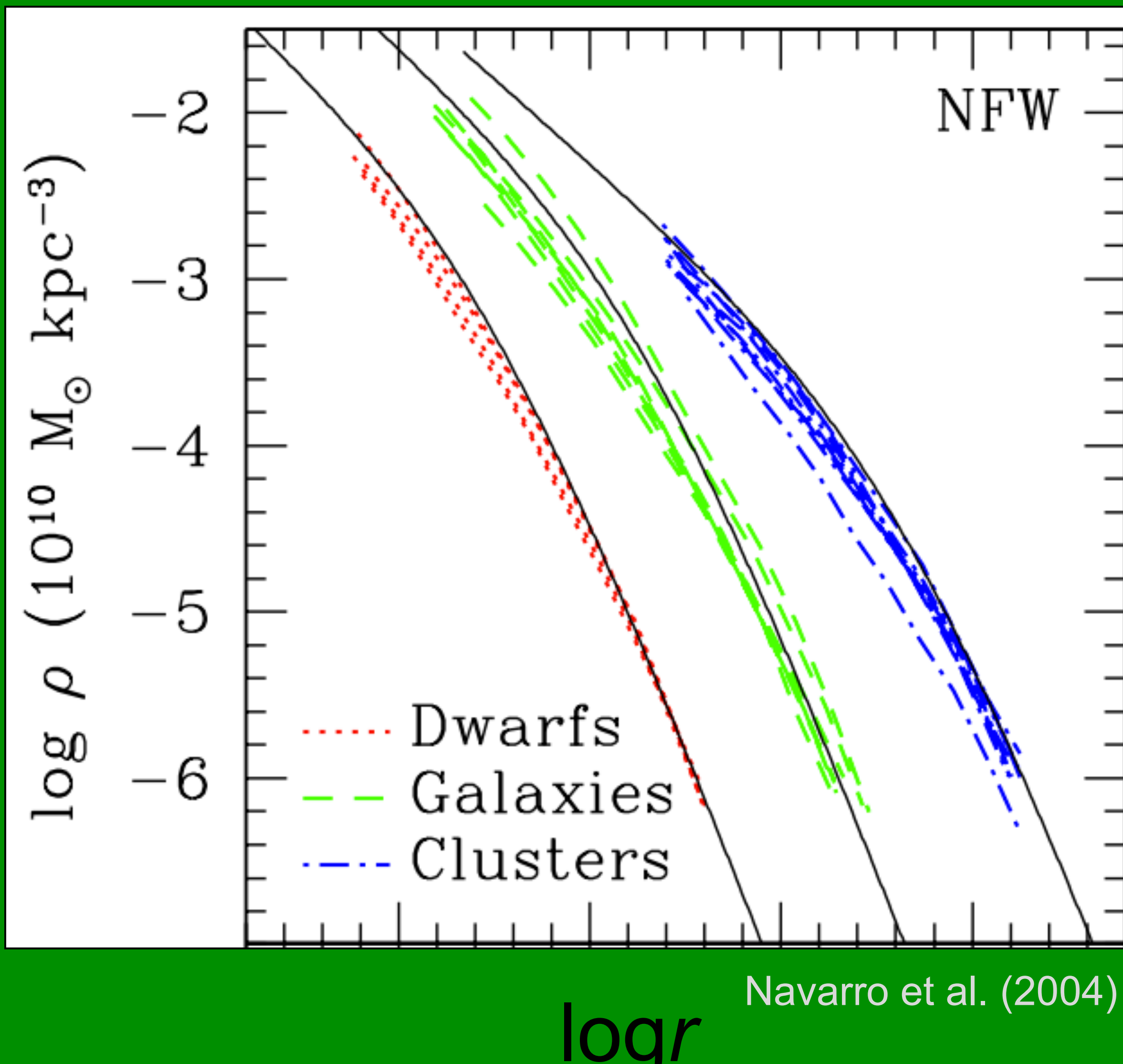


Seljak & Warren (2005)

Halo properties: Structure (density profile)



Halo properties: Structure (density profile)



Navarro et al. (2004)

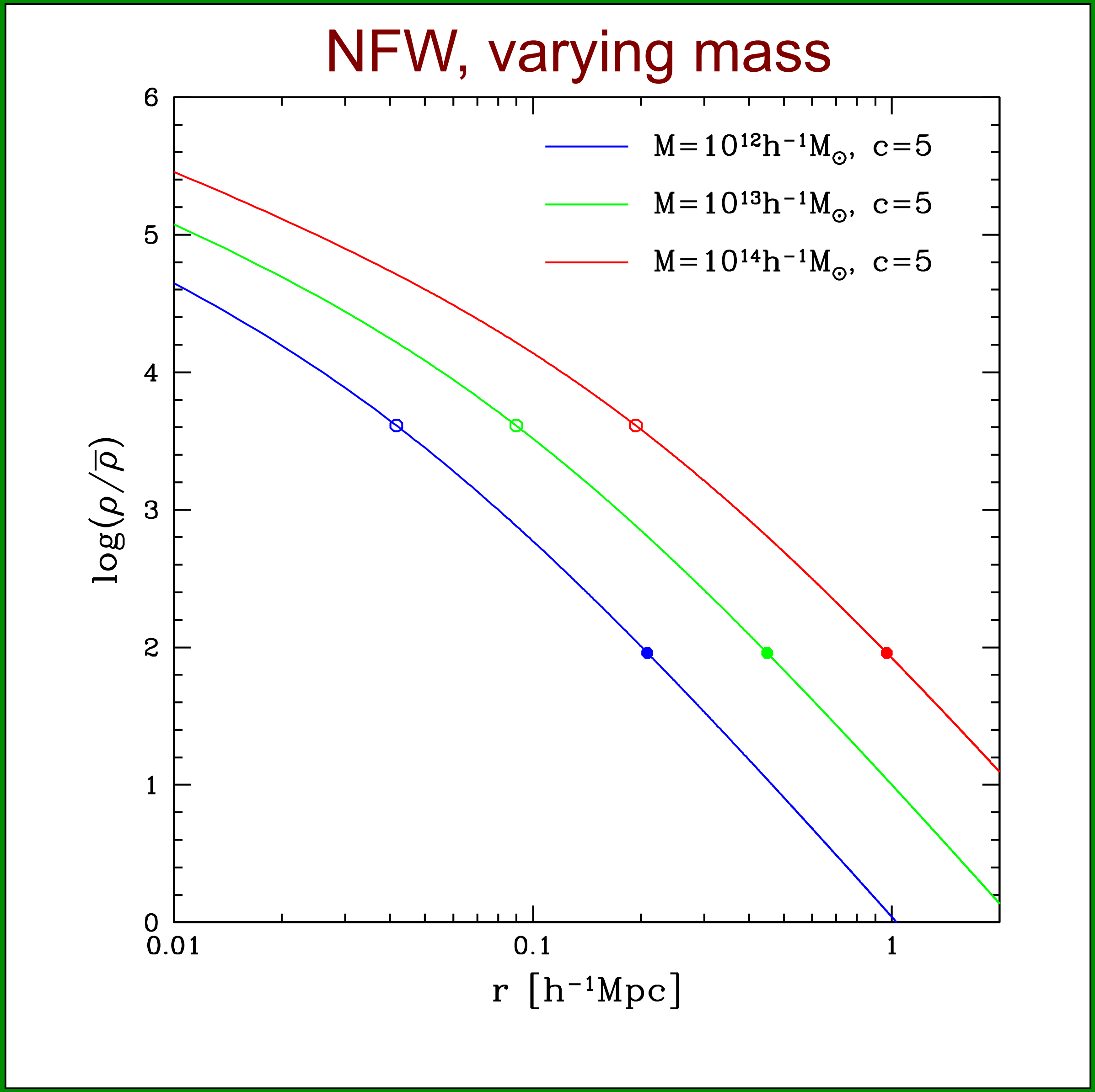
Navarro, Frenk & White (NFW)

$$\rho(r) = \frac{\rho_s}{(1 + r/r_s)^2 (r/r_s)}$$

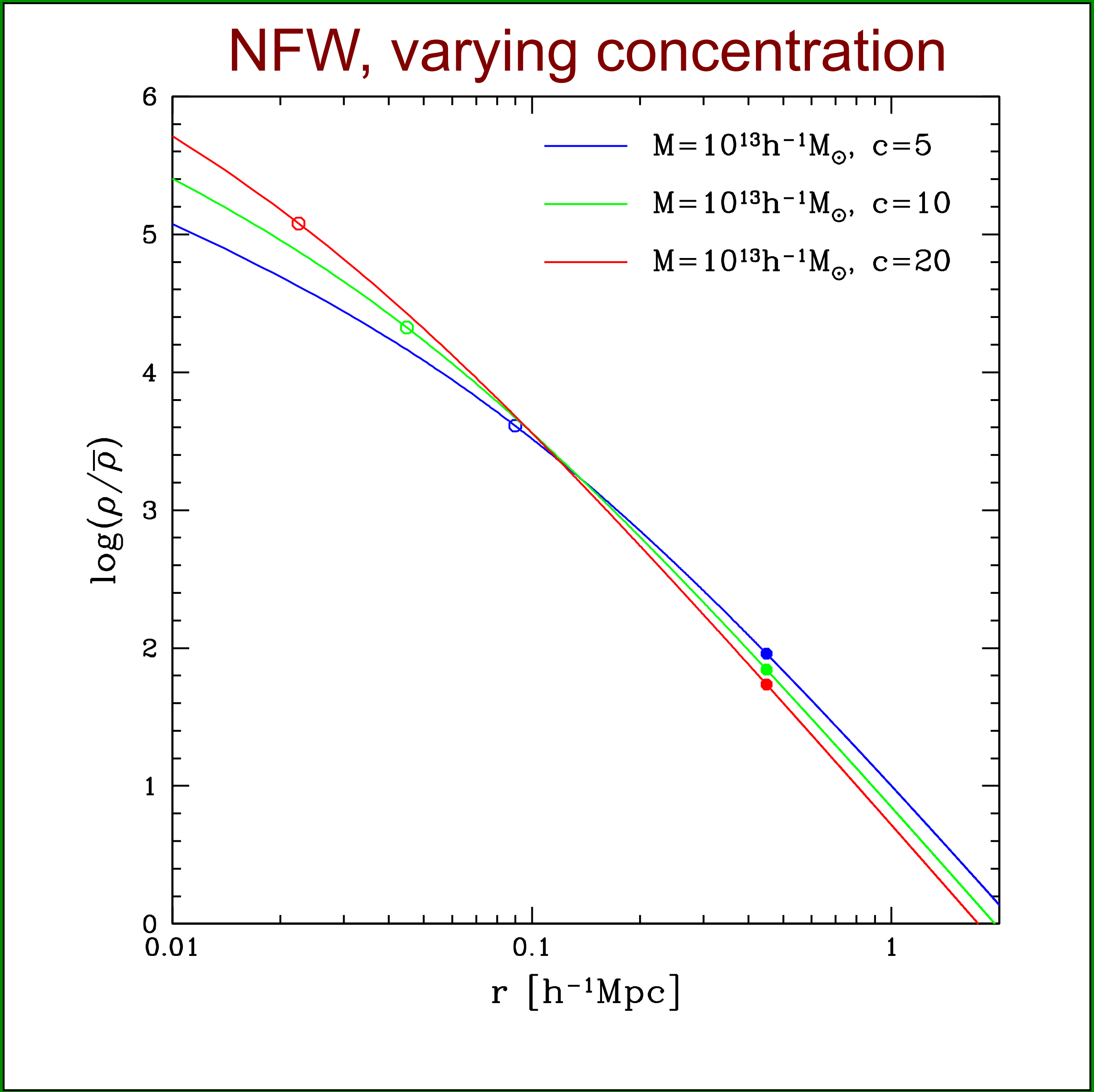
$$(\rho_s, r_s) \Leftrightarrow (M_{vir}, c \equiv R_{vir}/r_s)$$

$$M_{vir} = \frac{4}{3}\pi R_{vir}^3 \Delta_{vir} \bar{\rho}$$

Halo properties: Structure (density profile)

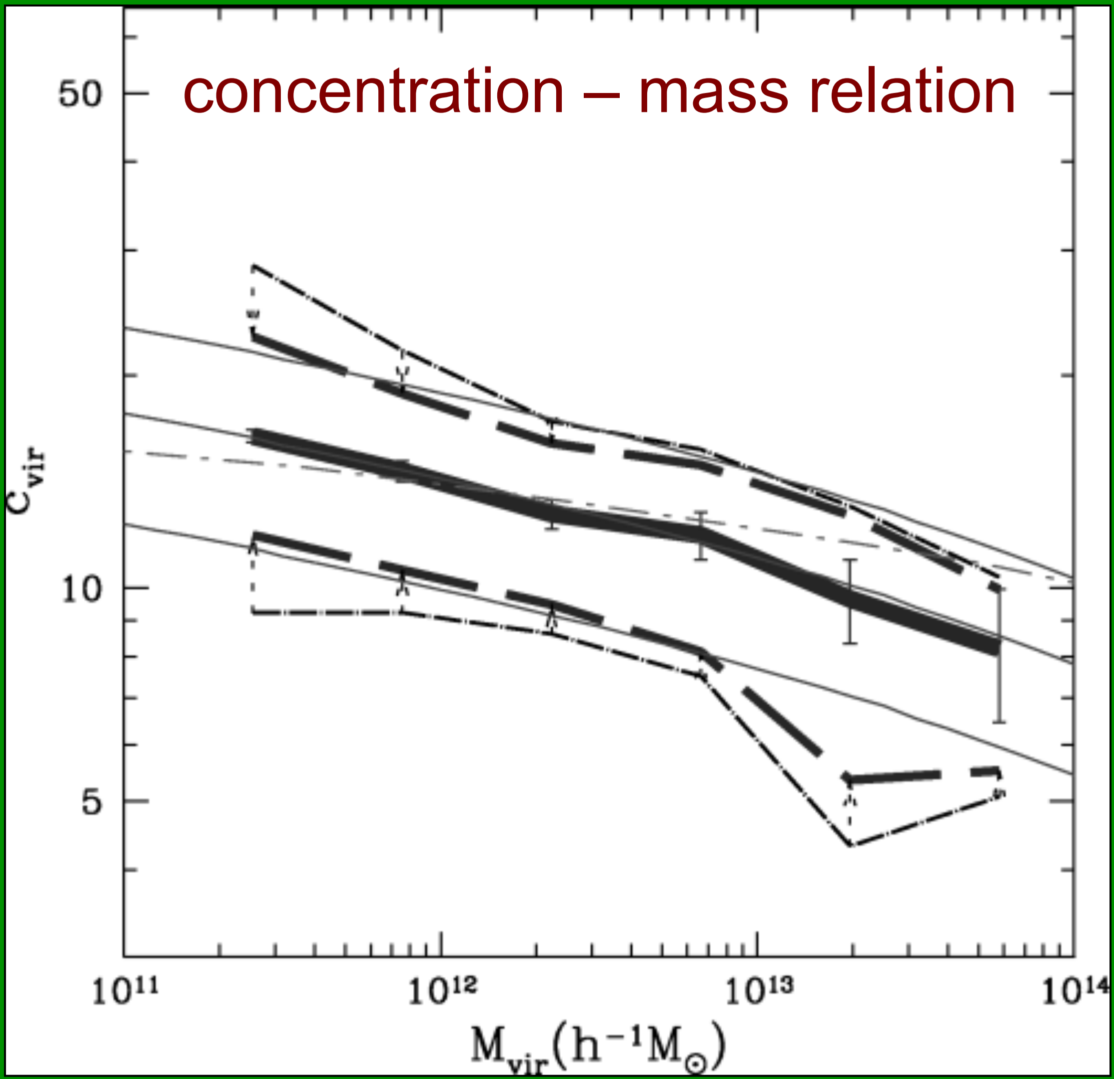


Halo properties: Structure (density profile)



Halo properties: Structure (density profile)

c



M_{vir}

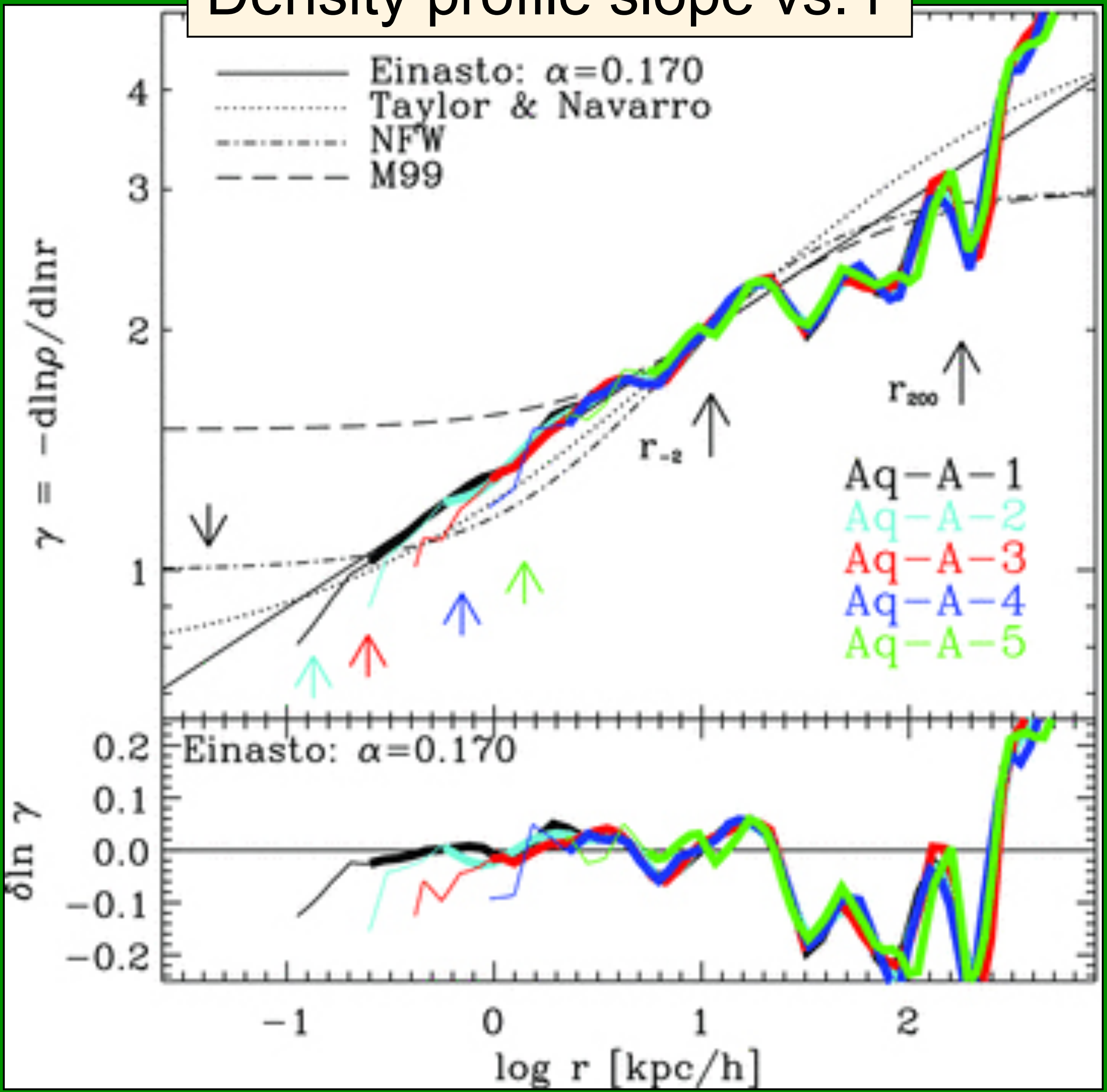
Bullock et al. (2001)

$$c \equiv R_{vir}/r_s$$

$$c \approx \frac{c_*}{(1+z)}\left(M/M_*\right)^{-0.13}$$

Halo properties: Structure (density profile)

Density profile slope vs. r



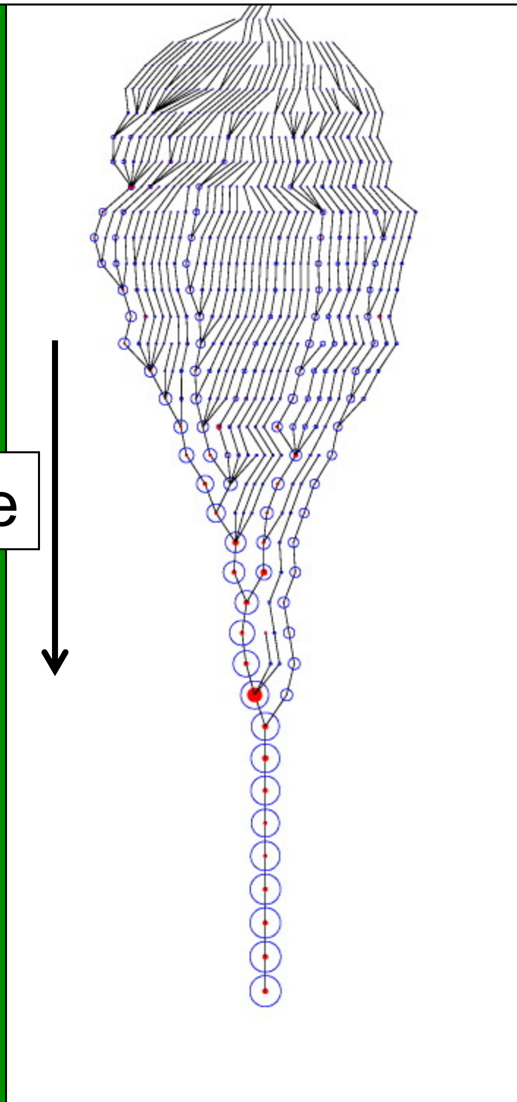
Navarro et al. (2010)

Einasto profile

$$\rho = \rho_{-2} e^{-2/\alpha} e^{\left[\left(r/r_{-2}\right)^{\alpha} - 1\right]}$$

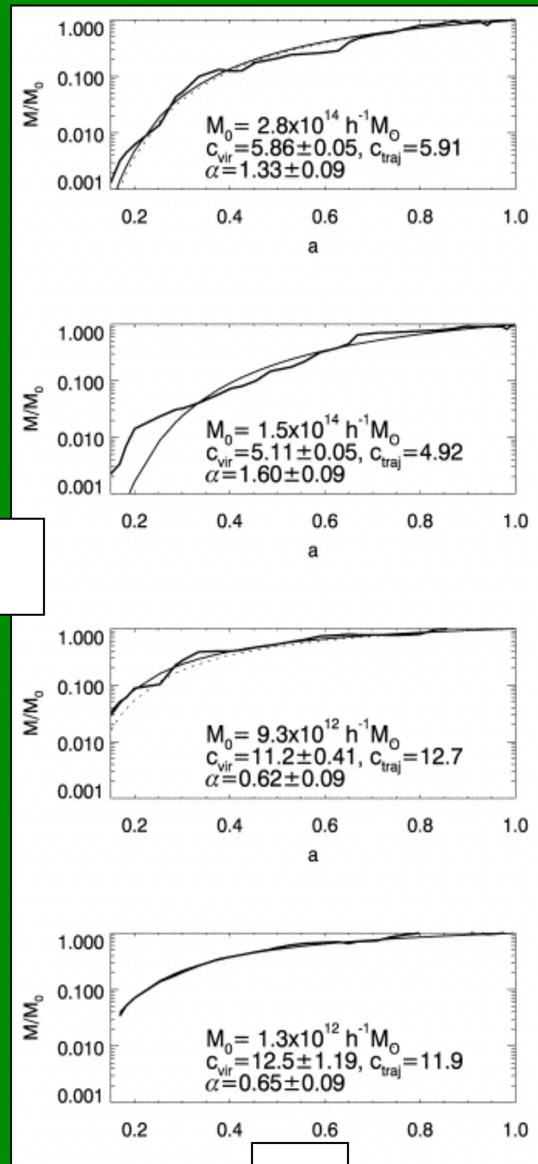
Halo properties: History (merger tree)

Assembly History



time

M

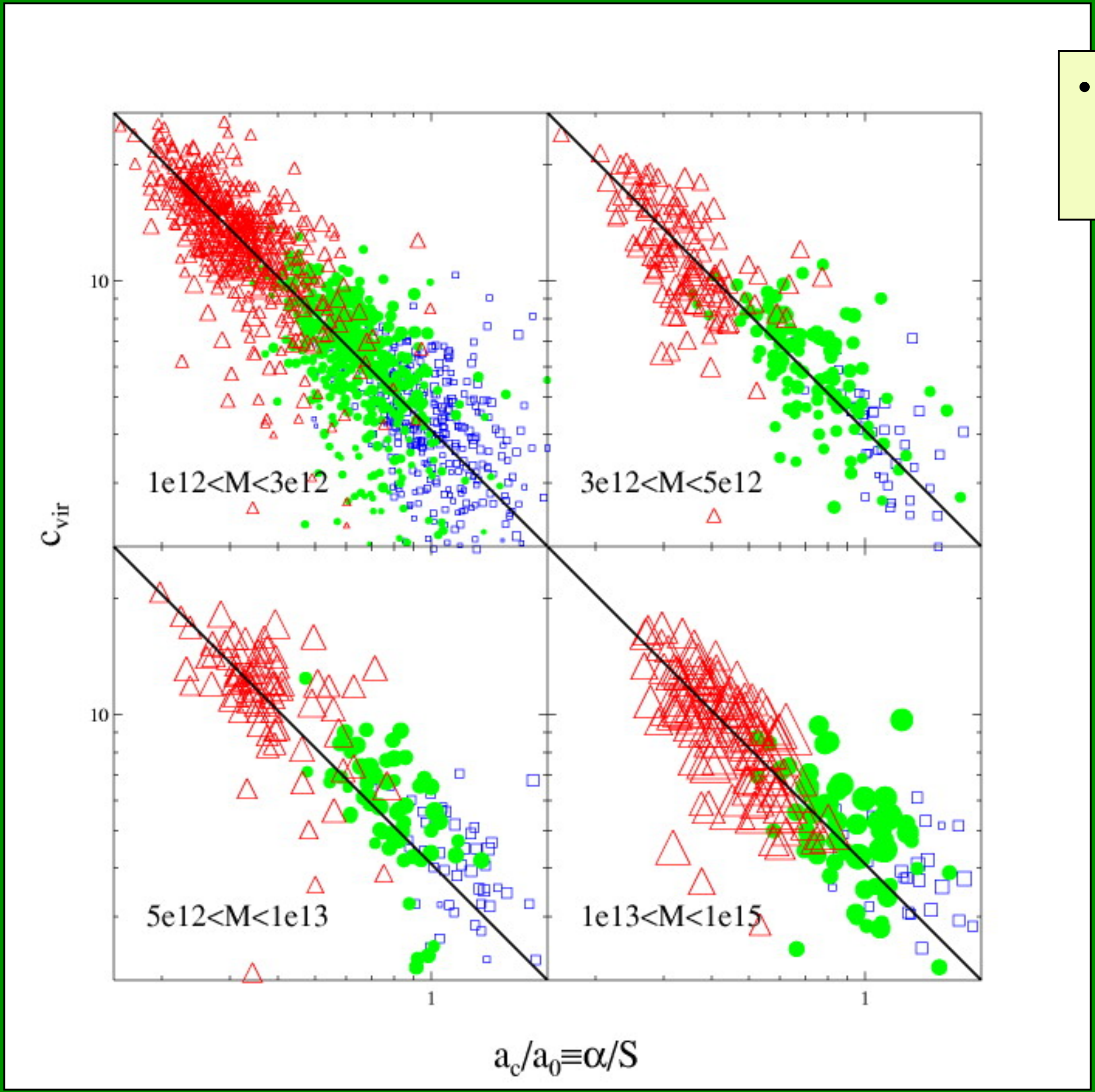


a

- High mass halos have accreted more of their mass recently relative to low mass halos.

Wechsler et al. (2002)

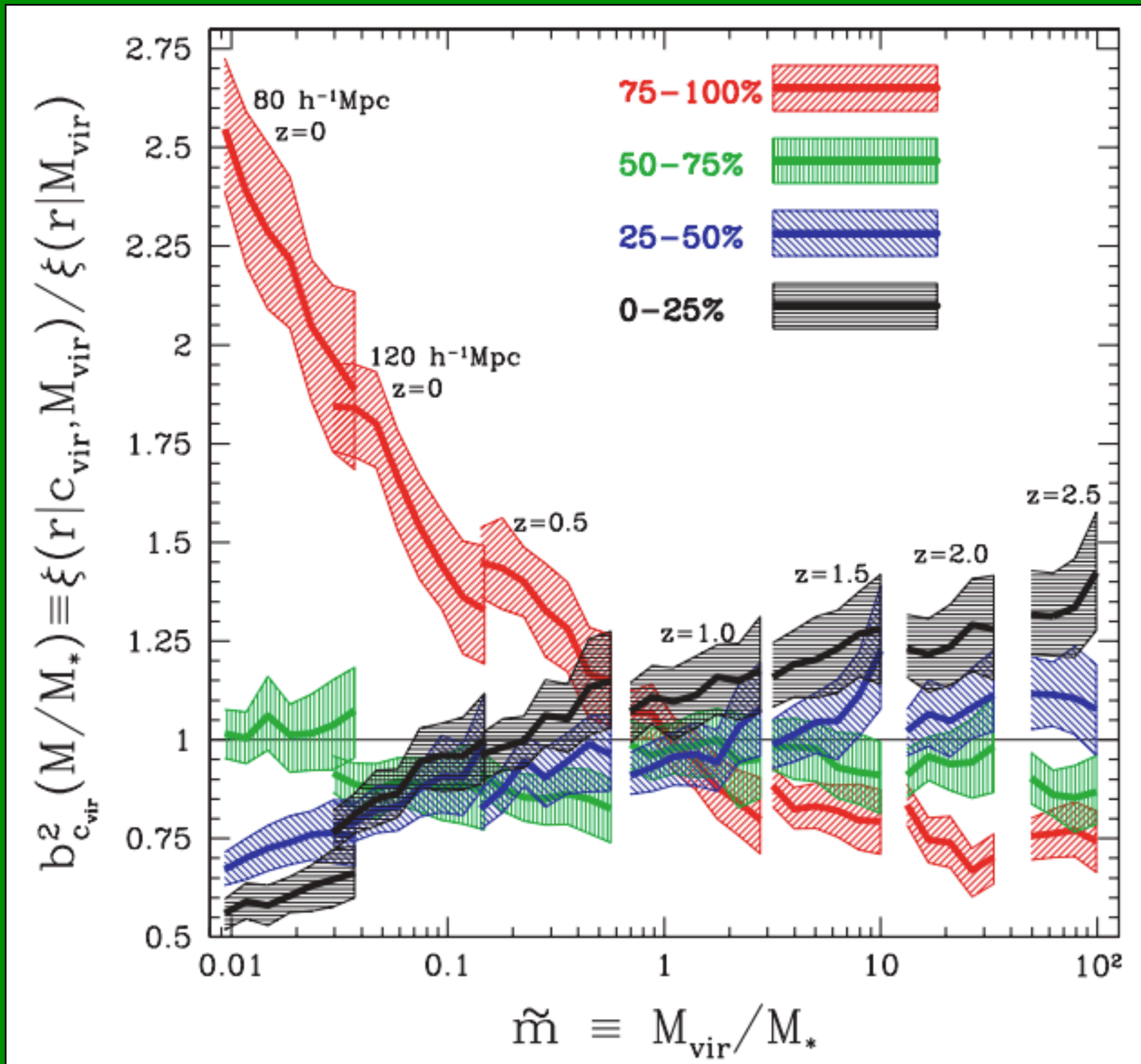
Halo properties: History (merger tree)



Wechsler et al. (2002)

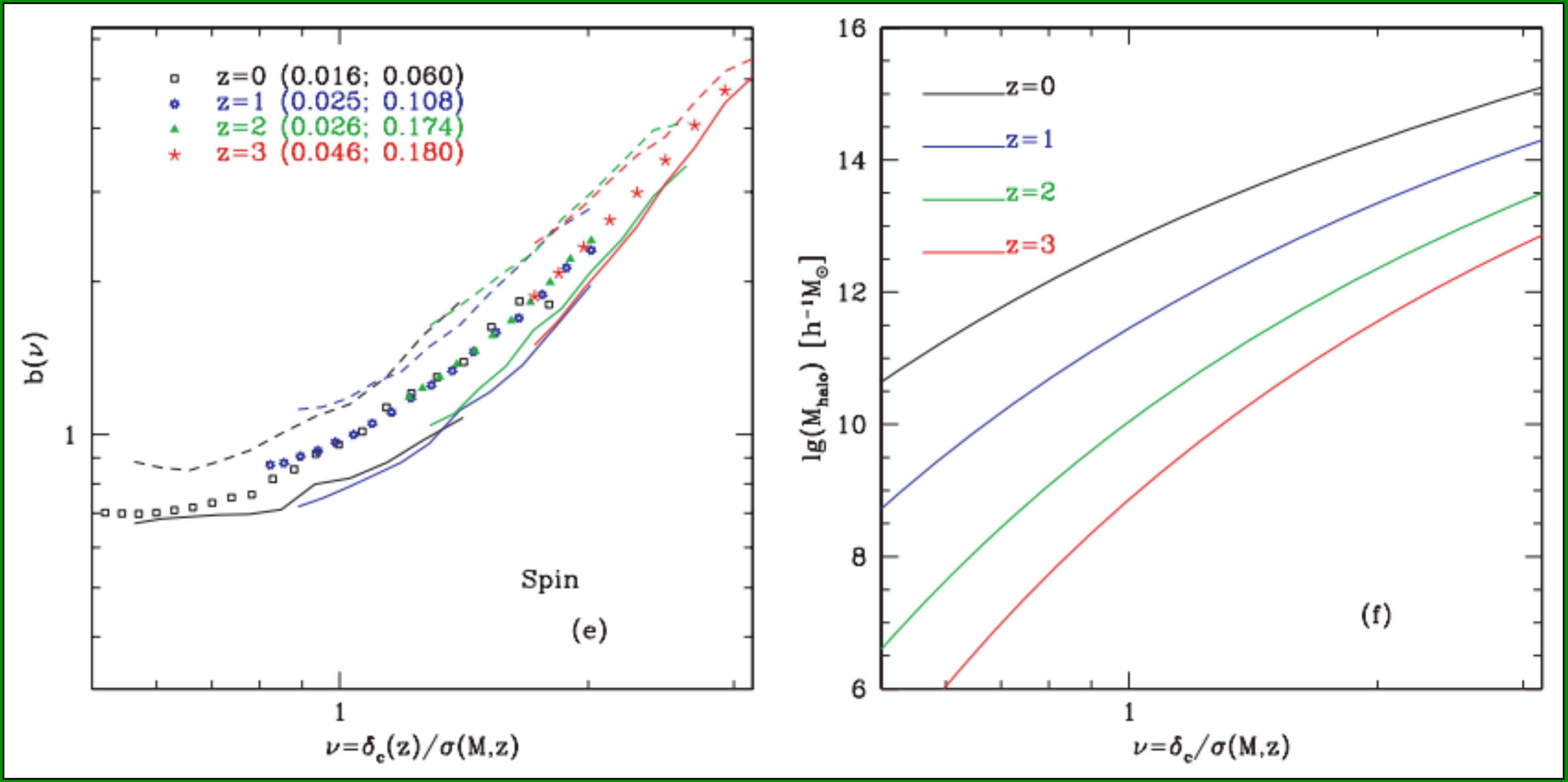
- Halo concentrations are determined by their accretion history.

Halo properties: Assembly Bias



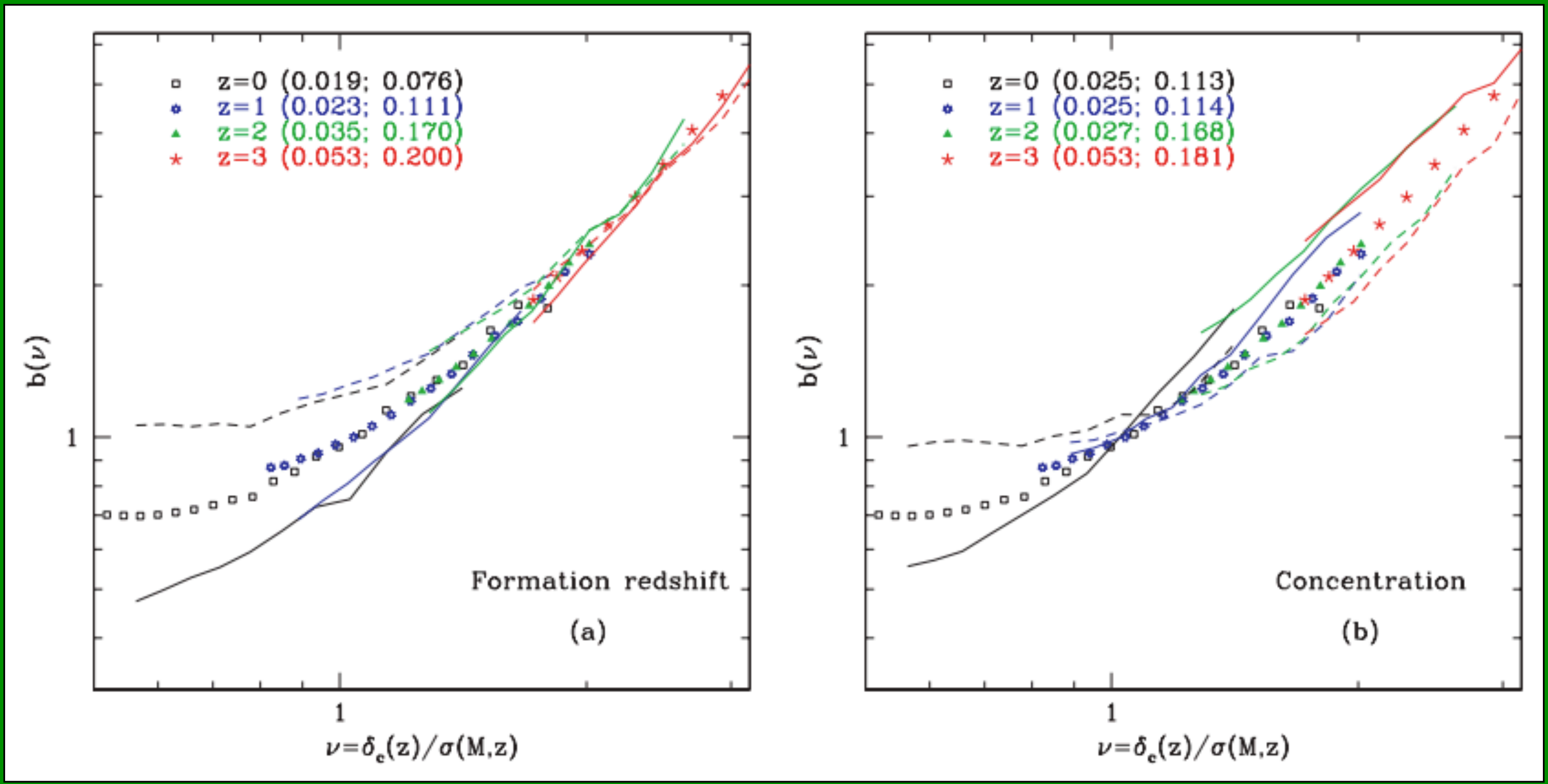
Wechsler et al. (2006)

Halo properties: Assembly Bias



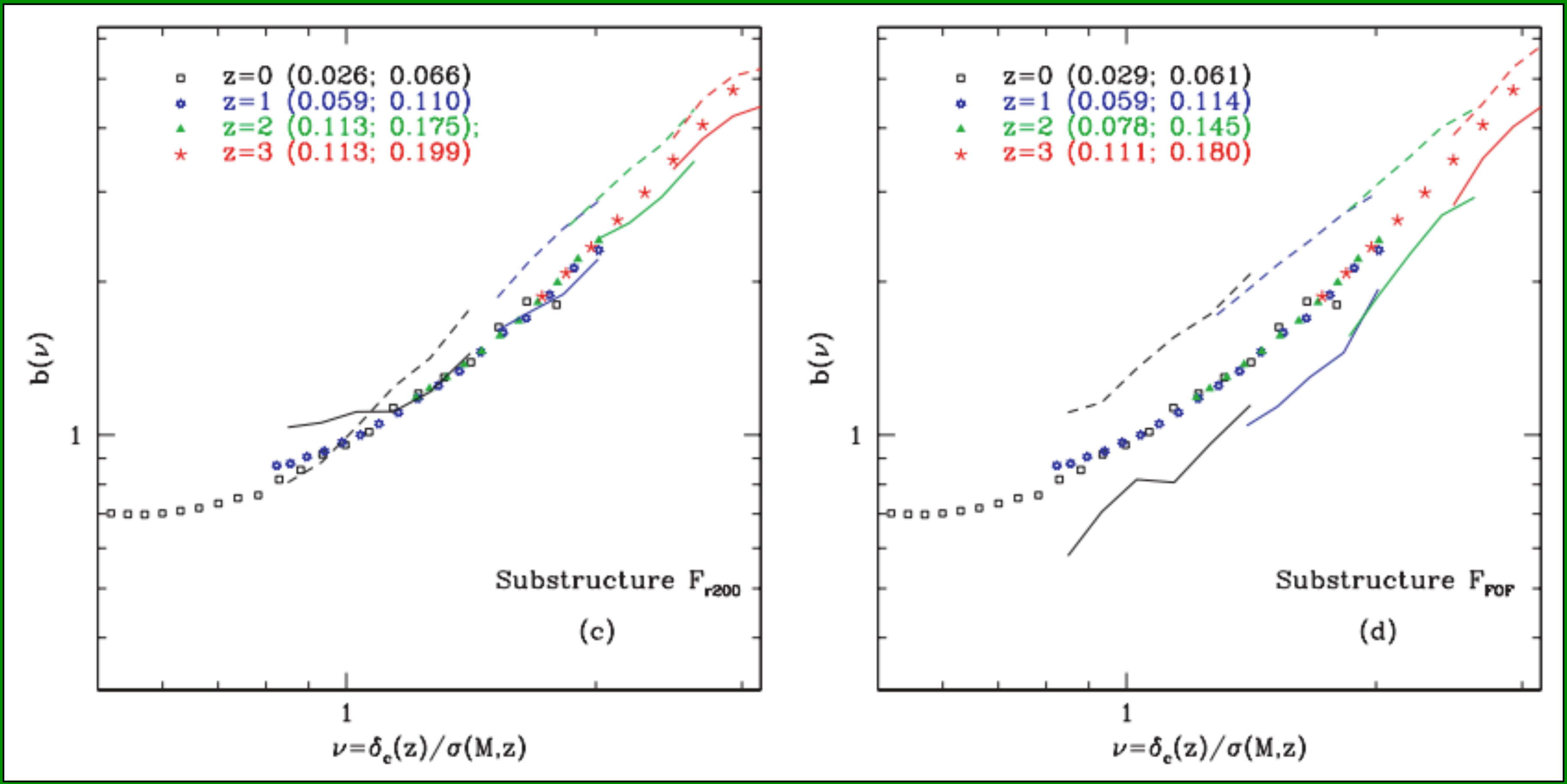
Gao & White (2007)

Halo properties: Assembly Bias



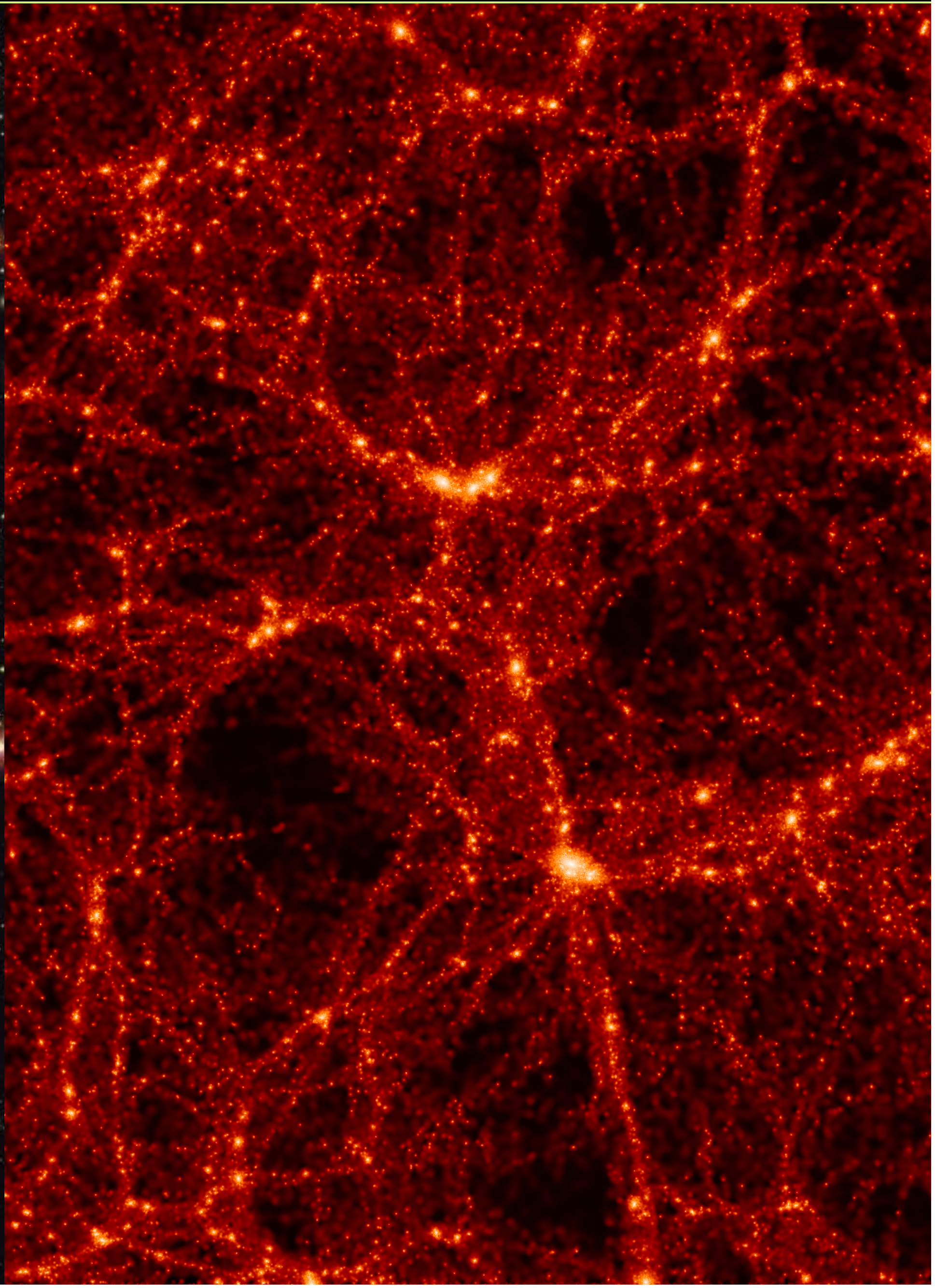
Gao & White (2007)

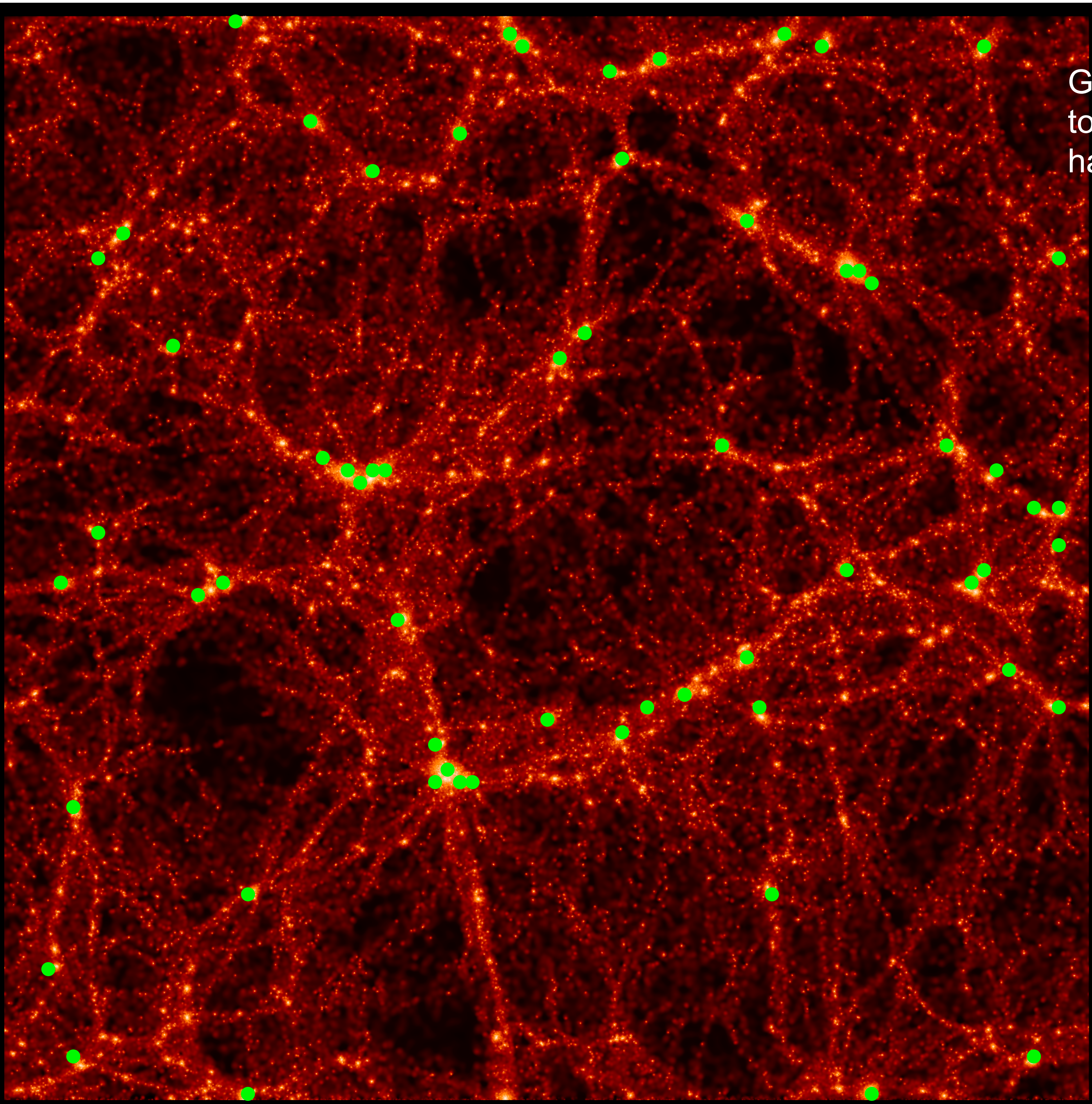
Halo properties: Assembly Bias



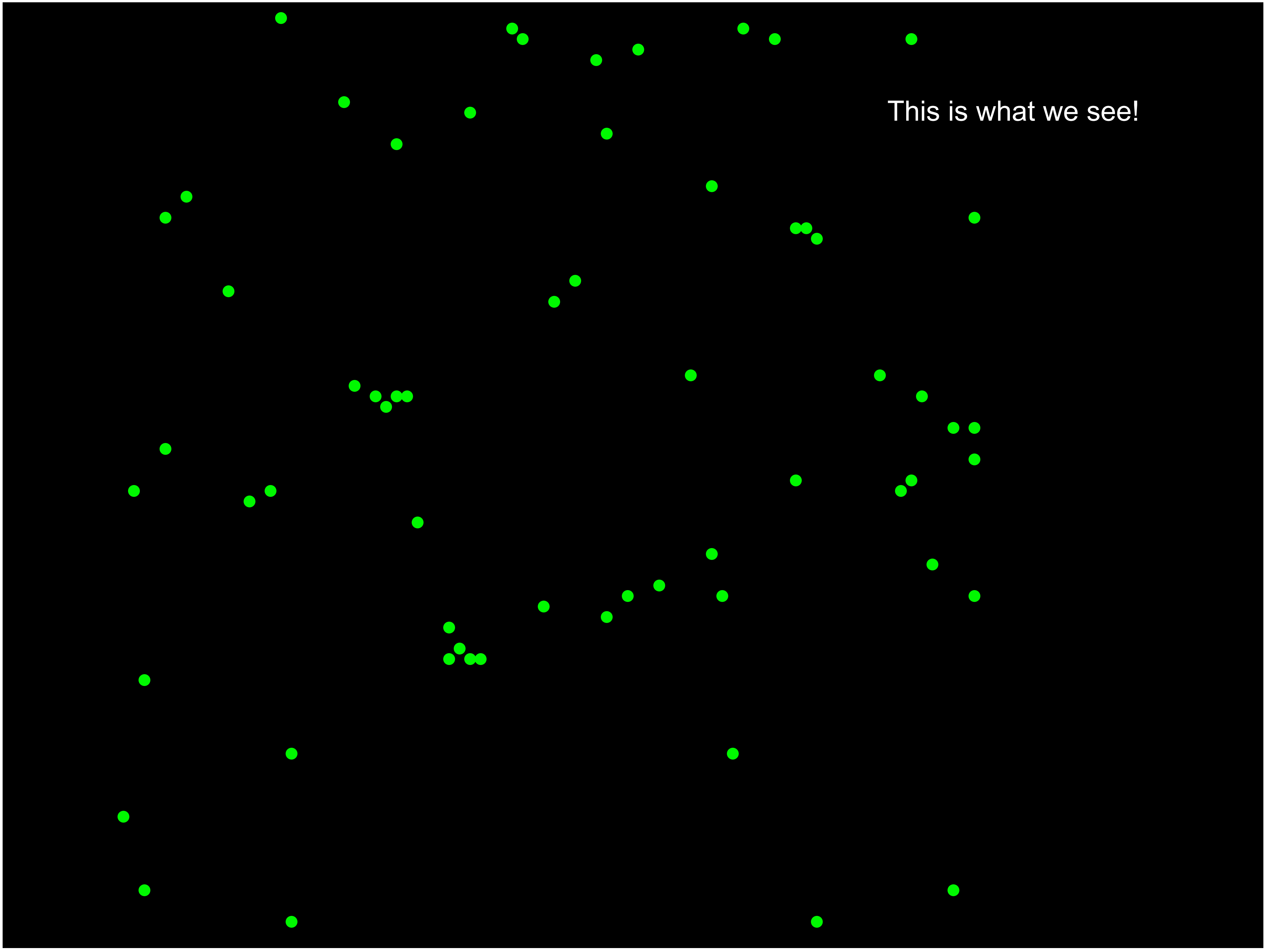
Gao & White (2007)

From dark matter to galaxies



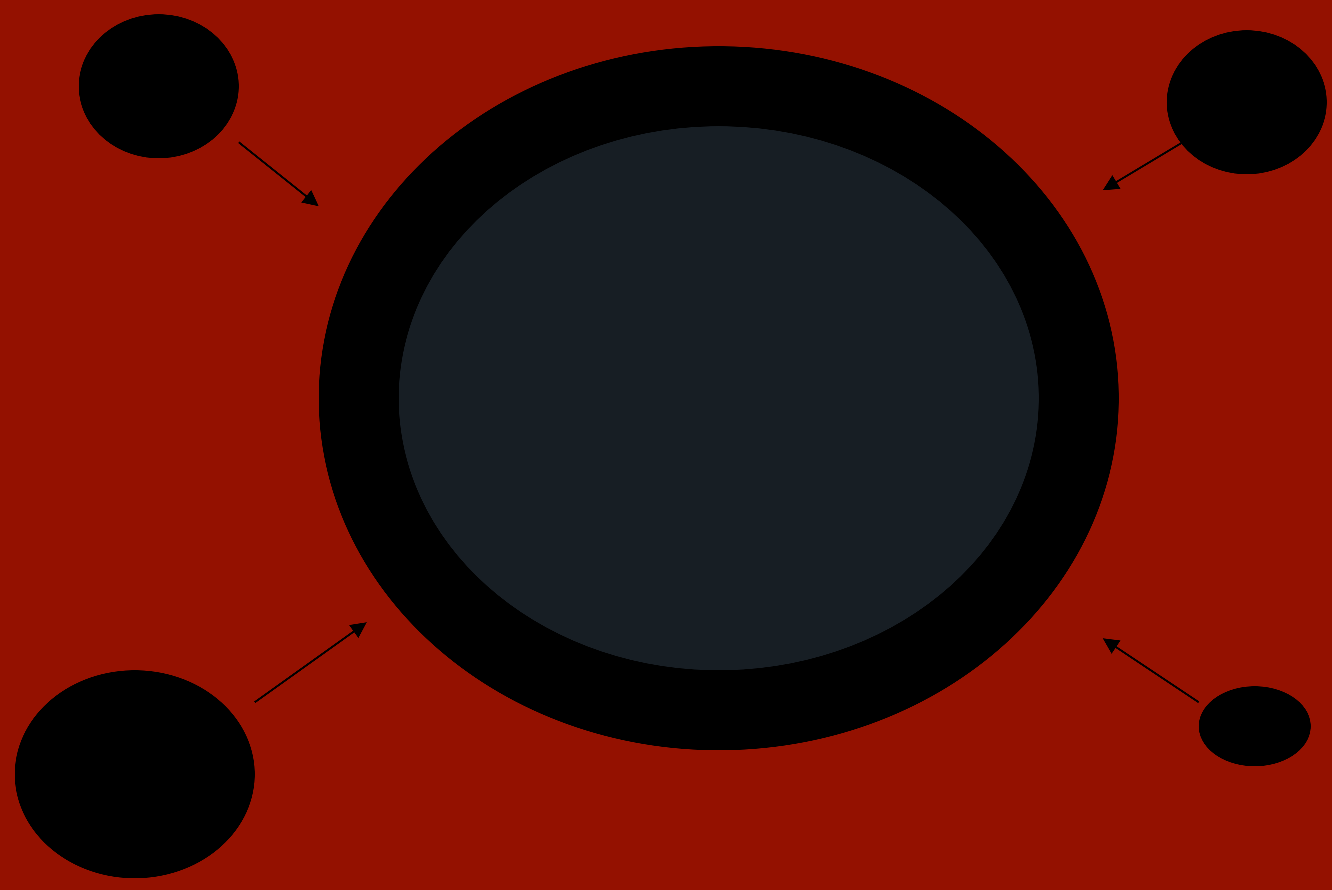


Galaxies are assumed to form in dark matter halos.


This is what we see!

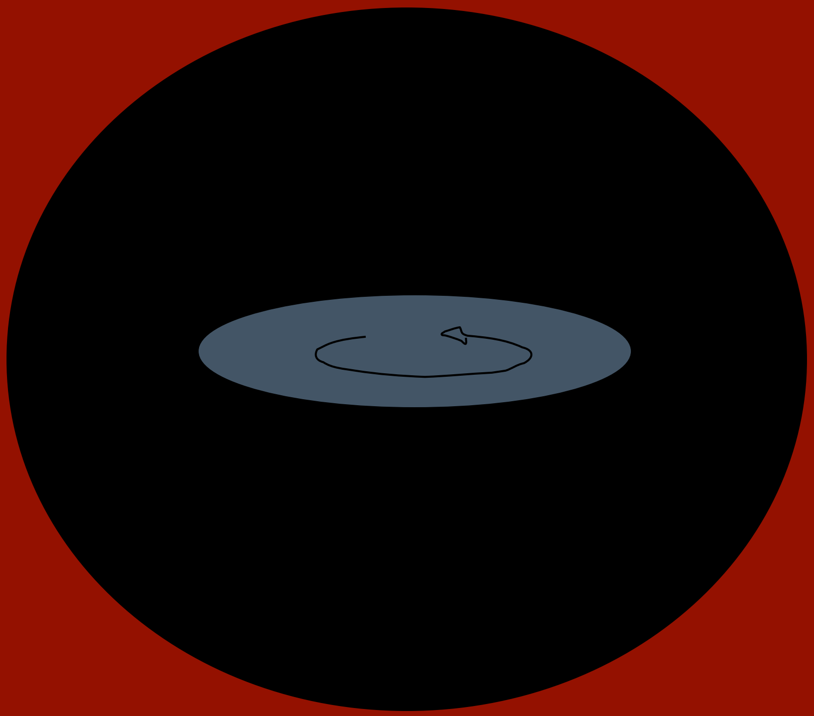
Galaxy formation theory in a nutshell

A dark matter halo forms. Inside the halo is hot/warm gas.
The gas has some angular momentum.



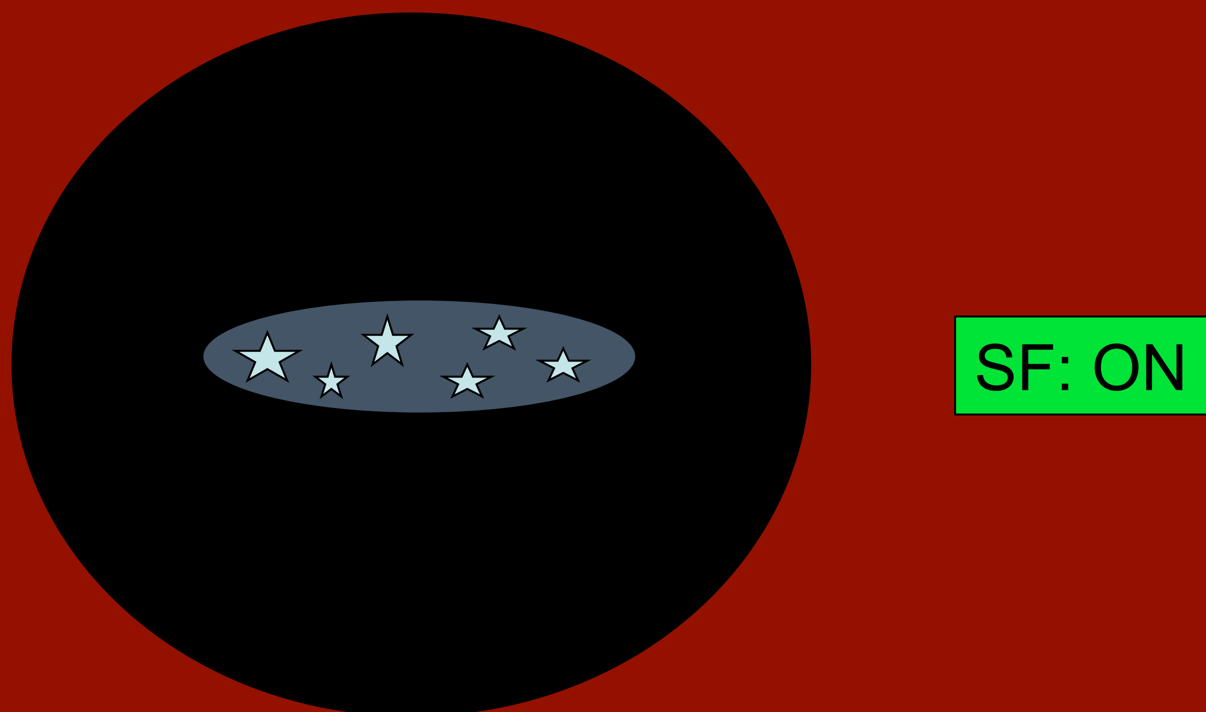
Galaxy formation theory in a nutshell

Gas cools inside the halo and settles into a rotating disk.



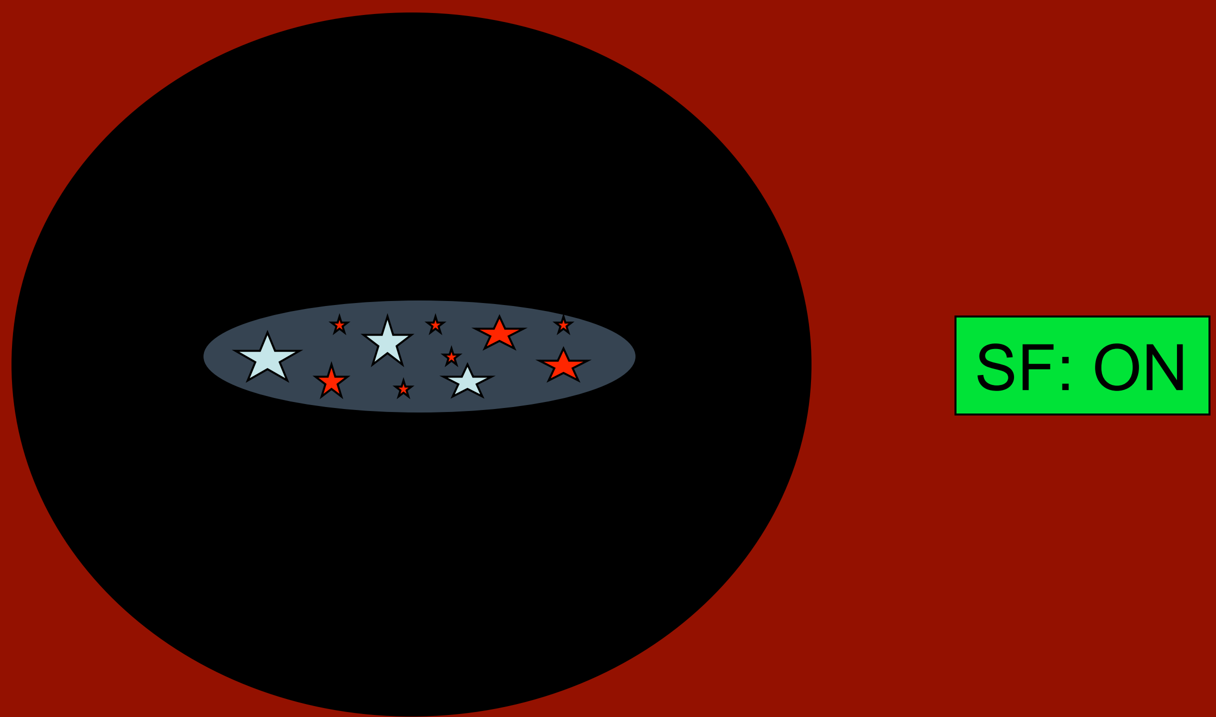
Galaxy formation theory in a nutshell

Stars form from the cold dense gas.



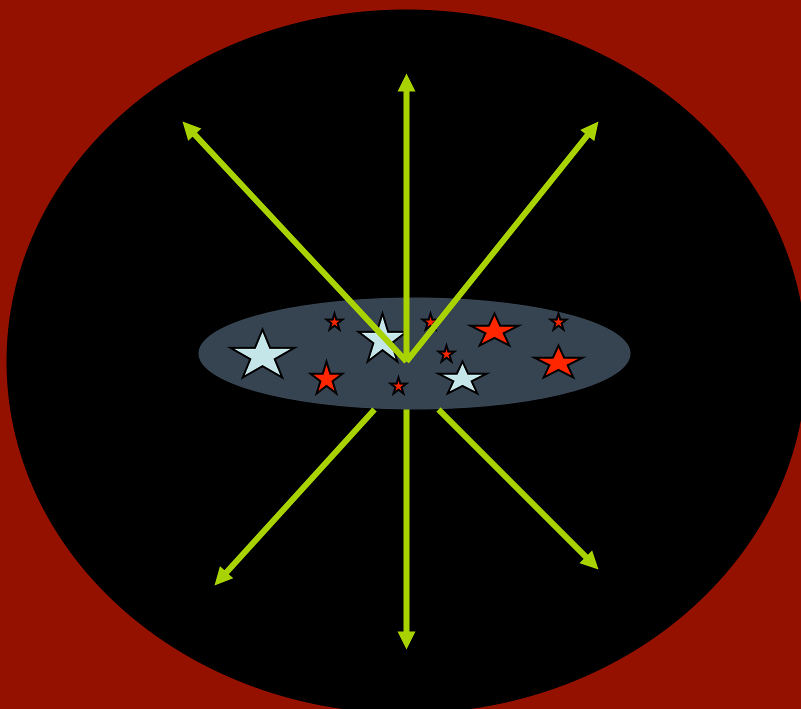
Galaxy formation theory in a nutshell

Stars grow old and fade, new stars are born.
Gas undergoes heating and cooling.



Galaxy formation theory in a nutshell

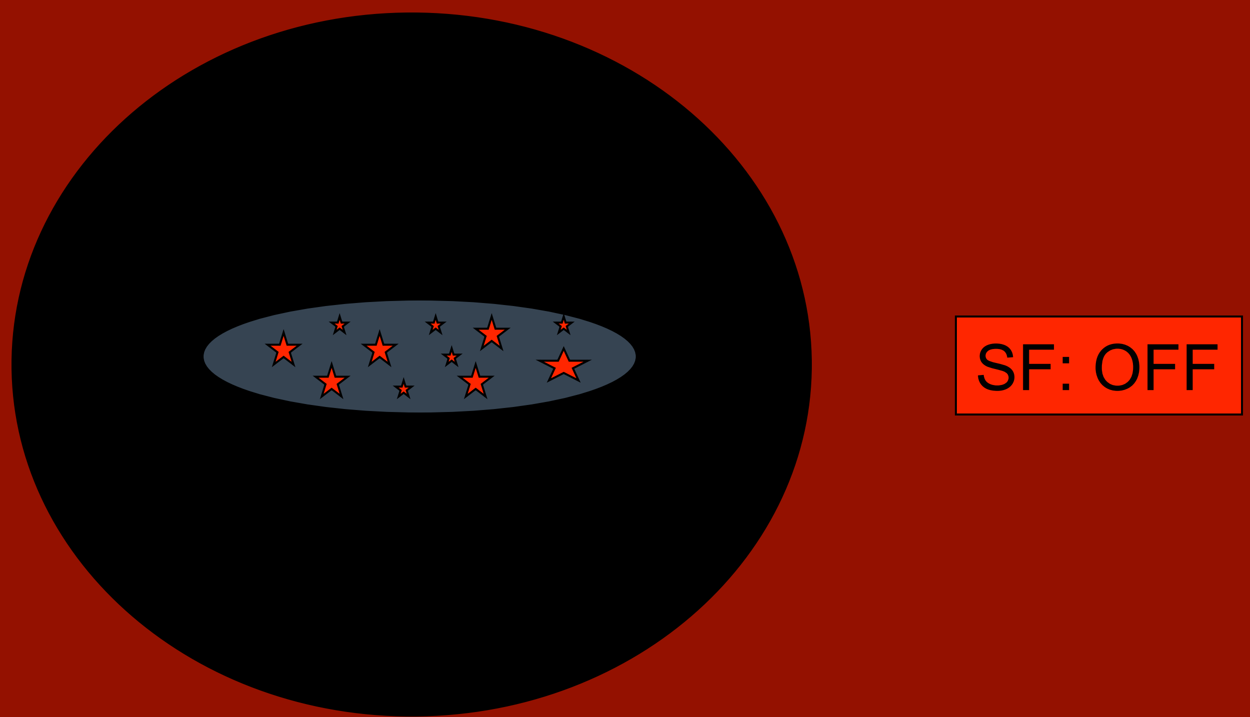
Energy feedback due to supernovae or a massive central black hole can reheat the gas or blow it out of the galaxy. This can end star formation.



SF: OFF

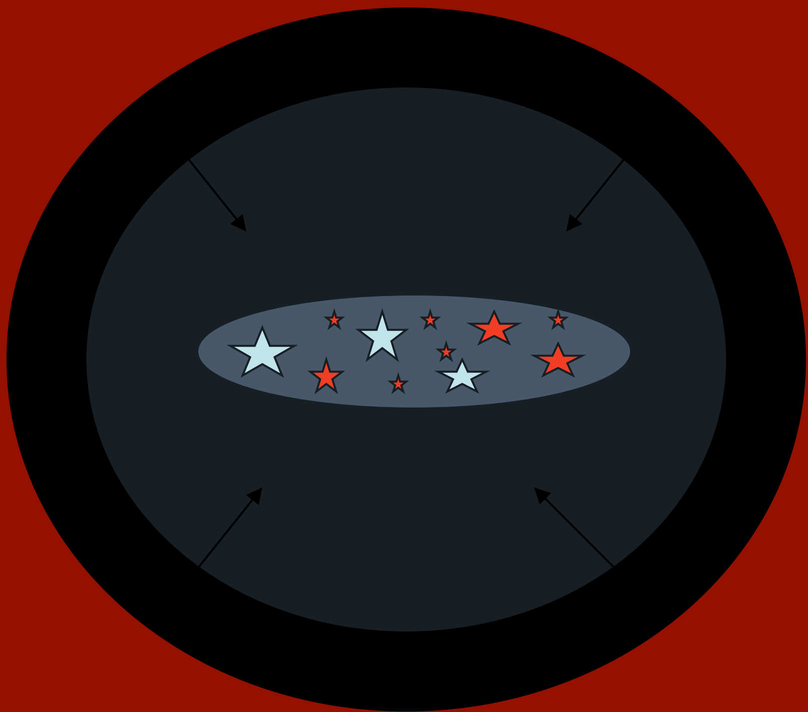
Galaxy formation theory in a nutshell

The galaxy gets dimmer and redder.



Galaxy formation theory in a nutshell

If the halo is in a gas-rich environment, more gas can fall into the halo from the inter-galactic medium.



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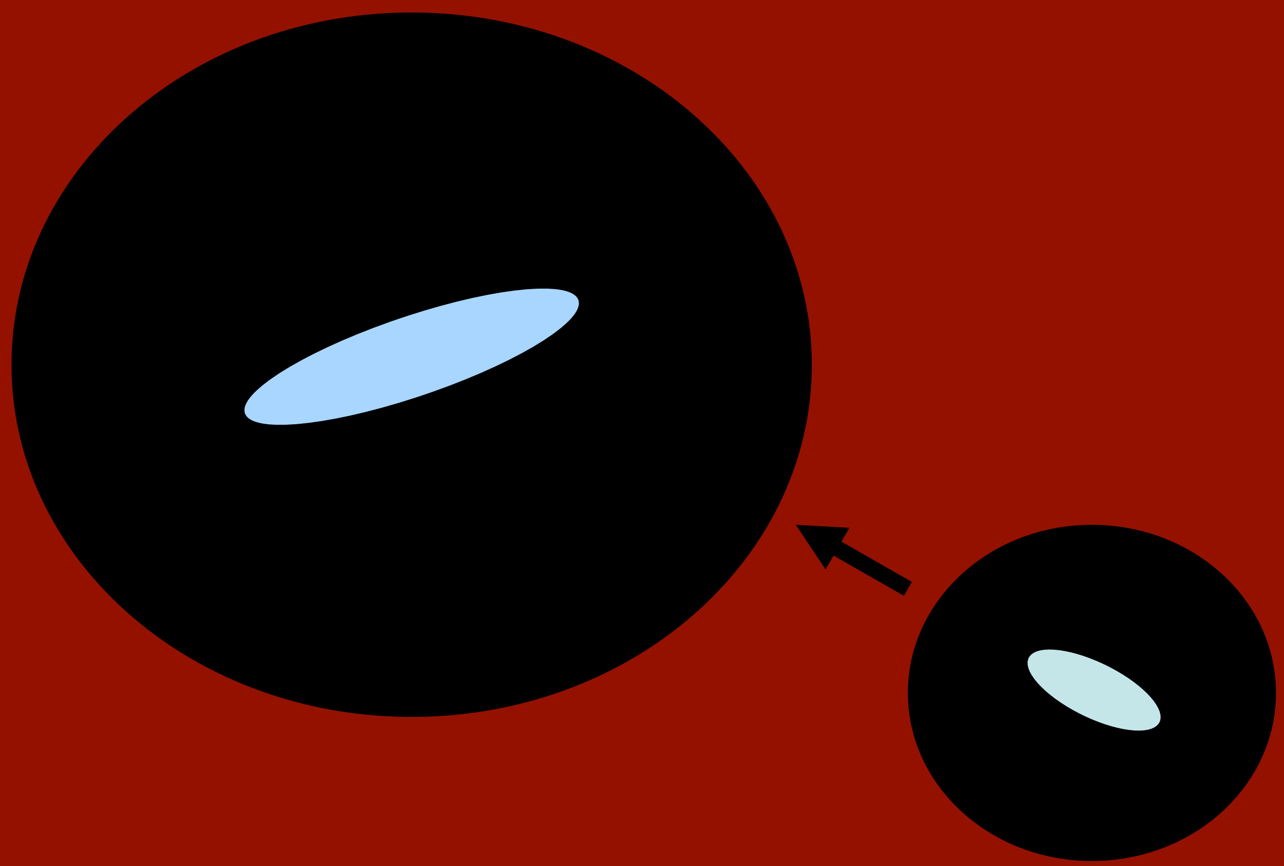
Galaxy formation theory in a nutshell

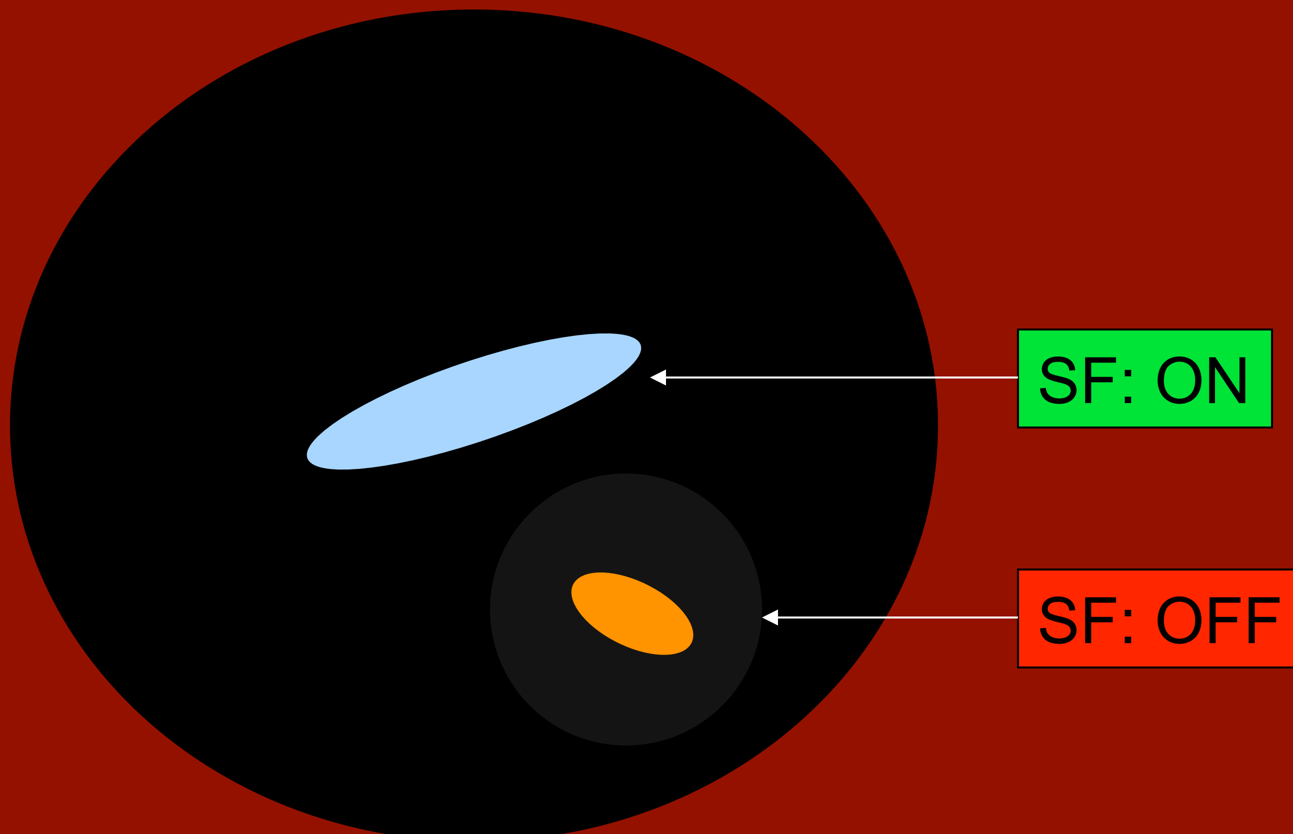
If the halo merges with another halo containing a galaxy, the smaller galaxy becomes a satellite.



Galaxy formation theory in a nutshell

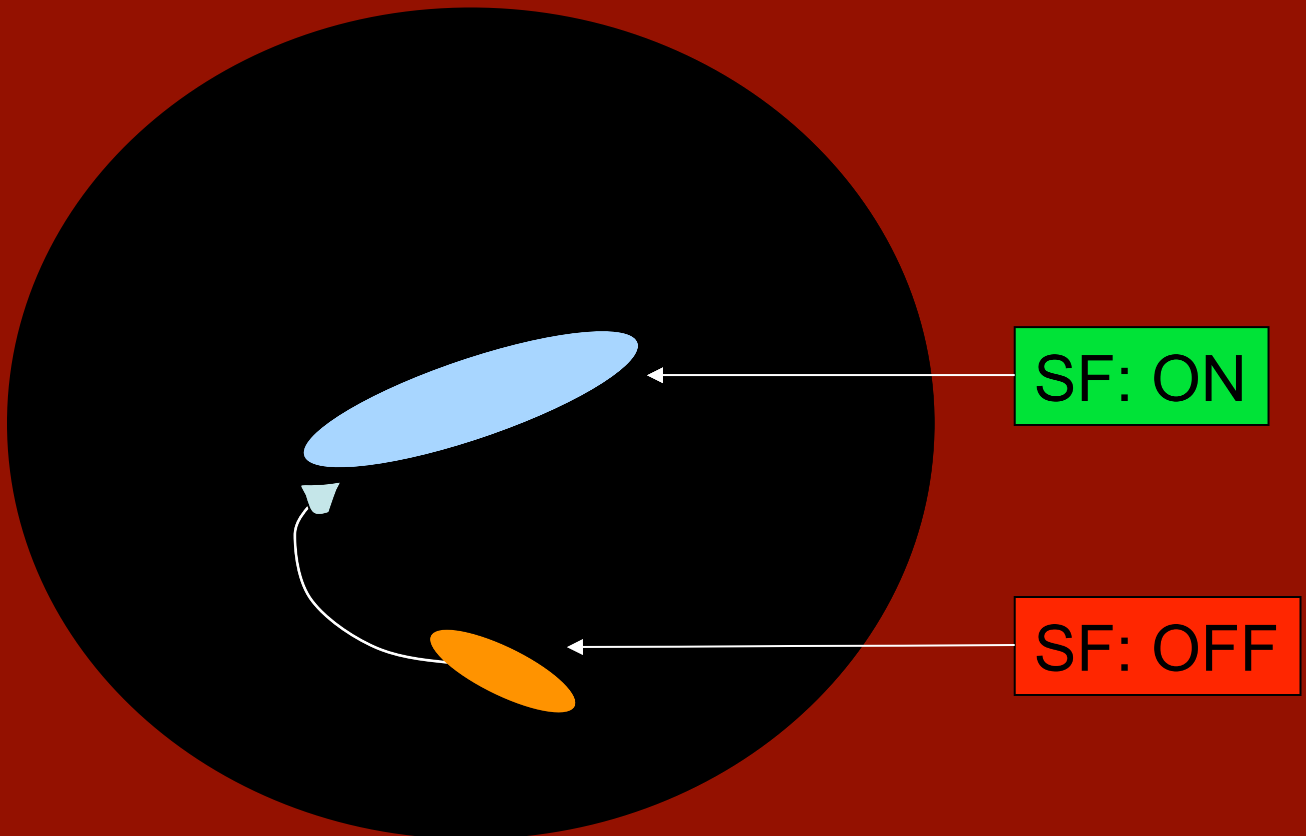
If the halo merges with another halo containing a galaxy, the smaller galaxy becomes a satellite.

If the main halo is massive enough, it can contain hot gas that might ram-pressure strip the gas out of the satellite galaxy, thereby ending its star formation.



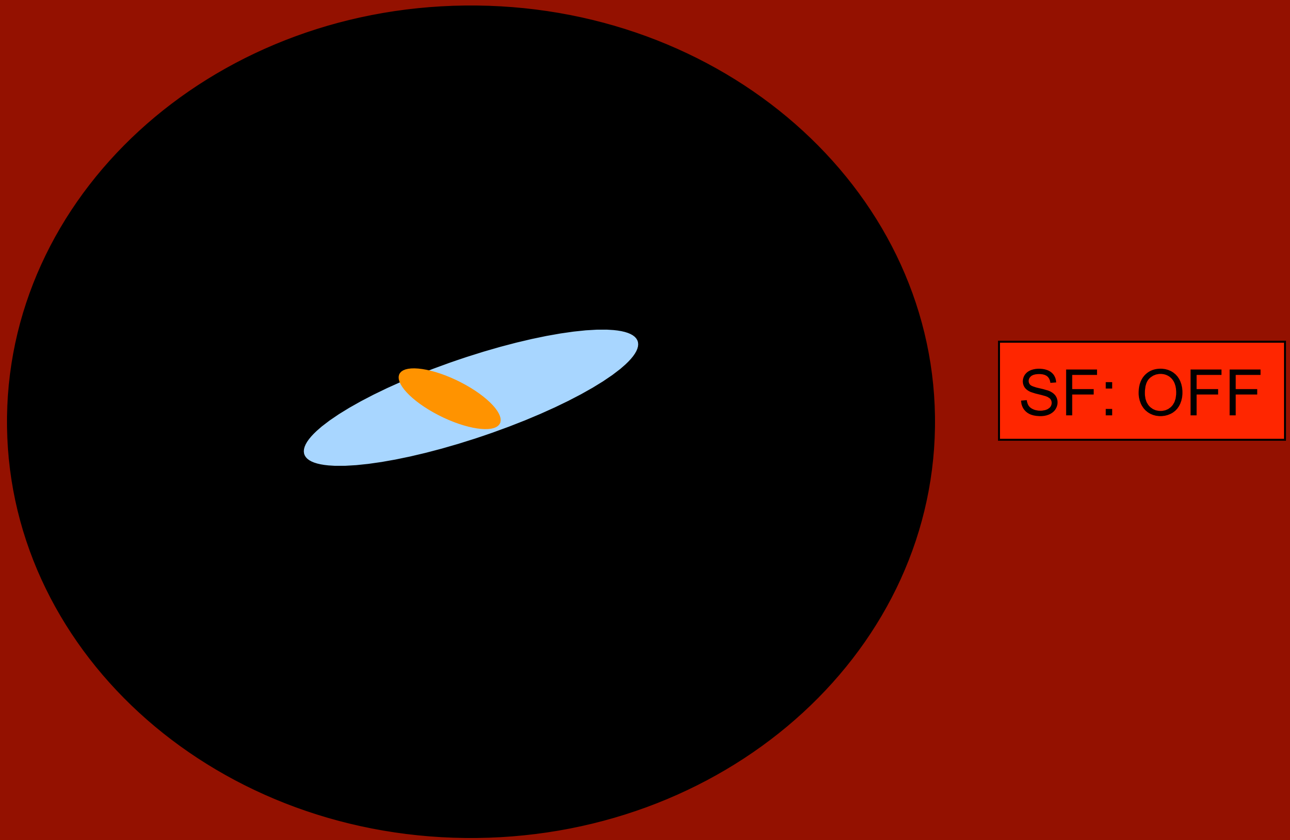
Galaxy formation theory in a nutshell

Eventually, the subhalo will be destroyed via tidal stripping and the satellite galaxy will spiral in due to dynamical friction.



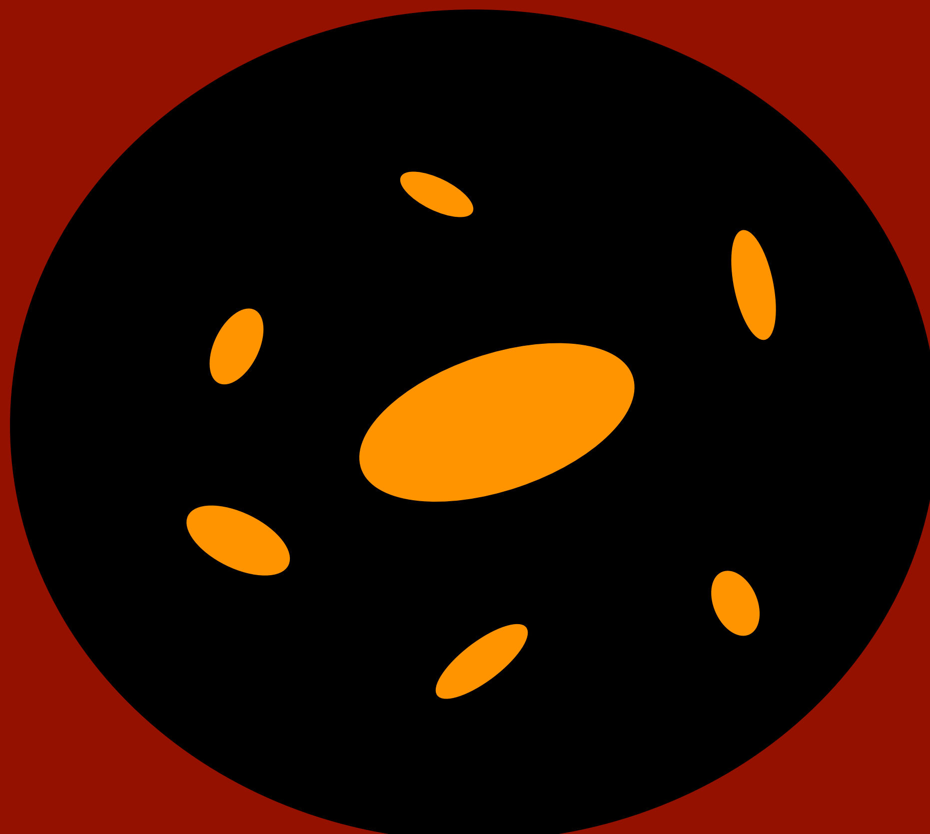
Galaxy formation theory in a nutshell

The galaxy merger can cause shocks in any remaining gas, which can trigger starbursts and exhaust the gas supply. Mergers can also affect the galaxy morphology.



Galaxy formation theory in a nutshell

In high mass halos, halo mergers happen frequently, but dynamical friction timescales are long, resulting in galaxy clusters.



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Galaxy formation theory in a nutshell

Key questions:

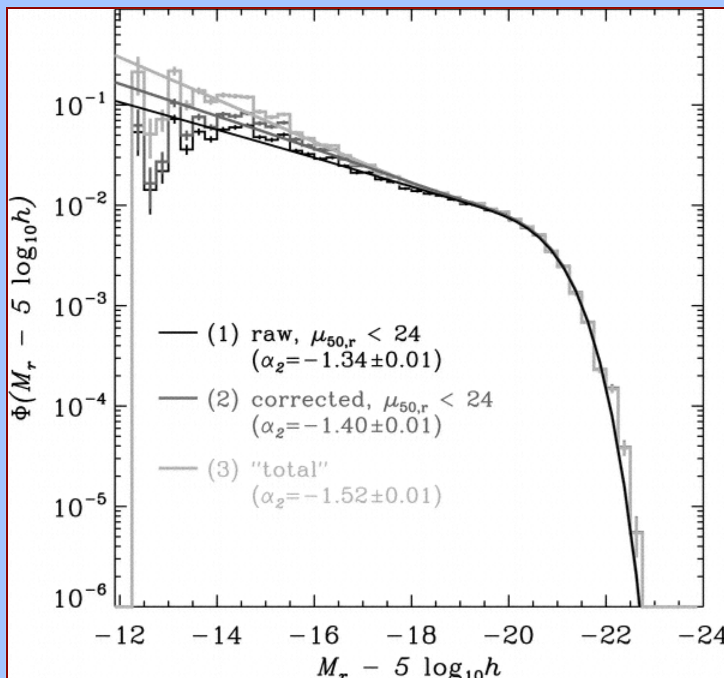
- What is the initial distribution of gas within halos and how does it cool? (e.g., multiphase medium)
- How do stars form exacty? (e.g., conditions for star formation, dependence of IMF on environment, metallicity)
- How does supernova feedback work? (e.g., thermal vs. kinetic energy injection, efficiency)
- How does AGN feedback work?
- How does merging affect galaxies’ star formation and morphology?
- How does fresh gas in the IGM feed galaxies?

Lots of unknowns!

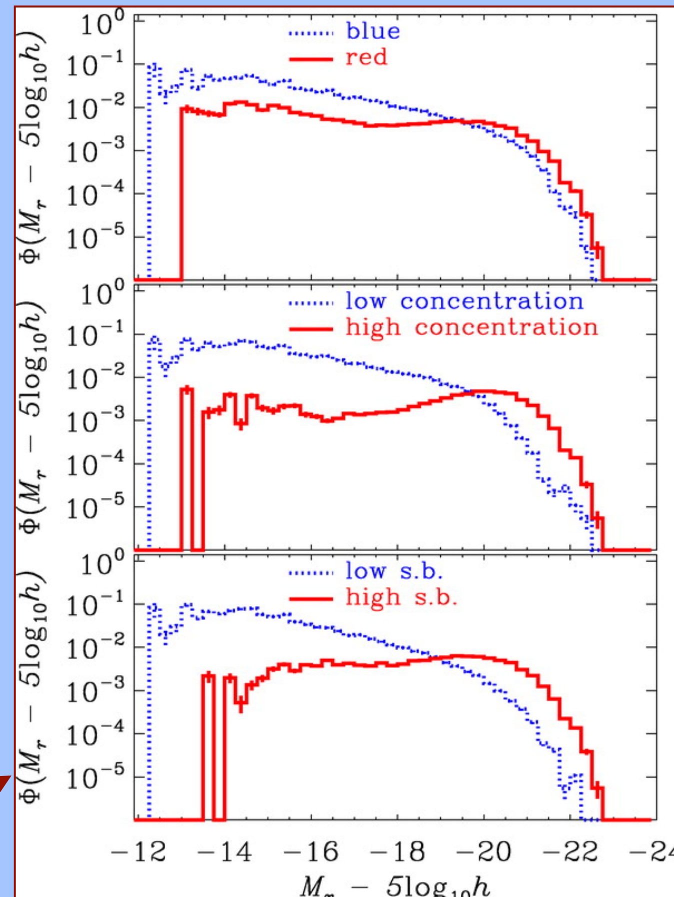
But also lots of data describing the distribution of galaxies!

Large surveys: Measurements of Galaxy Clustering

First Moments $\langle \delta(\vec{x}) \rangle$



Blanton et al. (2005)

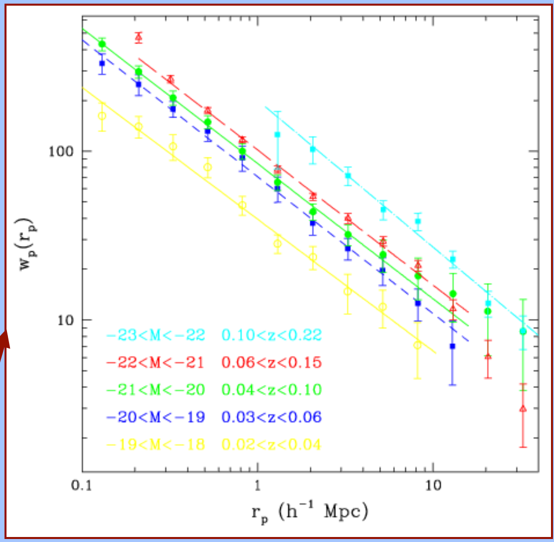


Luminosity Functions

Large surveys: Measurements of Galaxy Clustering

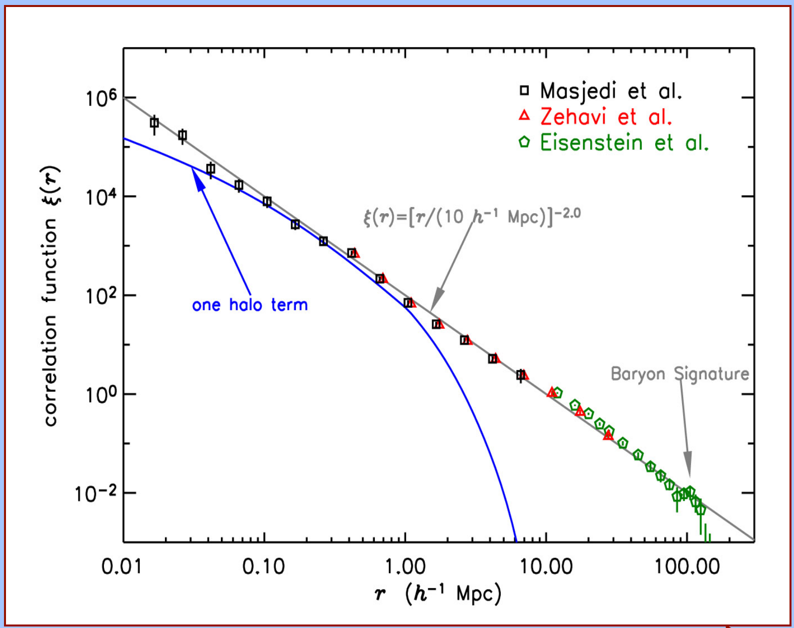
Second Moments $\langle \delta(\vec{x})\delta(\vec{x}+r) \rangle$

$w_p(r_p)$

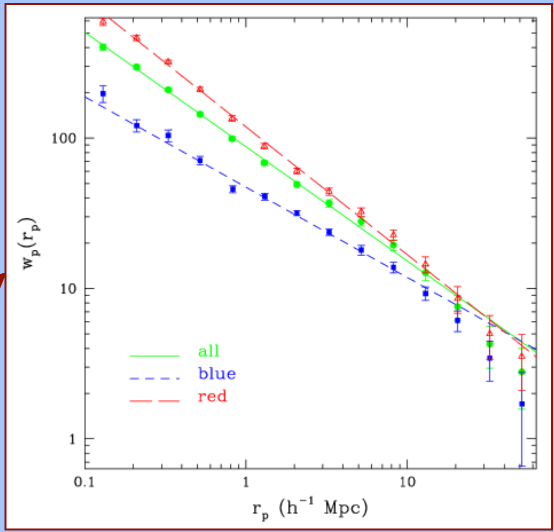


Zehavi et al. (2005)

$\xi_{gg}(r)$



Masjedi et al. (2006)

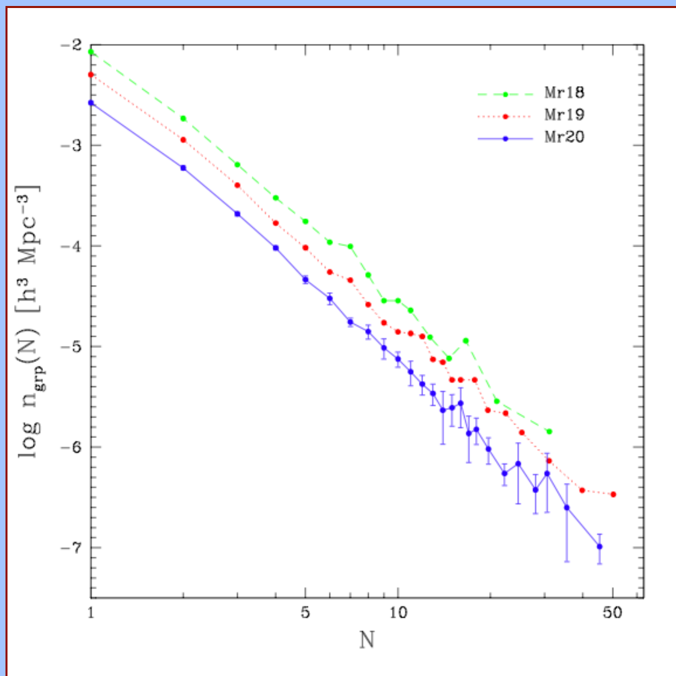


Correlation Functions

Large surveys: Measurements of Galaxy Clustering

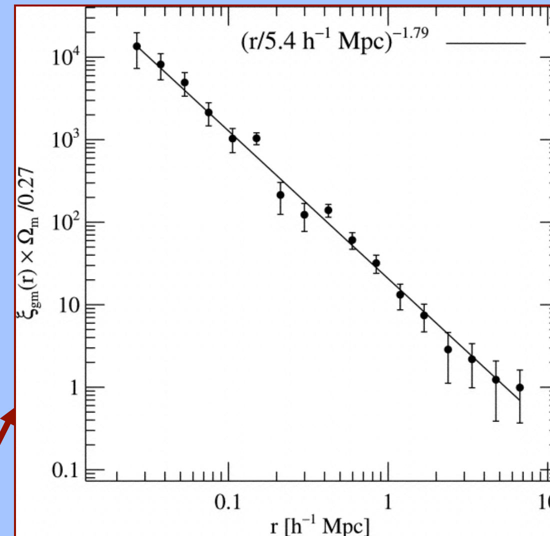
Higher Moments and Other Statistics

$n_{grp}(N)$



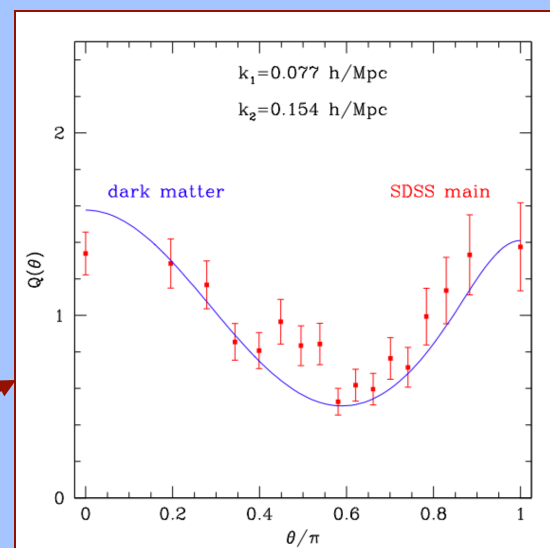
Berlind et al. (2006)

$\xi_{gm}(r)$



Sheldon et al. (2005)

$Q(\vartheta)$



Scoccimarro et al. (2006)

Galaxy-Mass Correlations

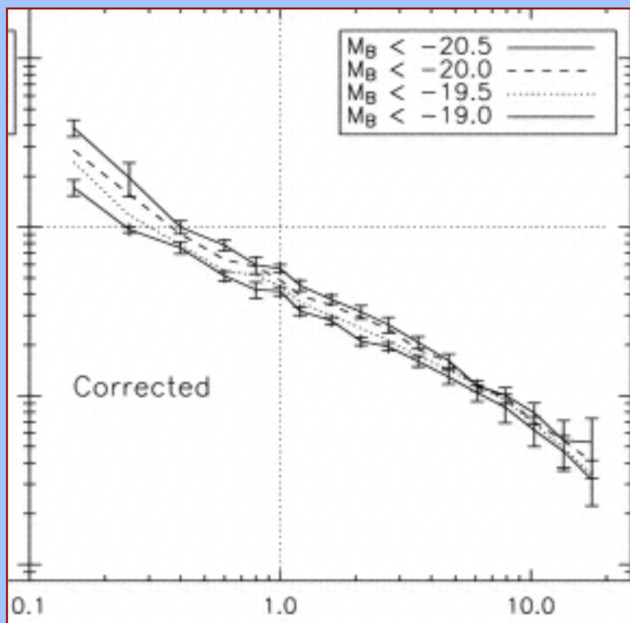
Group Multiplicity Function

Bispectrum

Large surveys: Measurements of Galaxy Clustering

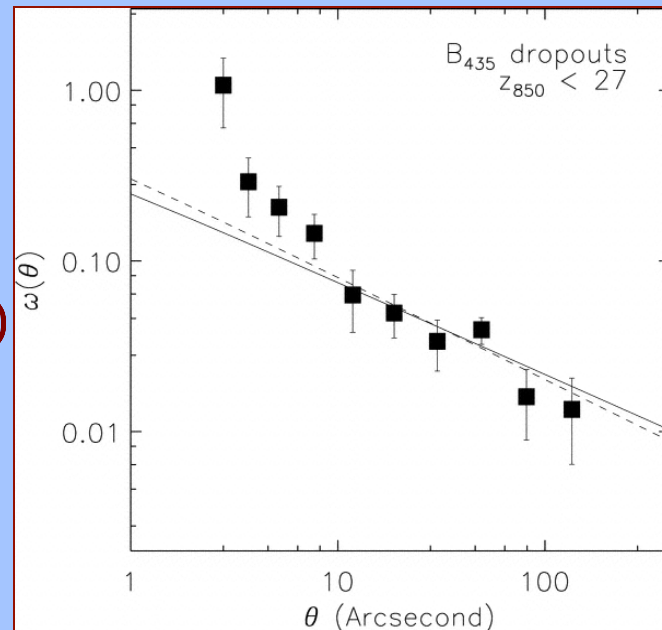
High Redshift

GOODS



$w_p(r_p)$

Coil et al. (2006)



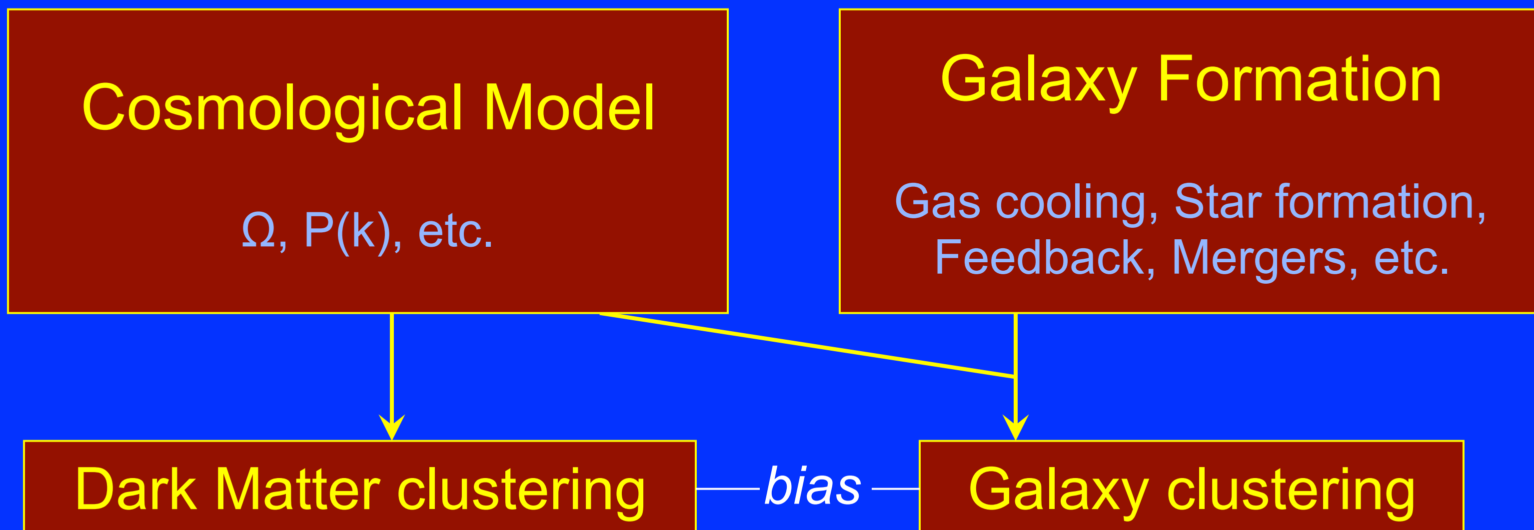
$\omega(\theta)$

Lee et al. (2006)

Z ~ 1

z ~ 4

Galaxy clustering data contains information about cosmology and galaxy formation/evolution.



How can we extract this information from the data?

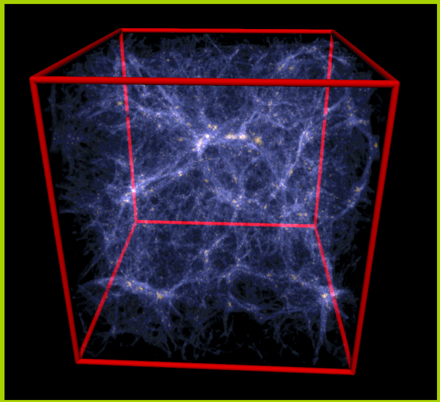
e.g., what does a particular shape of $\xi(r)$ for bright red galaxies tell us about how these galaxies formed? Can we use this statistic to constrain cosmological parameters?

Ab-initio Predictions

Hydrodynamic Simulations of Dark Matter + Gas

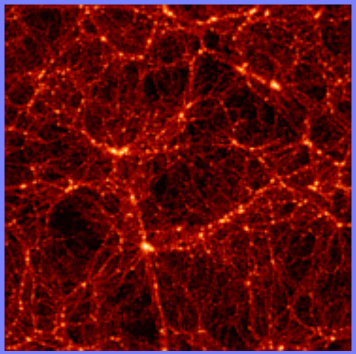
Gravity and Hydrodynamics + Heating and Cooling + Prescriptions for Star Formation and Feedback =

(Sub-grid physics)



Springel et al.

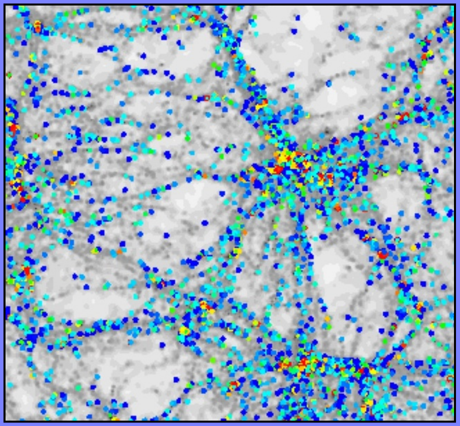
Semi-Analytic Models



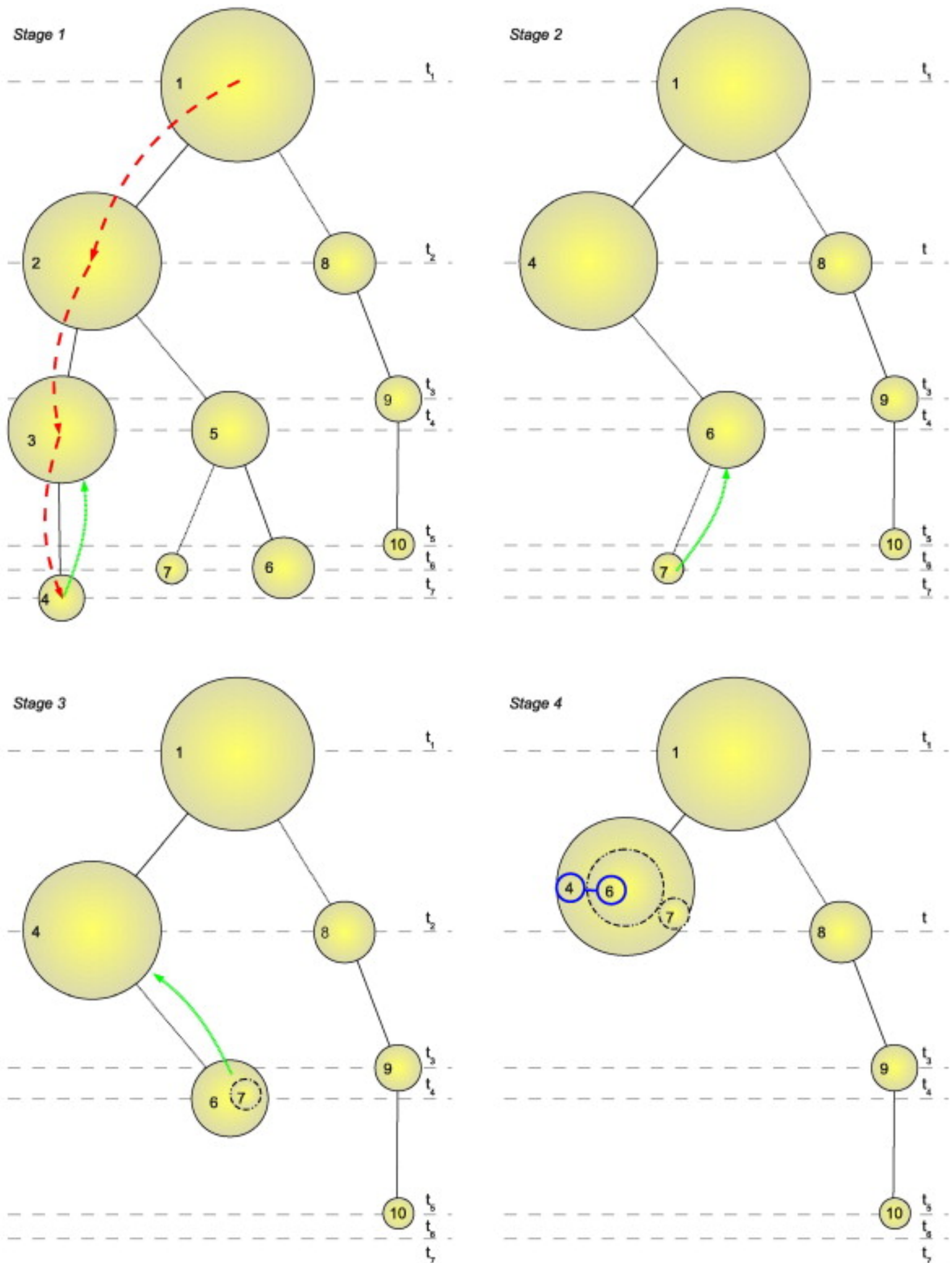
+

Prescriptions for:
Gas distribution, Gas Cooling,
Star Formation, Feedback,
Galaxy Mergers + more
in DM halos

=



Virgo Consortium



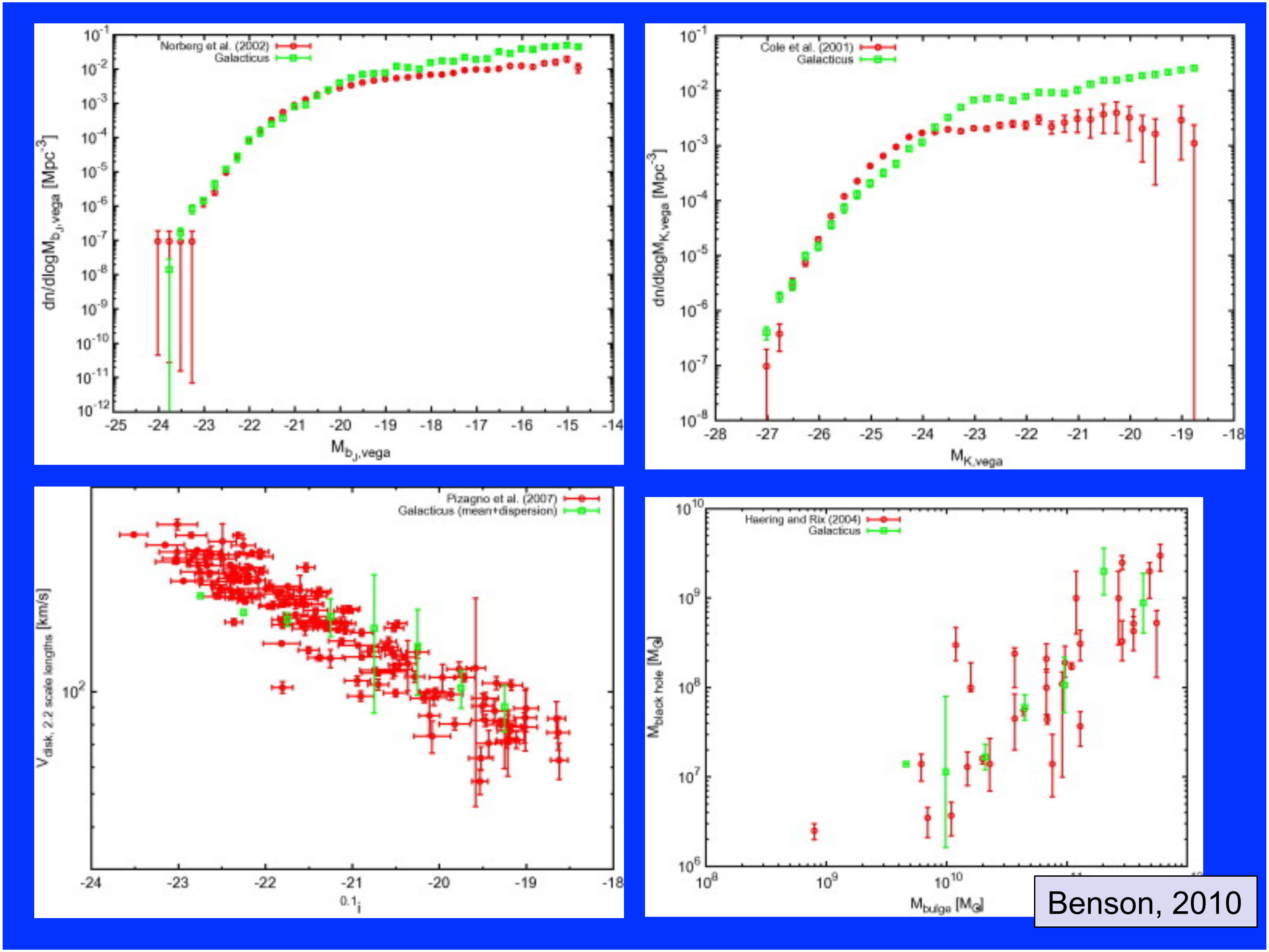
Benson, 2010

Parameter	Value	Reference
[H_0]	70.2 km/s	§4.2; (Komatsu et al., 2010)
[Omega_0]	0.2725	§4.2; (Komatsu et al., 2010)
[Omega_DE]	0.7275	§4.2; (Komatsu et al., 2010)
[Omega_b]	0.0455	§4.2; (Komatsu et al., 2010)
[T_CMB]	2.72548 K	§4.2; (Komatsu et al., 2010)
[accretionDisksMethod]	ADAF	§4.3
[adafAdiabaticIndex]	1.444	§4.3
[adafEnergyOption]	pure ADAF	§4.3
[adafRadiativeEfficiency]	0.01	§4.3
[adafViscosityOption]	fit	§4.3
[adiabaticContractionGnedinA]	0.8	§4.8
[adiabaticContractionGnedinOmega]	0.77	§4.8
[barInstabilityMethod]	ELN	§4.7
[blackHoleSeedMass]	100	§3.1.2
[blackHoleWindEfficiency]	0.001	§3.1.2
[bondiHoyleAccretionEnhancementHotHalo]	1	§3.1.2
[bondiHoyleAccretionEnhancementSpheroid]	1	§3.1.2
[bondiHoyleAccretionTemperatureSpheroid]	100	§3.1.2
[coolingFunctionMethod]	atomic CIE Cloudy	§4.5.1
[coolingTimeAvailableAgeFactor]	0	§4.5.5
[coolingTimeSimpleDegreesOfFreedom]	3	§4.5.4
[darkMatterProfileMethod]	NFW	§4.6.1
[darkMatterProfileMinimumConcentration]	4	§3.8.2
[diskOutflowExponent]	2	§4.23
[diskOutflowVelocity]	200 km/s	§4.23
[effectiveNumberNeutrinos]	4.34	§4.4.2
[galacticStructureRadiusSolverMethod]	adiabatic	§4.8
[haloMassFunctionMethod]	Tinker2008	§4.4.6
[haloSpinDistributionMethod]	Bett2007	§4.6.3
[hotHaloOutflowReturnRate]	1.26	§3.2.2
[imfSalpeterRecycledInstantaneous]	0.39	§4.12.2
[imfSalpeterYieldInstantaneous]	0.02	§4.12.2
[imfSelectionFixed]	Salpeter	§4.12.1
[isothermalCoreRadiusOverVirialRadius]	0.1	§4.10

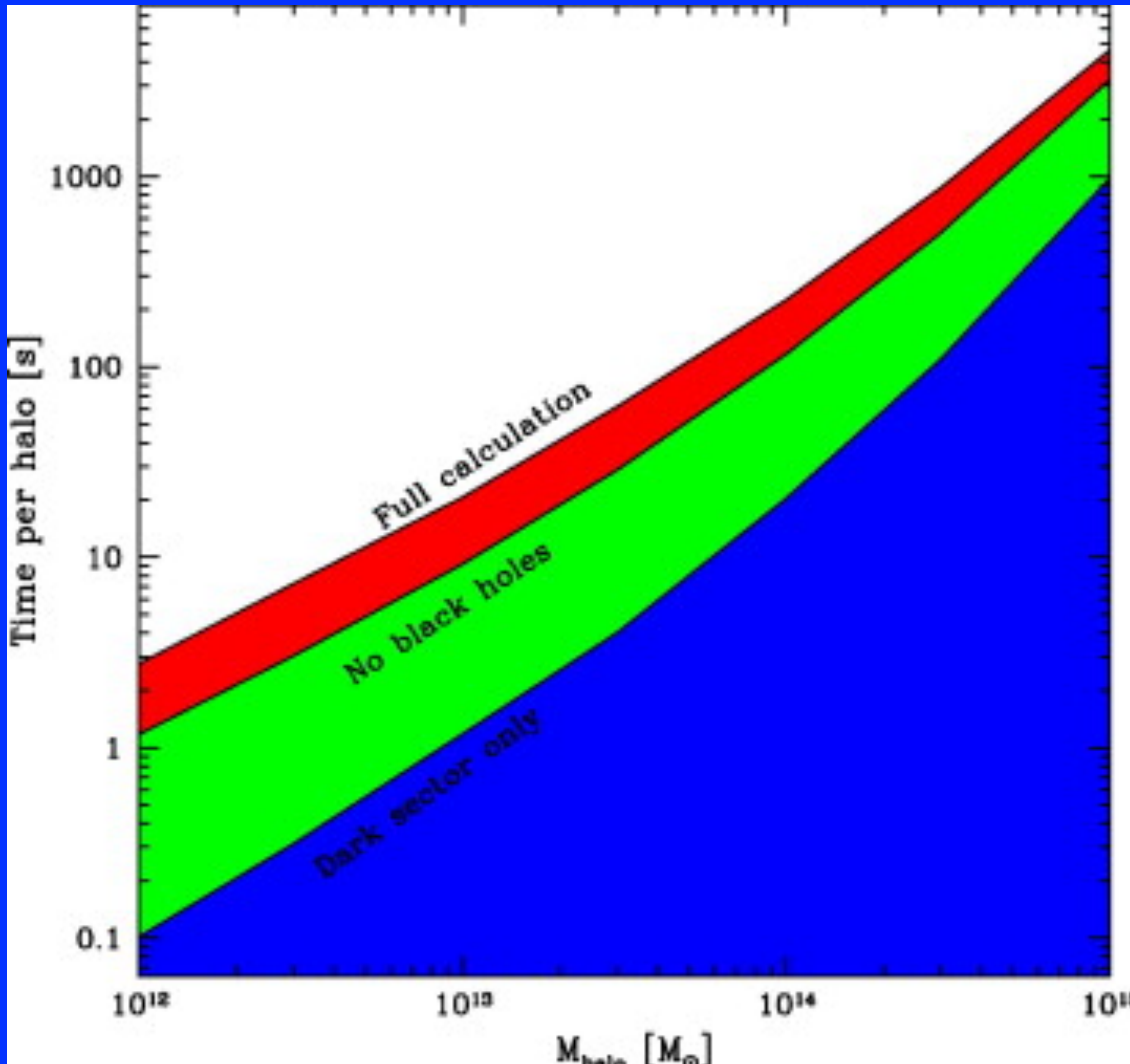
Benson, 2010

[majorMergerMassRatio]	0.1	§4.9.1
[mergerRemnantSizeOrbitalEnergy]	1	§4.9.2
[mergerTreeBuildCole2000AccretionLimit]	0.1	§4.16
[mergerTreeBuildCole2000MassResolution]	$5 \times 10^9 M_\odot$	§4.16
[mergerTreeBuildCole2000MergeProbability]	0.1	§4.16
[mergerTreeConstructMethod]	build	§4.14
[minorMergerGasMovesTo]	spheroid	§4.9.1
[modifiedPressSchechterFirstOrderAccuracy]	0.1	§4.15
[modifiedPressSchechterG0]	0.57	§4.15
[modifiedPressSchechterGamma1]	0.38	§4.15
[modifiedPressSchechterGamma2]	-0.01	§4.15
[powerSpectrumIndex]	0.961	§4.4.1; (Komatsu et al., 2010)
[powerSpectrumReferenceWavenumber]	1 Mpc^{-1}	§4.4.1; (Komatsu et al., 2010)
[powerSpectrumRunning]	0	§4.4.1; (Komatsu et al., 2010)
[randomSpinResetMassFactor]	2	§3.7.2
[reionizationSuppressionRedshift]	9	§4.1
[reionizationSuppressionVelocity]	30 km/s	§4.1
[satelliteMergingMethod]	Jiang2008	§4.22.1
[sigma_8]	0.807	§4.4.1 & §4.4.2
[spheroidEnergeticOutflowMassRate]	1	§3.4.2
[spheroidOutflowExponent]	2	§4.23
[spheroidOutflowVelocity]	50 km/s	§4.23
[spinDistributionBett2007Alpha]	2.509	§4.6.3
[spinDistributionBett2007Lambda0]	0.04326	§4.6.3
[stabilityThresholdGaseous]	0.9	§4.7
[stabilityThresholdStellar]	1.1	§4.7
[starFormationDiskEfficiency]	0.01	§4.17
[starFormationDiskMinimumTimescale]	0.001 Gyr	§4.17
[starFormationDiskVelocityExponent]	-1.5	§4.17
[starFormationSpheroidEfficiency]	0.1	§4.17
[starFormationSpheroidMinimumTimescale]	0.001 Gyr	§4.17
[starveSatellites]	true	§3.2.2
[stellarPopulationPropertiesMethod]	instantaneous	§4.18
[summedNeutrinoMasses]	0	§4.4.2
[transferFunctionMethod]	Eisenstein + Hu	§4.4.2
[virialDensityContrastMethod]	spherical top hat	§4.4.5

Benson, 2010


Norberg et al. (2002)
Galacticus
dn/dlogM_{bJ,vega} [Mpc^-3]
M_{bJ,vega}
Cole et al. (2001)
Galacticus
dn/dlogM_{K,vega} [Mpc^-3]
M_{K,vega}
Pizagno et al. (2007)
Galacticus (mean+dispersion)
V_{disk, 2.2 scale lengths} [km/s]
0.1i
Haering and Rix (2004)
Galacticus
M_{black hole} [M☉]
M_{bulge} [M☉]

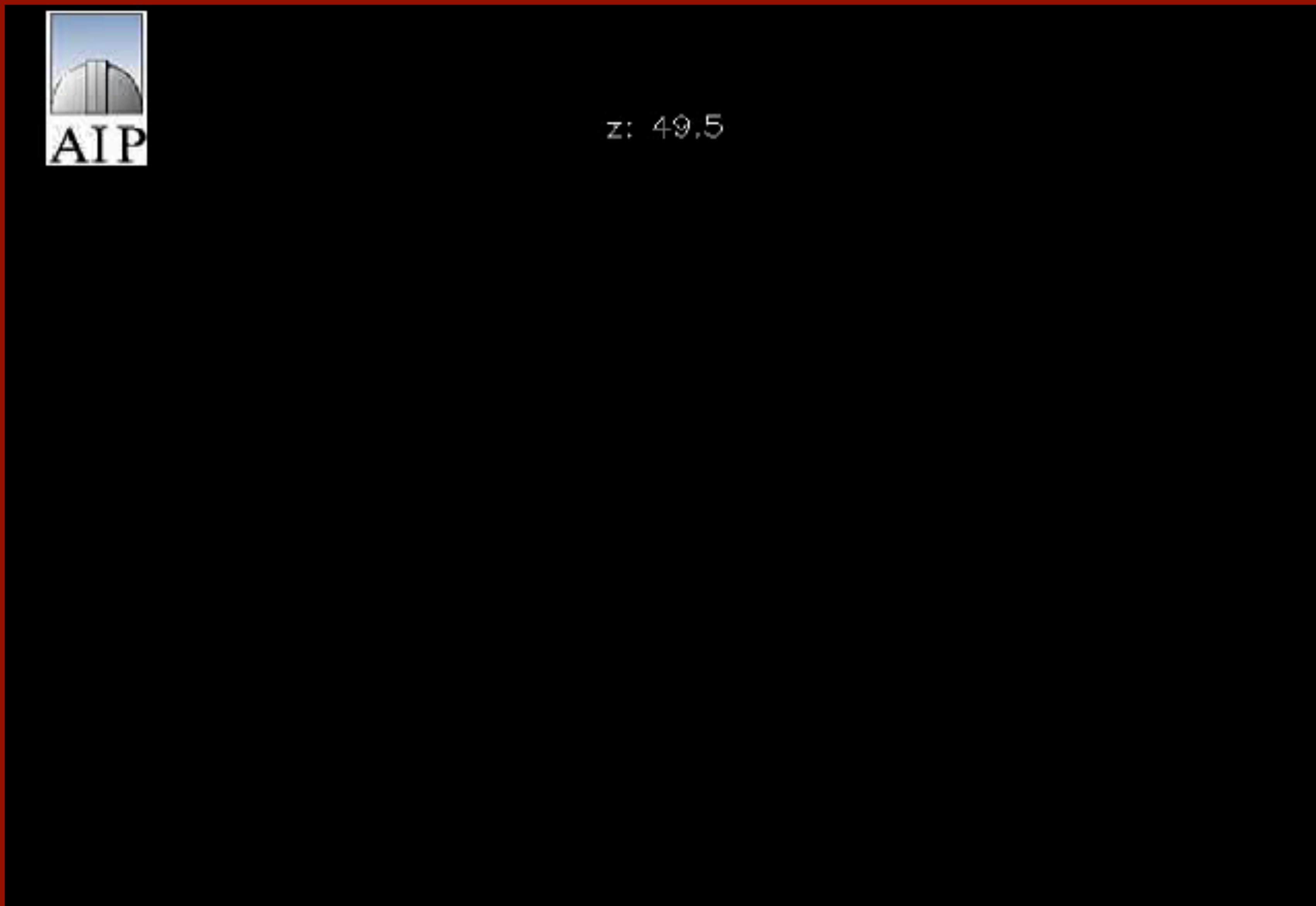
Benson, 2010



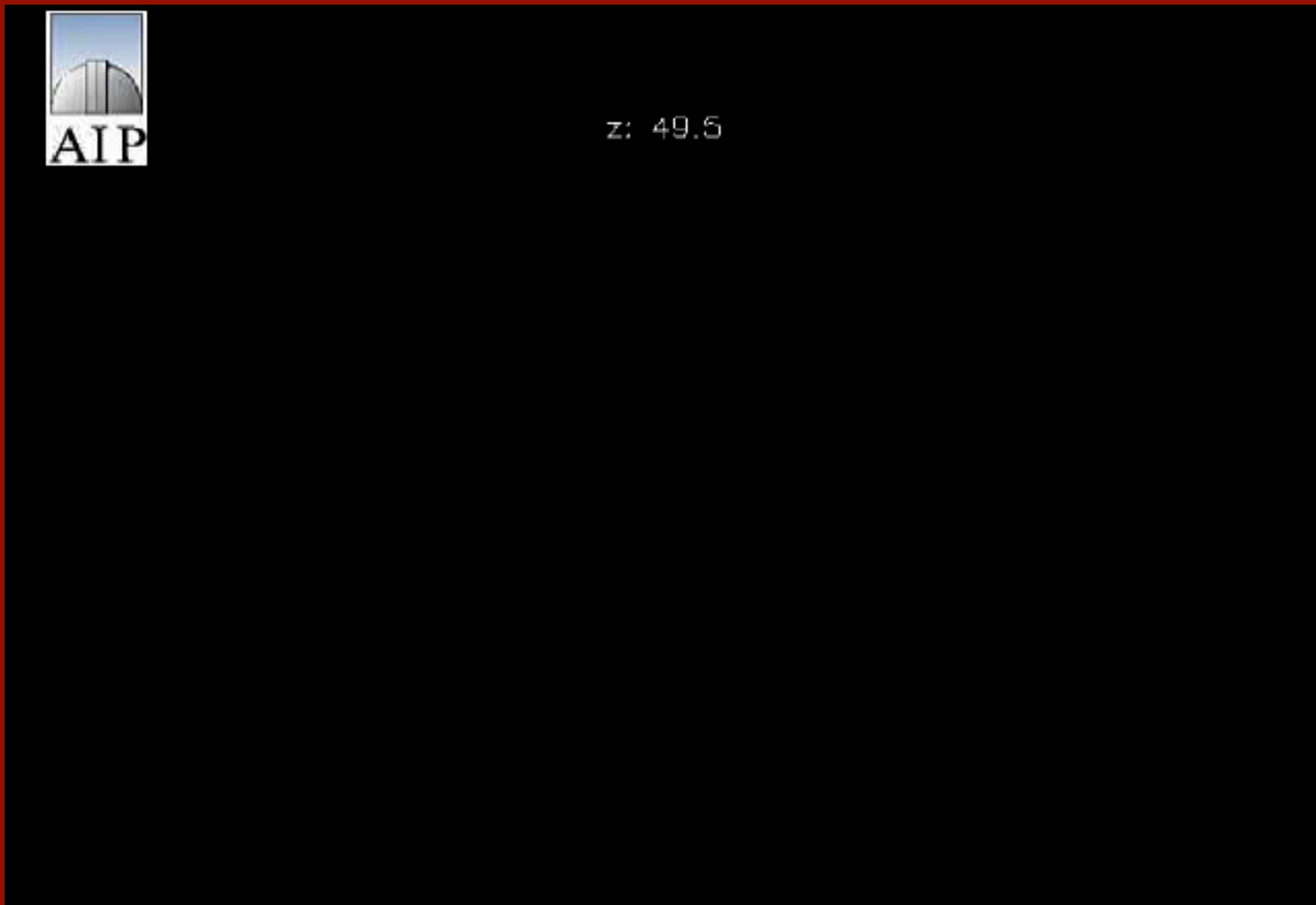
1000
100
10
1
0.1
Time per halo [s]
10^{12}
10^{13}
10^{14}
10^{15}
M_{halo} [$M_\odot$]
Full calculation
No black holes
Dark sector only

Benson, 2010

Galaxy formation theory in a nutshell



Galaxy formation theory in a nutshell



Forward approach is not practical

- Hydrodynamic simulations take too long + cannot resolve much of the important physics $\longrightarrow$ make many assumptions.
- Semi-analytic models have too many free parameters and do not necessarily include all the relevant physics.

For constraining cosmological parameters: too many uncertainties in galaxy formation physics. We can predict dark matter clustering to fairly high precision, but we have trouble going from DM to galaxies.

For constraining galaxy formation physics: difficult to understand how parameters affect clustering statistics. *e.g., what does it mean if a model predicts a 3-point correlation function that is too high for faint red galaxies?*

To constrain cosmology: parameterize our ignorance in order to bypass the need to understand galaxy formation.

"Environmental" bias

δ_g and δ_m are defined on some smoothing scale $\left(\delta \equiv \frac{\rho}{\bar{\rho}} - 1\right)$

$$\delta_g = b\delta_m \qquad \delta_g = f\left(\delta_m\right)$$

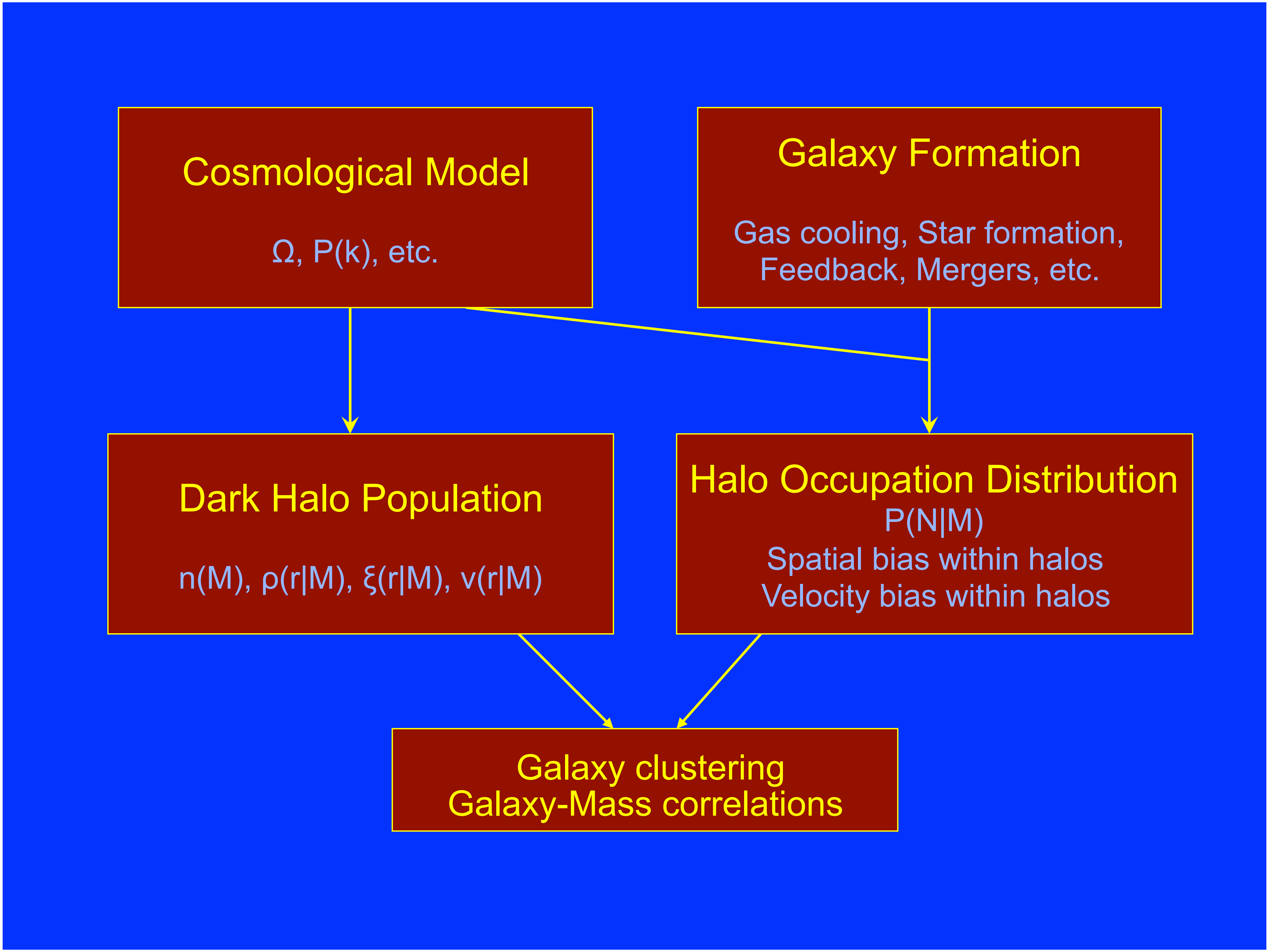
- Cannot describe bias on scales smaller than smoothing scale.
- Choice of smoothing scale is arbitrary.
- Bias is only simple (b=const) on very large scales.
- Only well-suited to describe some clustering statistics.
- δ_m is generally unobservable.

“Halo Occupation” Bias

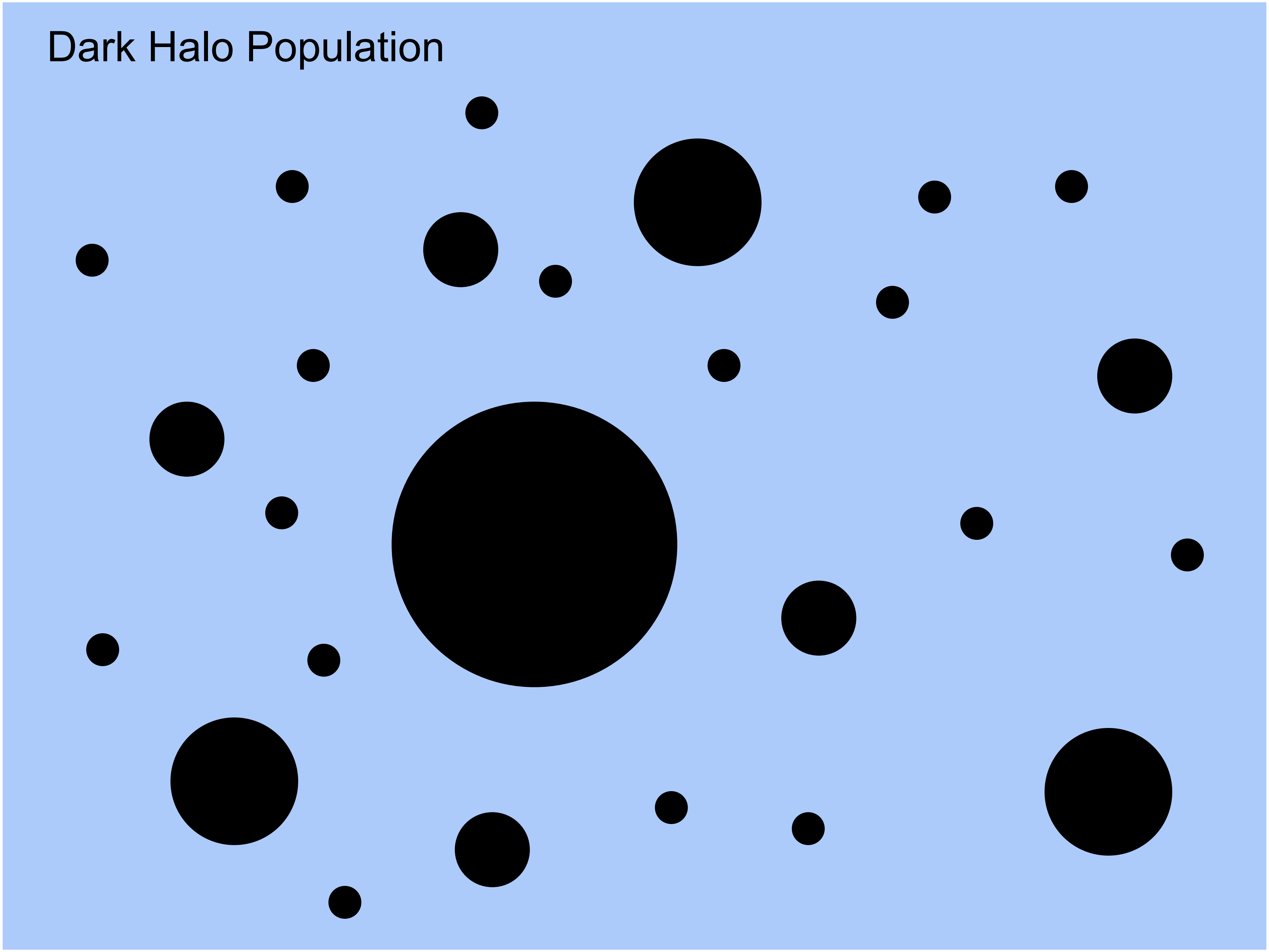
1. All galaxies live in DM halos
2. The galaxy content of a halo is statistically independent of the halo’s larger scale environment (depends only on mass)

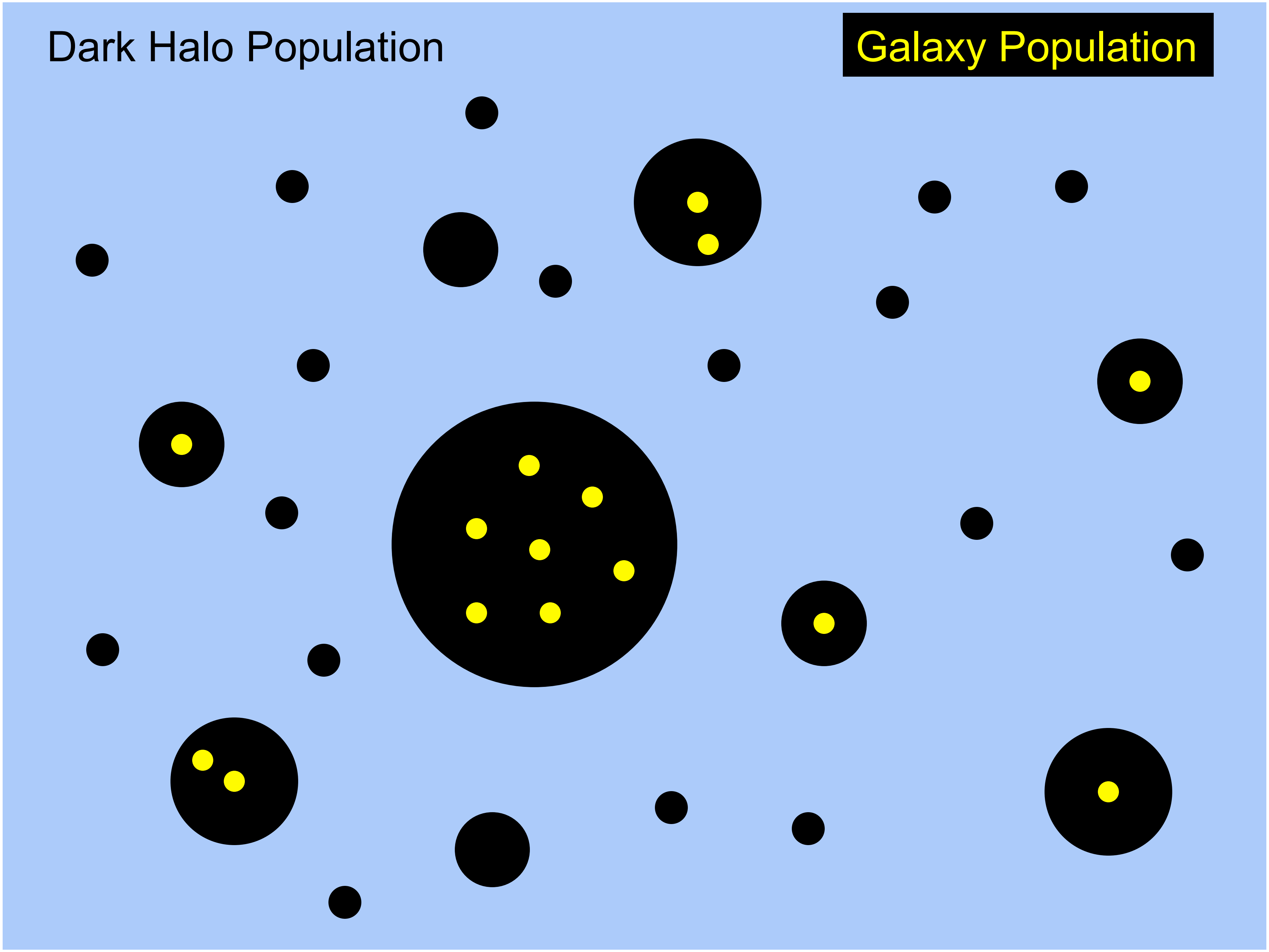
The bias of any class of galaxies (luminosity, type, etc.) is fully defined by the Halo Occupation Distribution (HOD):

- The probability distribution $P(N|M)$ that a halo of mass M contains N galaxies of that class.
- The relation between the spatial distributions of galaxies and DM within halos.
- The relation between the velocity distributions of galaxies and DM within halos.



Cosmological Model
Ω, P(k), etc.
Galaxy Formation
Gas cooling, Star formation,
Feedback, Mergers, etc.
Dark Halo Population
n(M), ρ(r|M), ξ(r|M), v(r|M)
Halo Occupation Distribution
P(N|M)
Spatial bias within halos
Velocity bias within halos
Galaxy clustering
Galaxy-Mass correlations


Dark Halo Population


Dark Halo Population
Galaxy Population

Why is the Halo Occupation Distribution (HOD) the right way to think about bias?

- **Complete:** It tells us everything a theory of galaxy formation has to say about galaxy clustering (all statistics, all scales).
- **Physically illuminating:** Discrepancies offer guidance about their physical origin.
- **Observationally powerful:** Description of bias at the level of systems in dynamic equilibrium, where methods can constrain mass.

Nice conceptual division between roles of "cosmological model" and "theory of Galaxy formation".

The basic approach.

- Develop machinery to compute galaxy clustering statistics given halo properties (mass function, etc.) + HOD.
- We know how to go from cosmological parameters to halo properties.
- Parameterize the HOD (and thus our ignorance about galaxy formation).
- Fit cosmological + HOD parameters (or HOD parameters at fixed cosmology) to galaxy clustering measurements.
- Use measured HODs to gain insight into galaxy formation.

How do we parameterize the HOD?

- Look at theoretical predictions for guidance.
- Interested in moments of *P(N|M)*, as well as radial and velocity distributions within halos.

$$\langle N \rangle_M = \sum_N N P(N|M)$$

$$\langle N(N-1) \rangle_M = \sum_N N(N-1) P(N|M)$$

$$\langle N(N-1)(N-2) \rangle_M = \sum_N N(N-1)(N-2) P(N|M)$$

How do we parameterize the HOD?

A note about the second moment of integer distributions

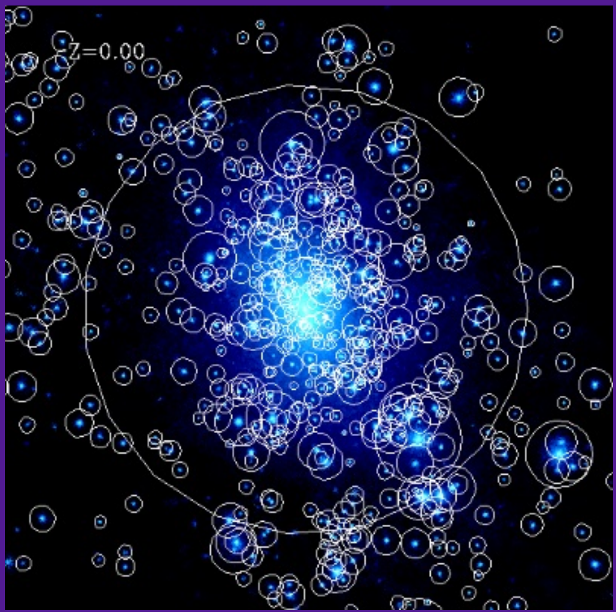
For a Poisson distribution: $$\langle N^2 \rangle = \langle N \rangle^2 + \langle N \rangle$$

The number of pairs is: $$\begin{aligned} \langle N(N-1) \rangle &= \langle N^2 - N \rangle \\ &= \langle N^2 \rangle - \langle N \rangle \\ &= \langle N \rangle^2 \end{aligned}$$

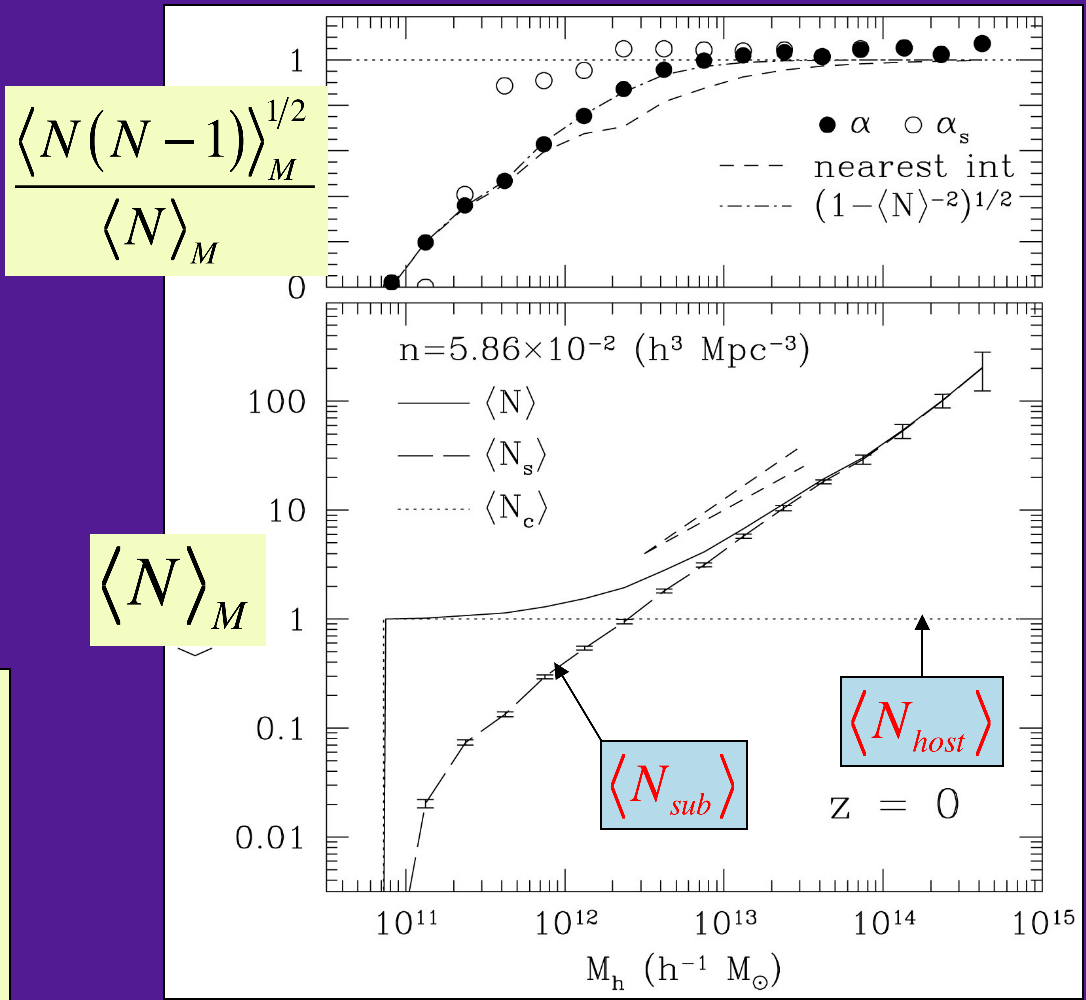
Narrower distributions have a smaller value than this and wider distributions have a larger value for the number of pairs.

How do we parameterize the HOD?

N-body



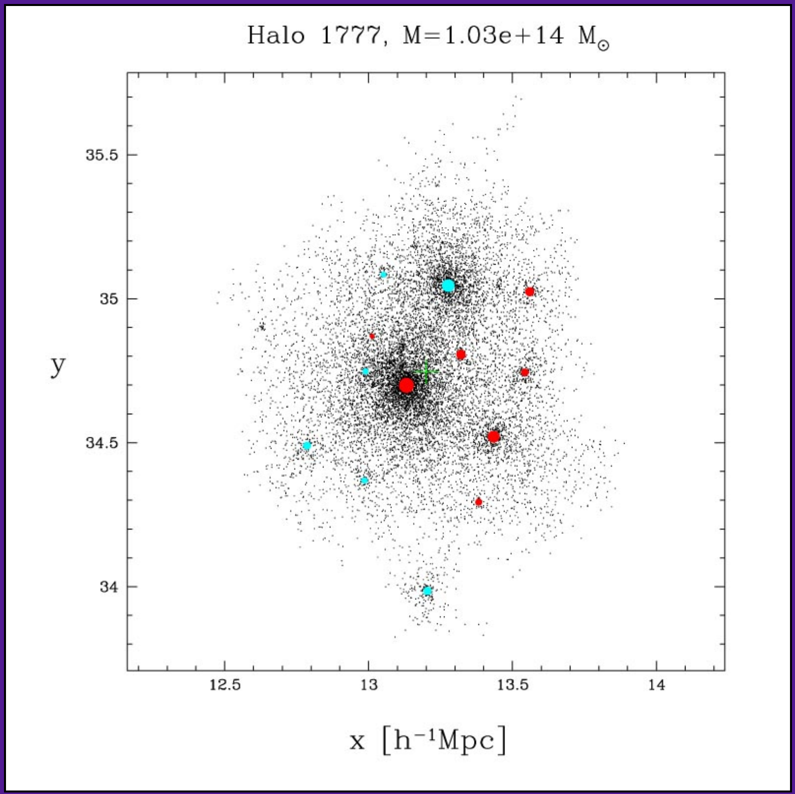
- HOD for halos + subhalos.
- <N$_{sub}$> is a power law with slope ~1.
- Distribution about <N$_{sub}$> is Poisson.

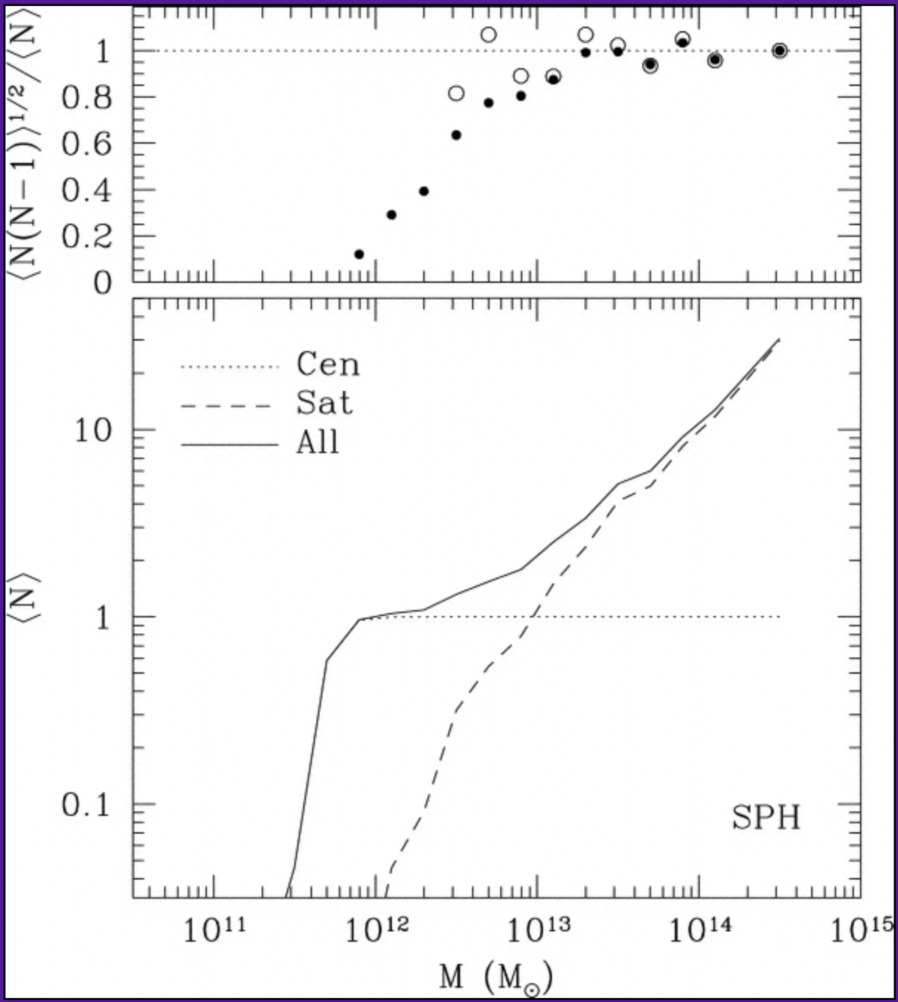


Kravtsov, Berlind et al. (2004)

How do we parameterize the HOD?

SPH

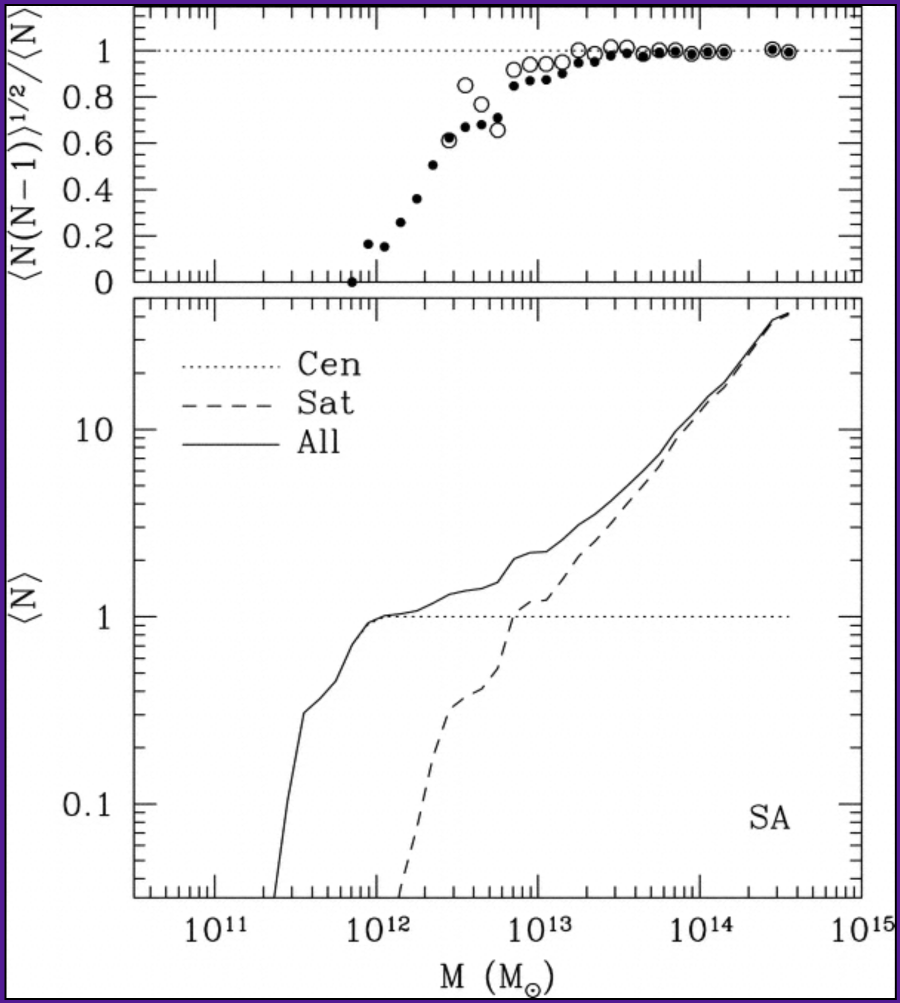




Zheng, Berlind et al. (2005)

How do we parameterize the HOD?

Semi-Analytic



Zheng, Berlind et al. (2005)

How do we parameterize the HOD?

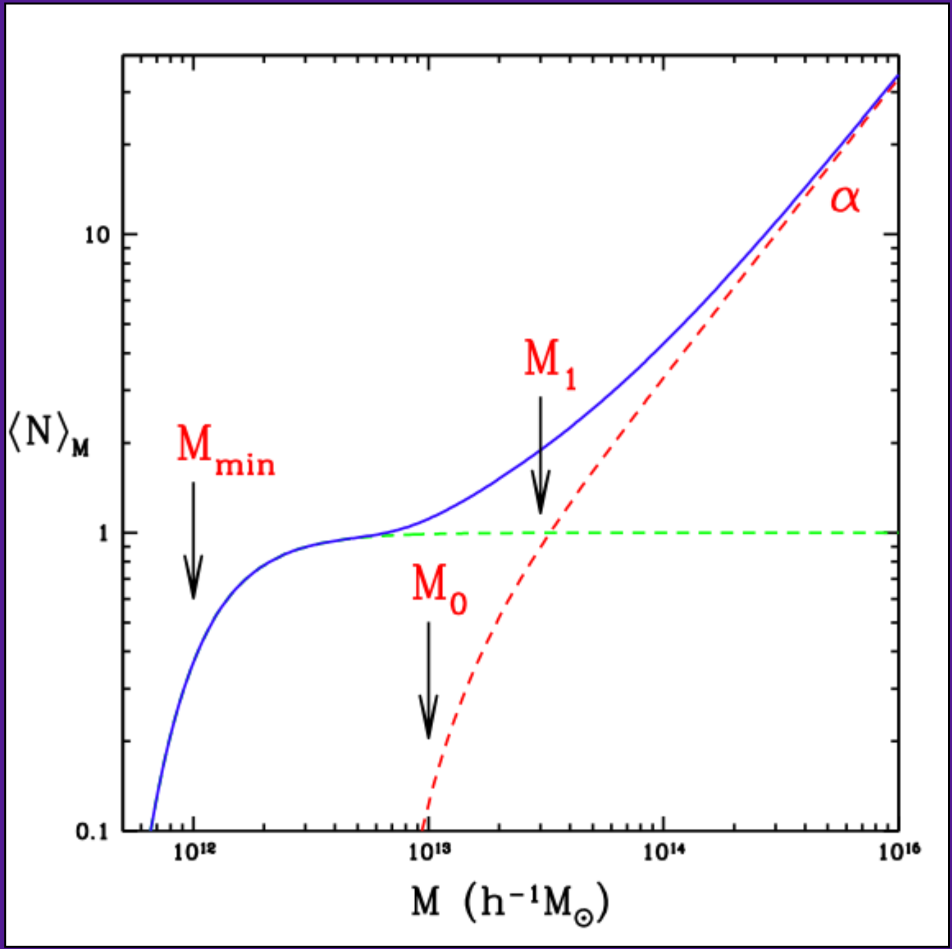
For luminosity/mass threshold samples:

$$N = N_{cen} + N_{sat}$$

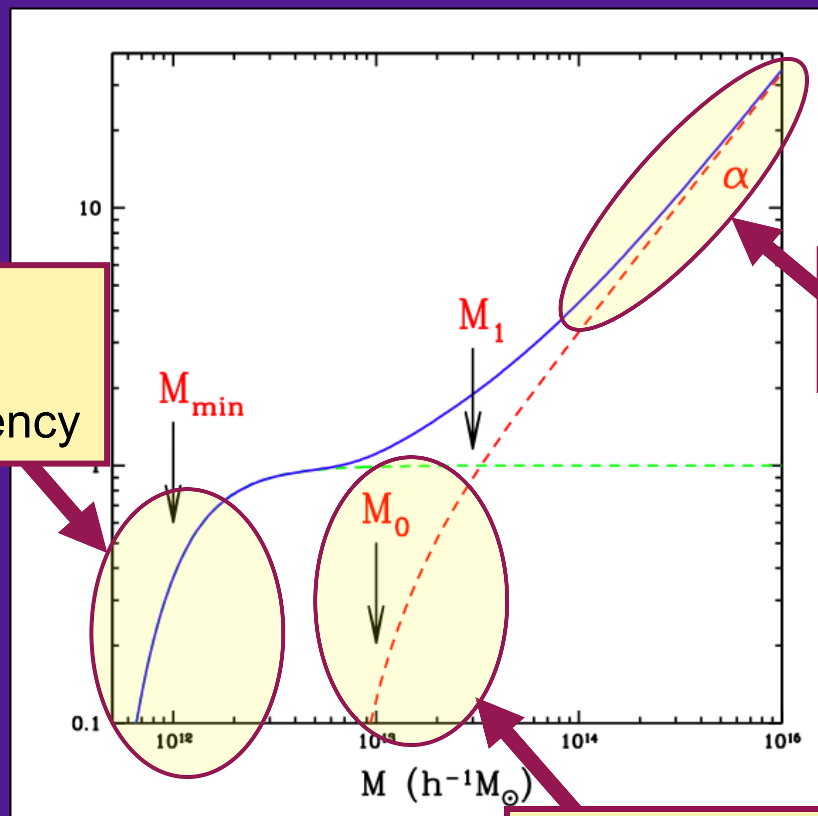
$$N_{cen} = \begin{cases} 0, & M \ll M_{\min} \\ 1, & M \gg M_{\min} \end{cases}$$

$$P\left(N_{sat} \middle| \langle N_{sat} \rangle\right) = \frac{\langle N_{sat} \rangle^{N_{sat}}}{N_{sat}!} e^{-\langle N_{sat} \rangle}$$

$$\langle N_{sat} \rangle = \begin{cases} 0, & M \ll M_0 \\ \left(\frac{M}{M_1}\right)^{\alpha}, & M \gg M_0 \end{cases}$$



The HOD contains information about physics!



Baryon/DM fraction
Gas cooling
Star formation efficiency

Dynamical friction
Tidal disruption

DM halo merger statistics

How do we compute clustering statistics?

Number density

$$n_g = \int_0^\infty dM \, \frac{dn}{dM} \langle N \rangle_M$$

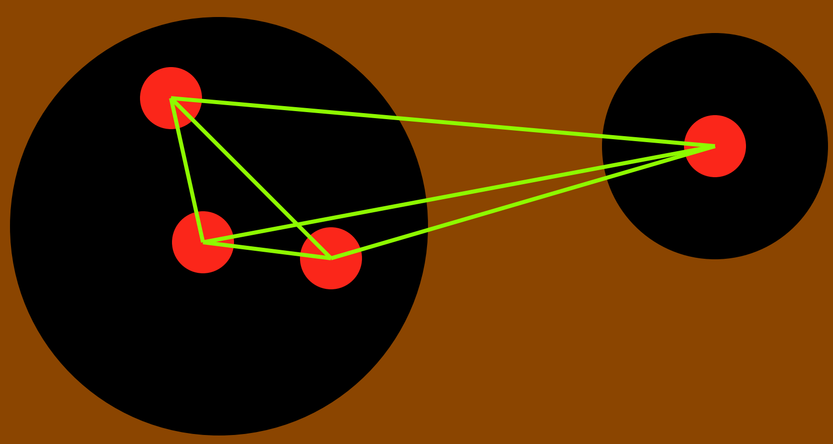
How do we compute clustering statistics?

2-pointCorrelation function

Small scales: All pairs come from same halo.
1-halo term

$$1+\xi_g^{1h}(r)=\left(2\pi r^2 n_g^2\right)^{-1}\int_0^\infty dM\,\frac{dn}{dM}\,\frac{\langle N(N-1)\rangle_M}{2}\,\lambda(r|M)$$

Large scales: Pairs come from separate halos.
2-halo term



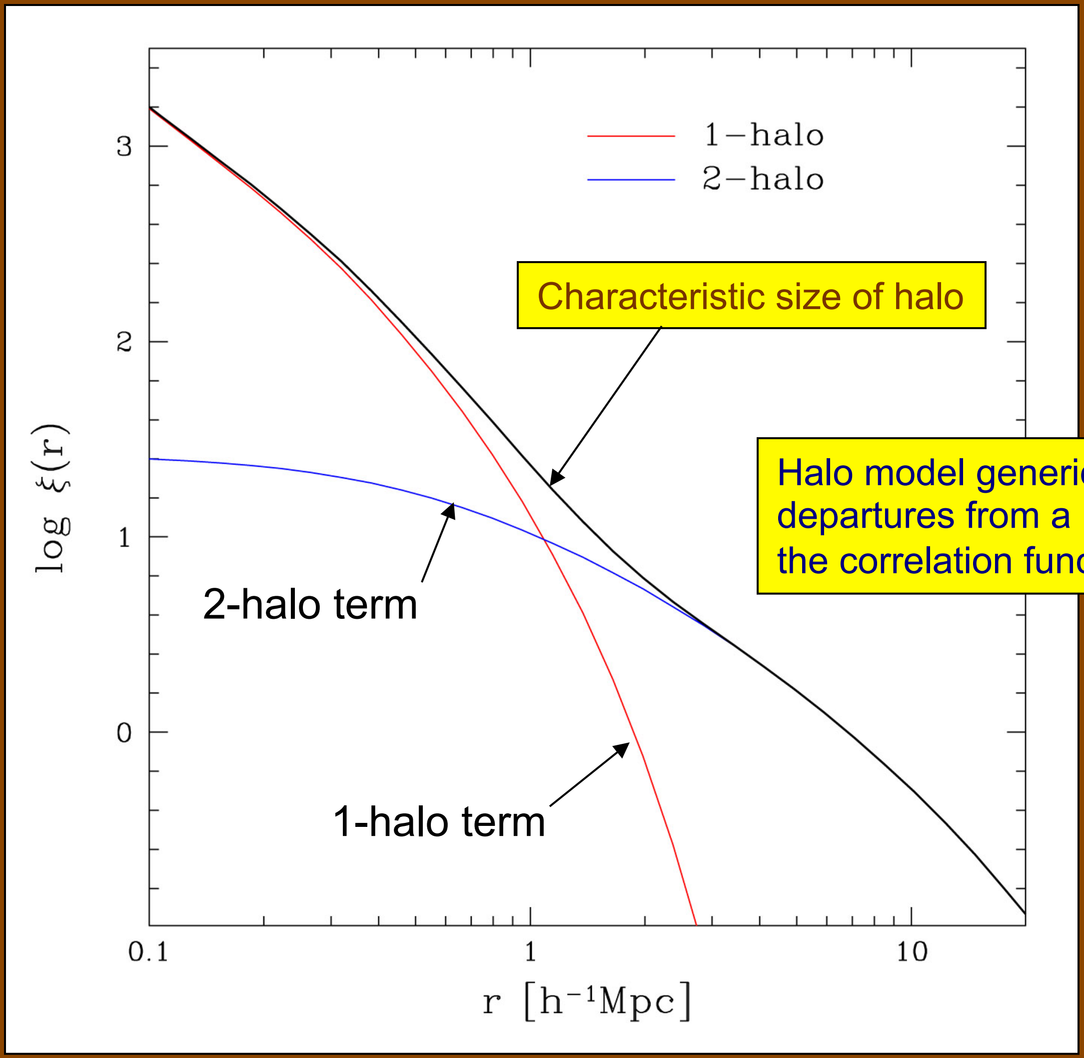
$$\xi_g(r)=b_g{}^2\xi_m(r)$$

$$b_g=n_g^{-1}\int_0^\infty dM\,\frac{dn}{dM}\langle N\rangle_M b_h(M)$$

Berlind & Weinberg (2002)

How do we compute clustering statistics?

2-point correlation function



Halo model generically predicts departures from a power law in the correlation function.

How do we compute clustering statistics?

N-point correlation functions

3-point function has 3 terms: 1-halo, 2-halo, 3-halo
1-halo term depends on <N(N-1)(N-2)>

Redshift-space and velocity statistics

Need model for velocity distribution in DM halo
+ velocity bias for galaxies

Luminosity function

$$\Phi(L) = \int_0^\infty dM \, \frac{dn}{dM} \langle N(M, L) \rangle$$

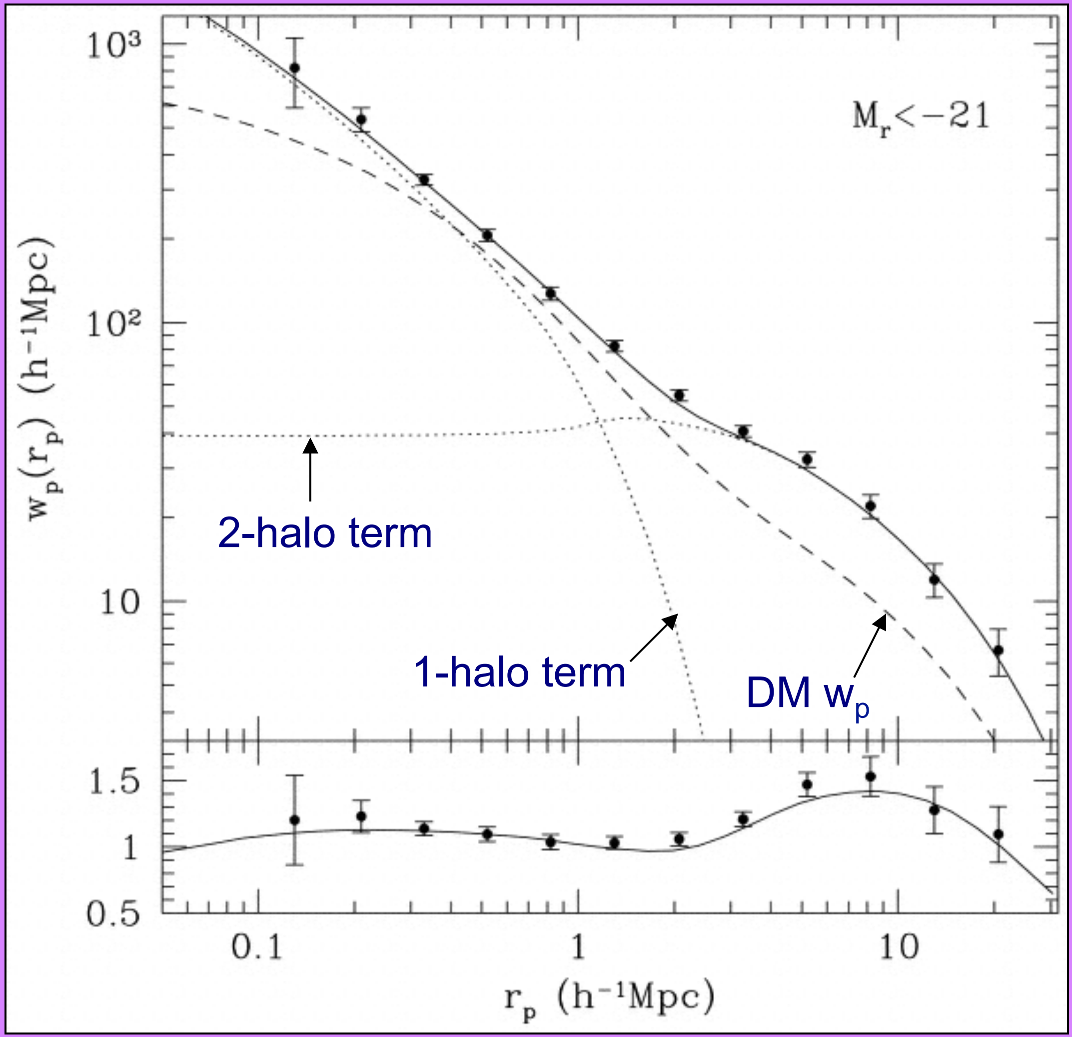
How do we compute clustering statistics?

Improvements to standard halo model

- Non-linear P(k) in 2-halo term
- Scale dependence of halo bias: b(M,r)
- Halo exclusion
- Non-spherical halos
- Non-NFW profiles
- Dependence of b(M) and/or P(N|M) on halo assembly history
- Parameterize P(N|M) for non-trivial galaxy populations

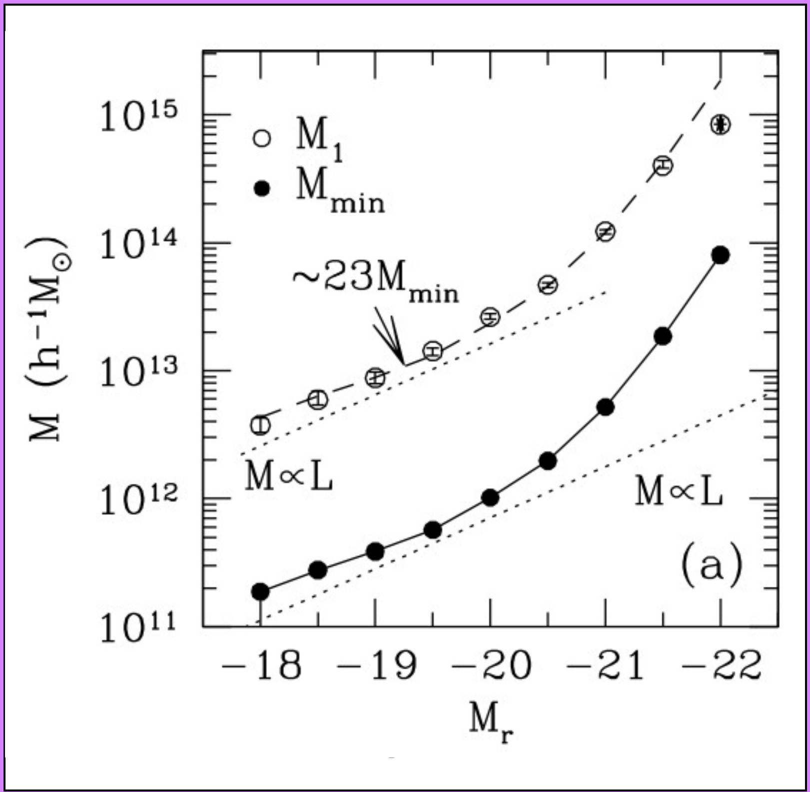
Measurements of the HOD

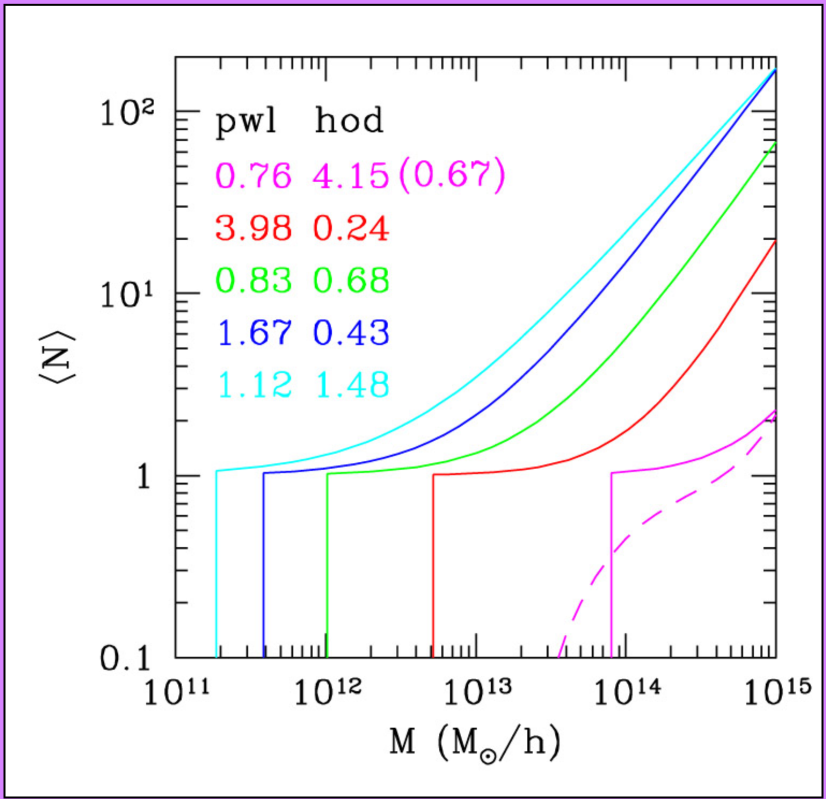
Deviation from power law detected. Halo model gives a good fit to the data. (χ^2*/dof = 0.93 vs. 6.12 for plaw*)



Zehavi et al. (2004)

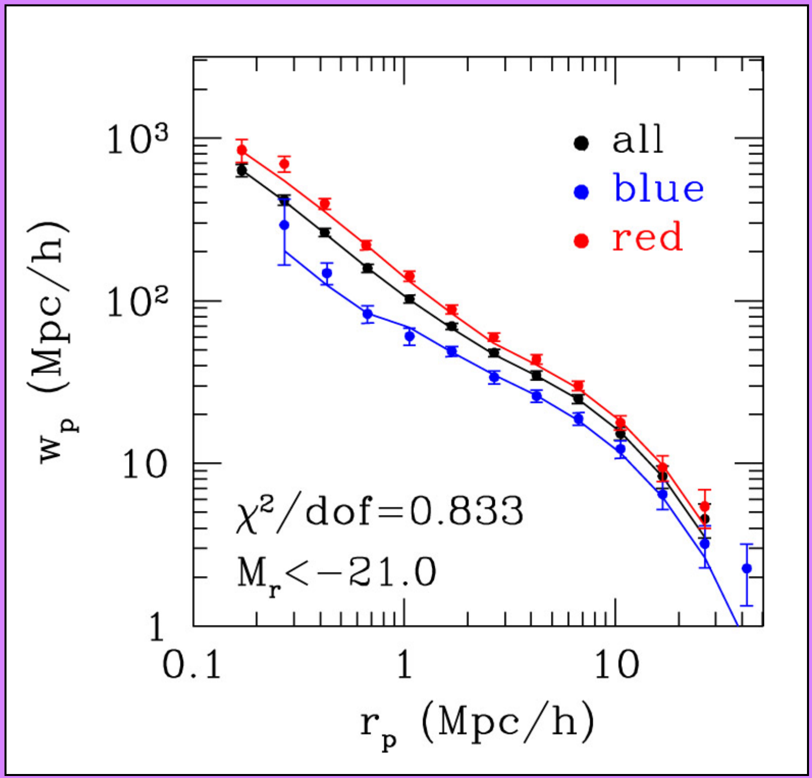
Measurements of the HOD

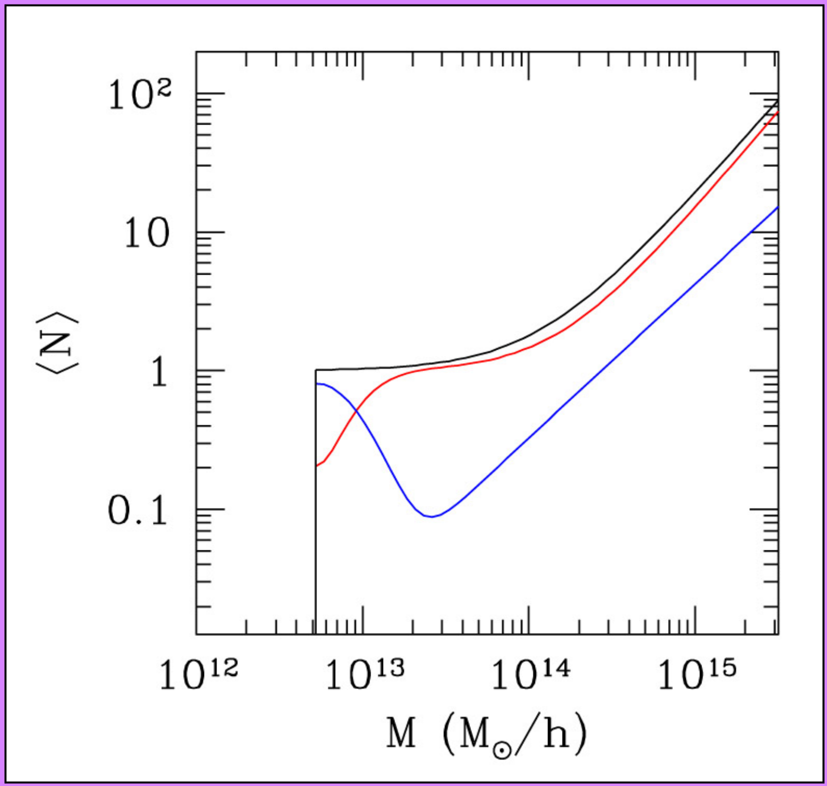




Zehavi et al. (2005)

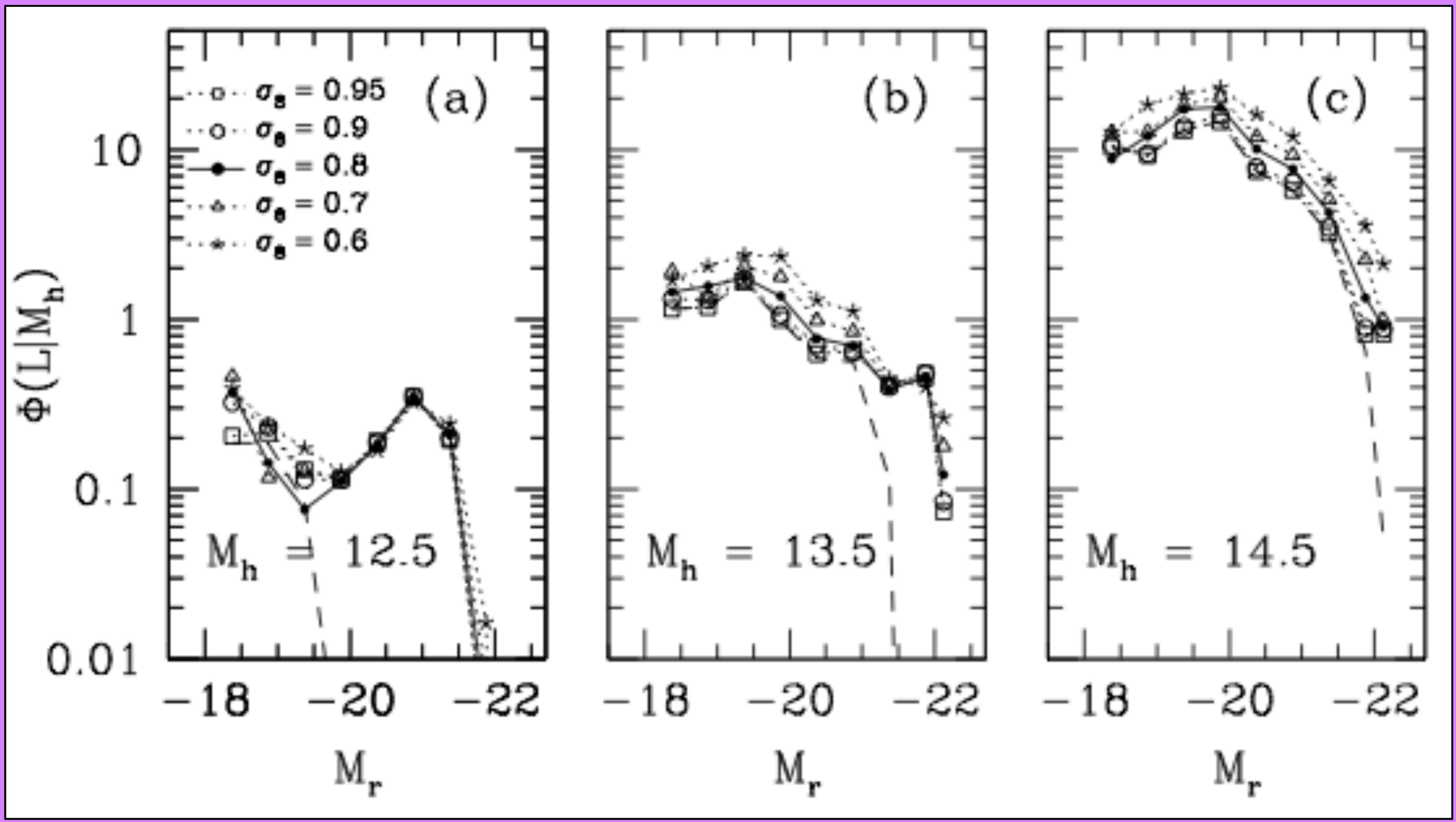
Measurements of the HOD





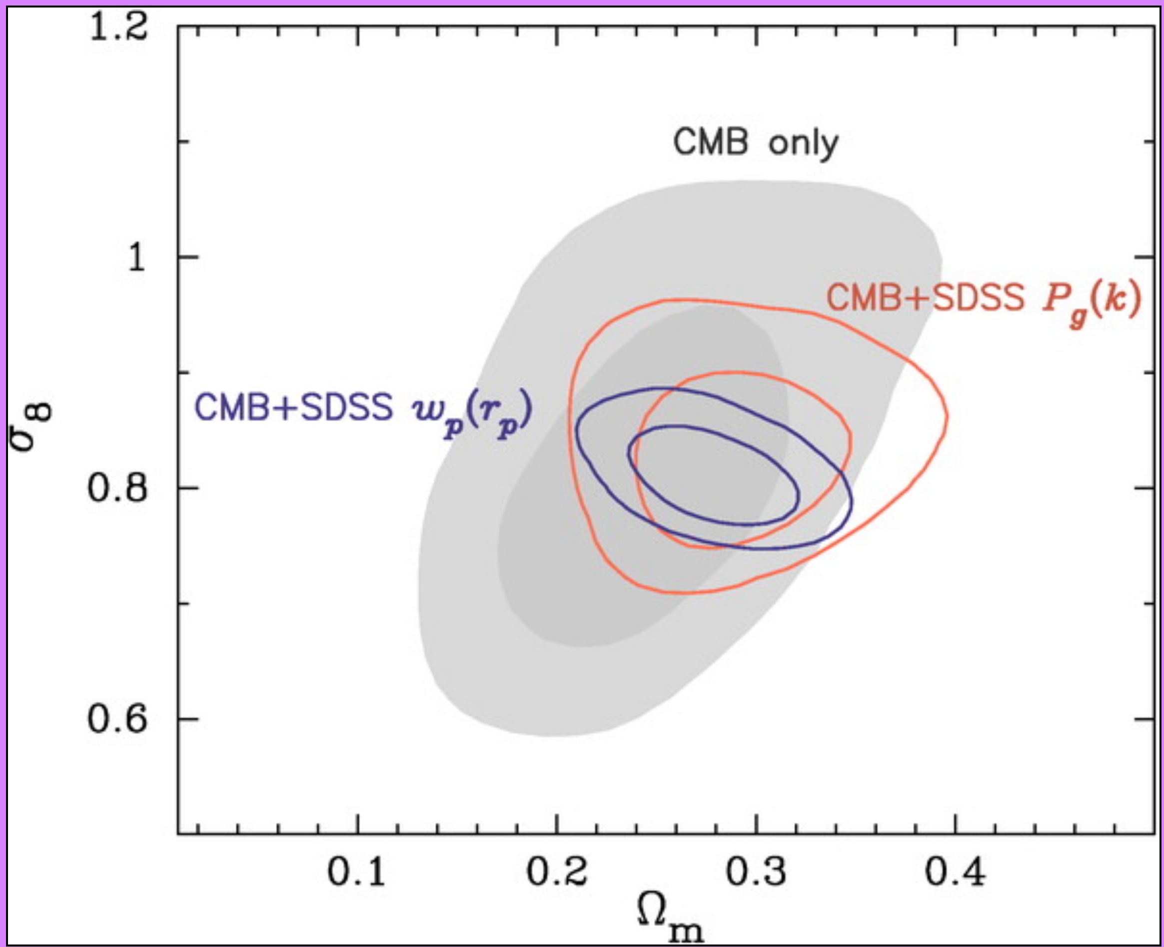
Zehavi et al. (2005)

Measurements of the HOD

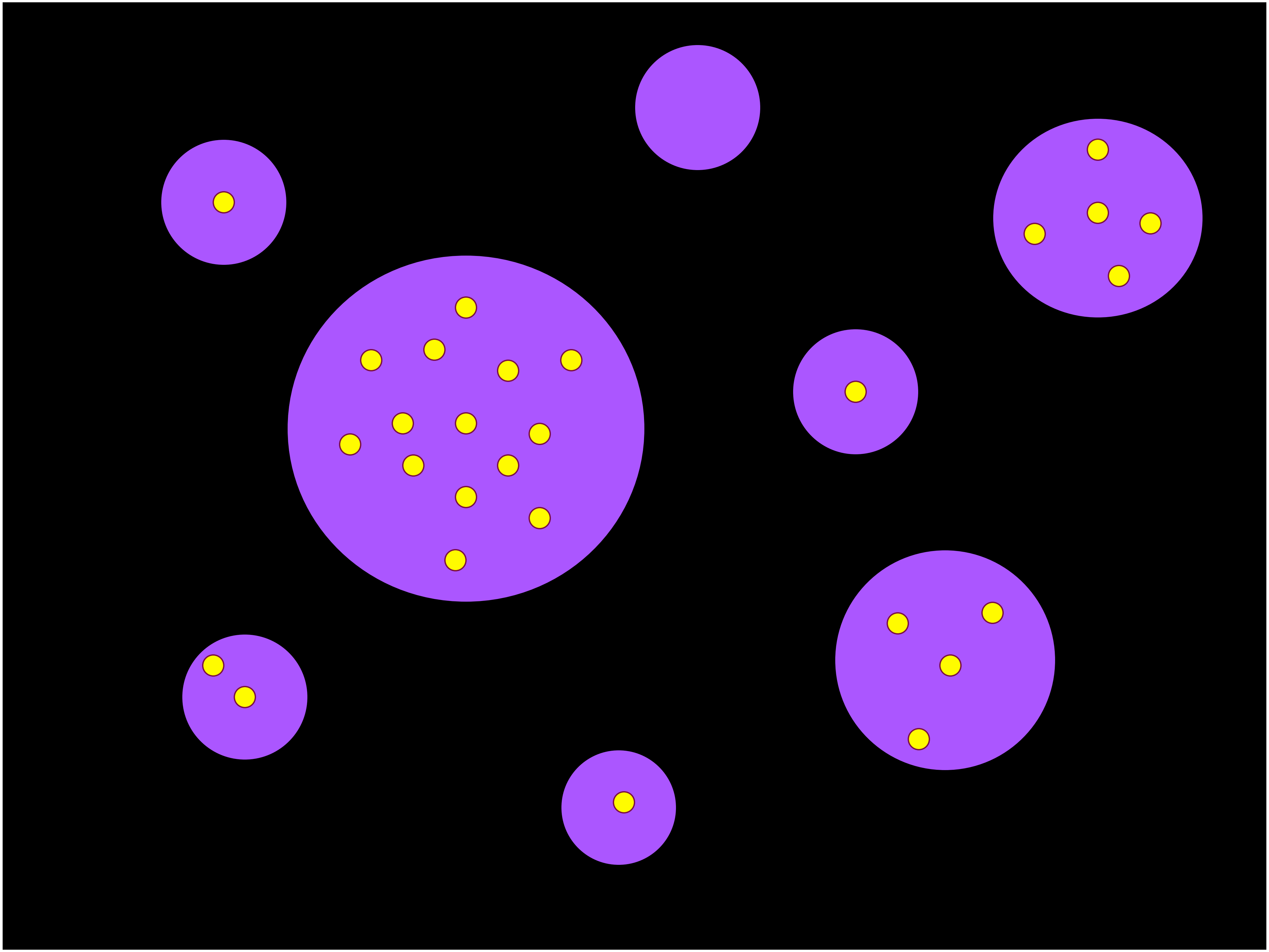


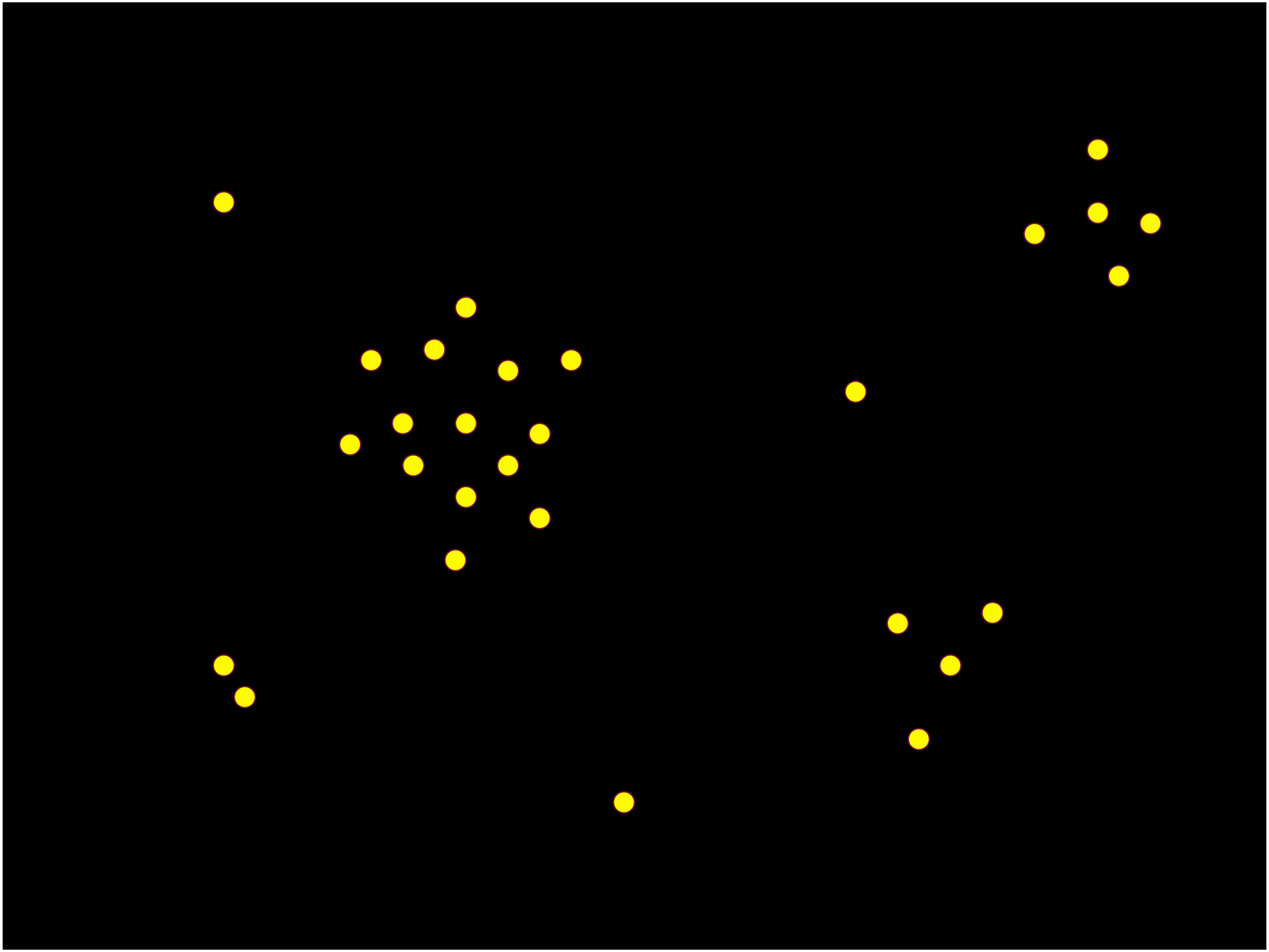
Tinker et al. (2003)

Measurements of cosmology



Abazajian et al. (2003)





How do we compute clustering statistics?

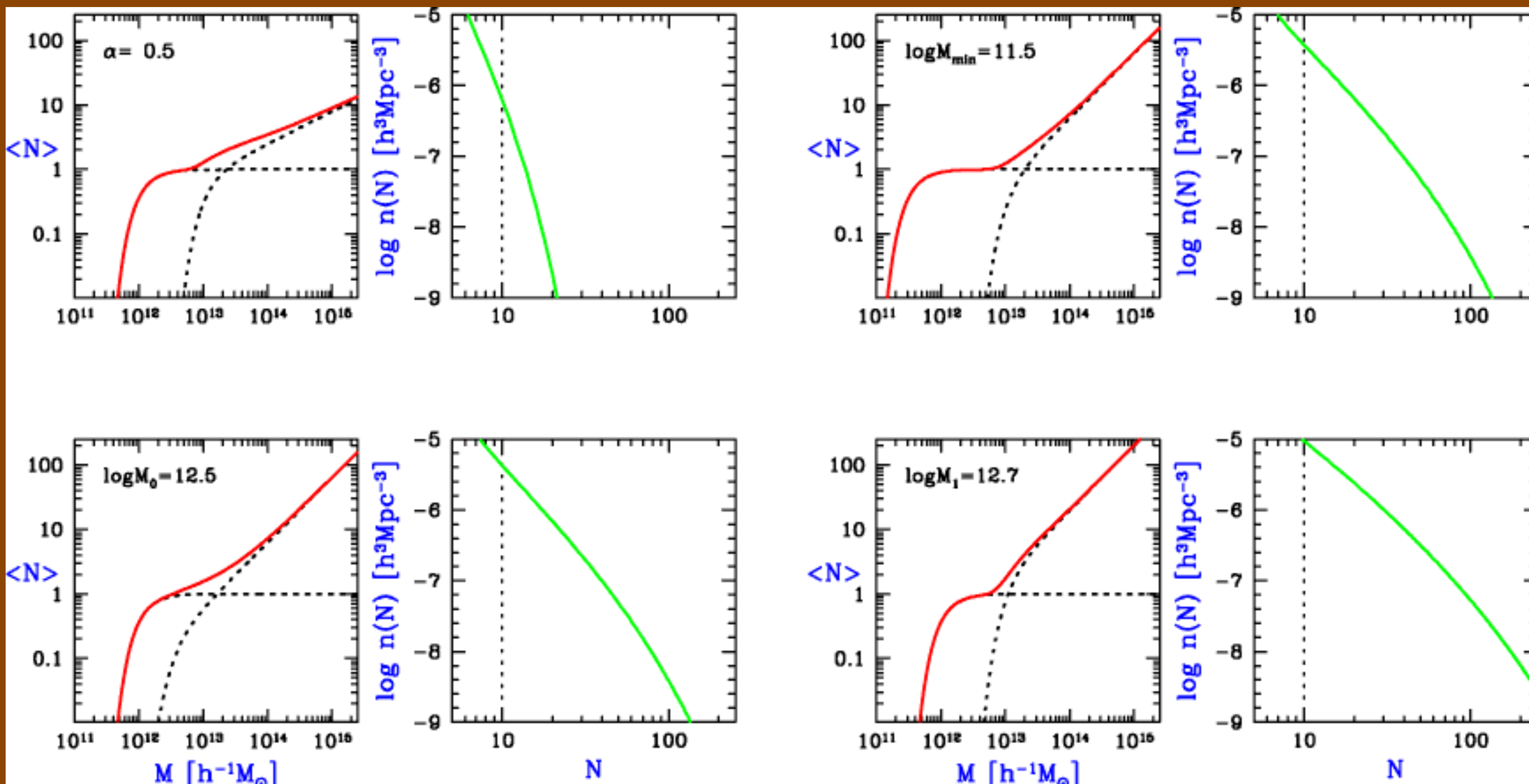
Group/cluster multiplicity function

$$n(N) = \int_0^\infty dM \, \frac{dn}{dM} \, P(N|M)$$

Berlind & Weinberg (2002)

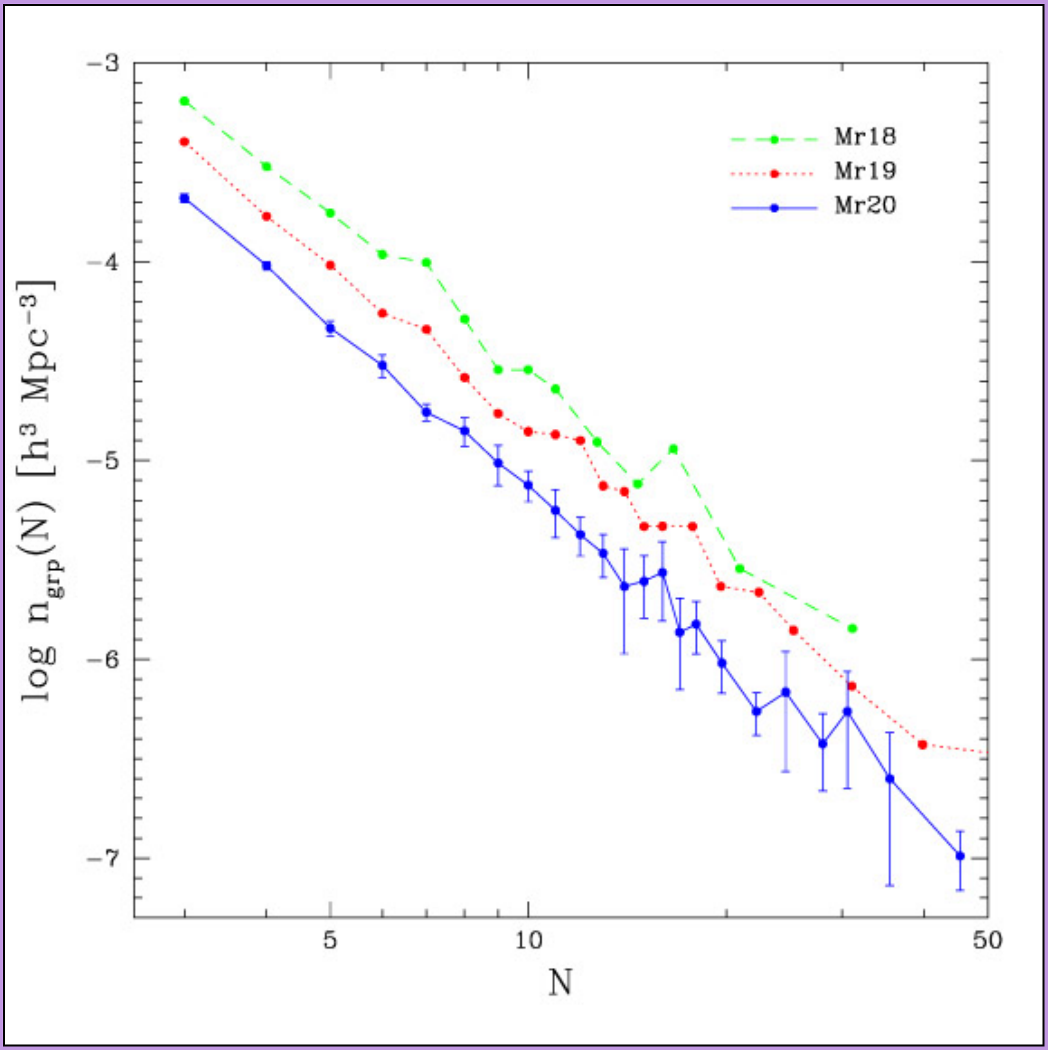
How do we compute clustering statistics?

α M_{min}



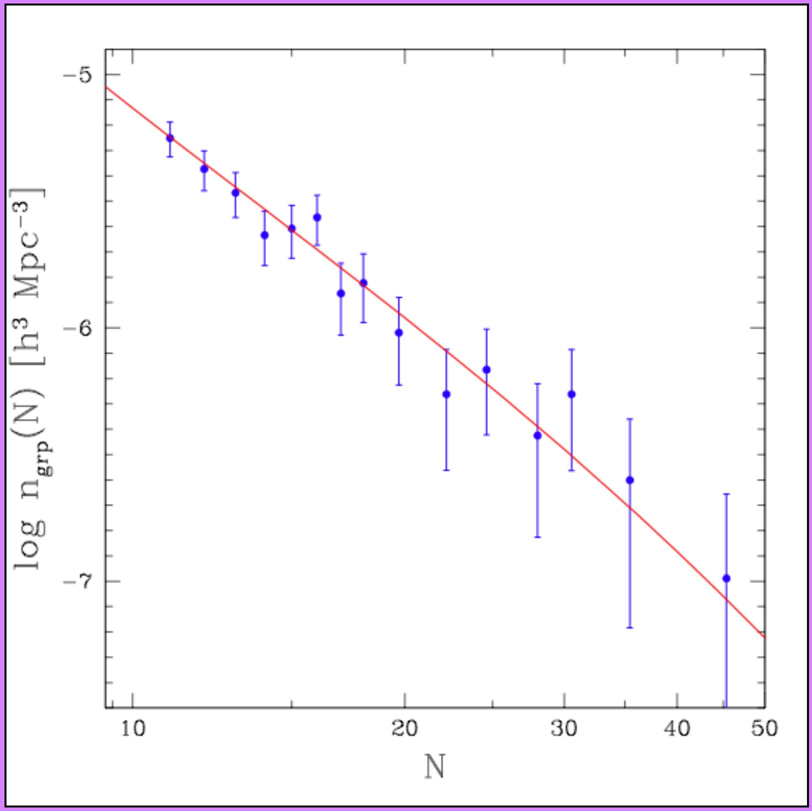
M_0 M_1

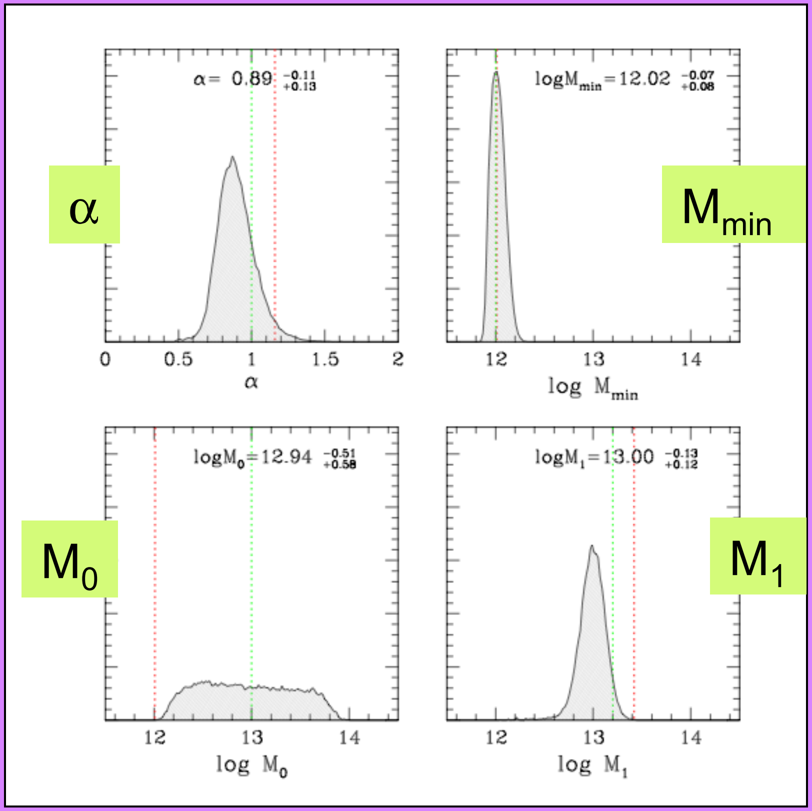
Group Multiplicity Function



Berlind et al. (2006)

Measurements of the HOD





Berlind et al. in prep.

Testing the halo model assumptions

1. All galaxies live in halos. ✔

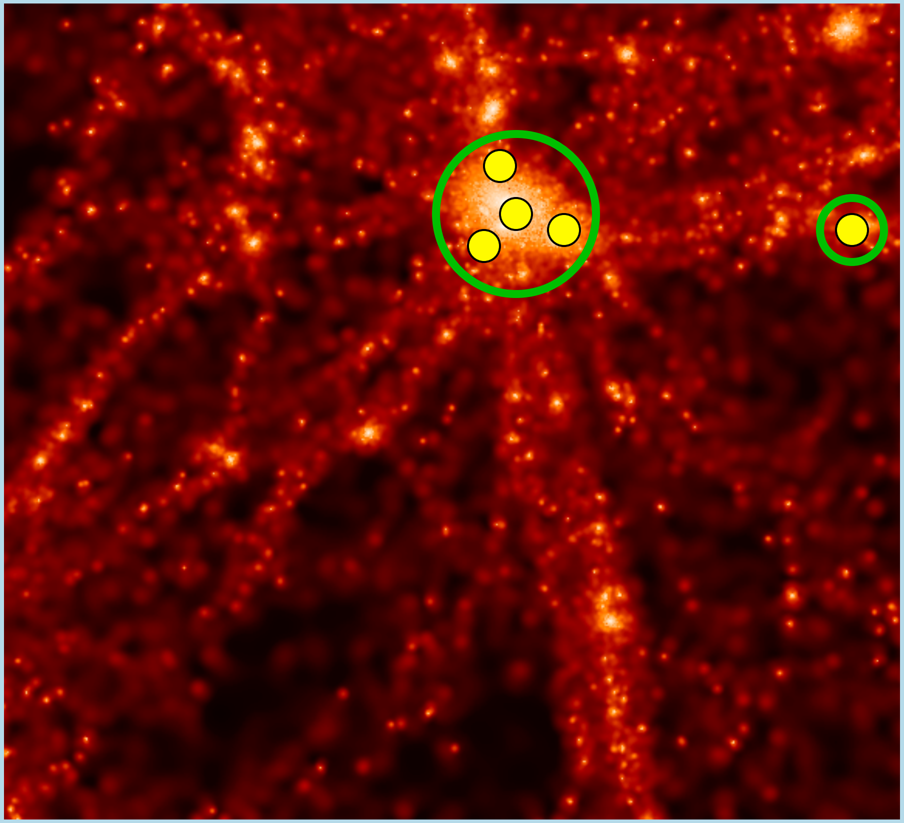
2. The statistical content of halos depends only on halo mass.
   i.e., P(N|M) is sufficient, as opposed to P(N|M,X)

   Recent work shows that halo bias $b_h(M)$ depends on halo assembly history at fixed mass. If $\langle N(M) \rangle$ also shows this dependence, then standard halo model will be incorrect.

   Preliminary work shows that this effect is not greater than ~5% for *L** galaxies. This systematic effect needs to be addressed if galaxy clustering is to be used for precision cosmology.

Alternative to the analytic halo model approach

Populate an N-body simulation with an HOD to compute clustering statistics instead of using analytic formulas.



Alternative to the analytic halo model approach

Populate an N-body simulation with an HOD to compute clustering statistics instead of using analytic formulas.

Advantages:

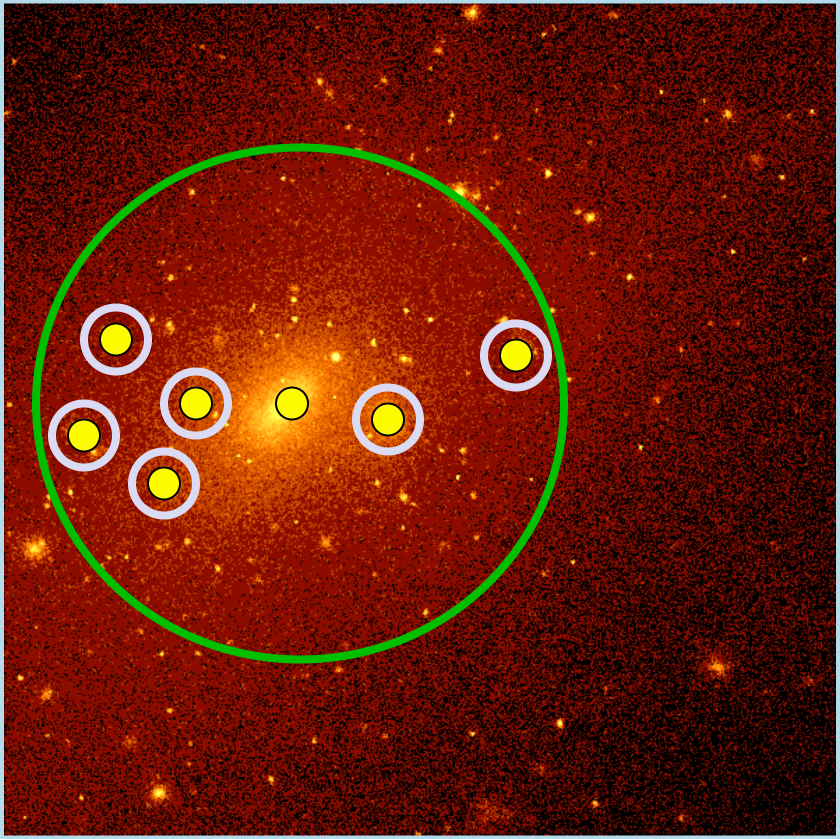
- Halo clustering, abundances, and profiles are correct on all scales above the simulation's resolution limit.
- Can calculate any clustering statistic.

Disadvantages:

- Not good for very small scale clustering.
- Much slower.

Alternatives to the halo model / HOD approach

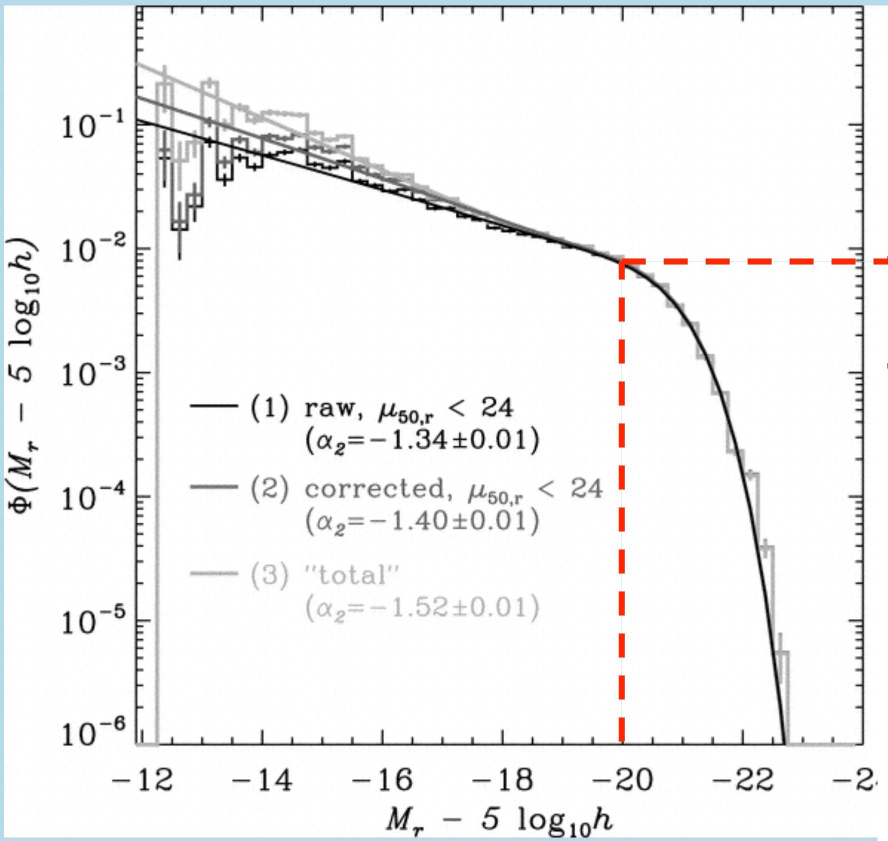
Use a high resolution N-body simulation to place galaxies in halos + subhalos, assuming relations between galaxy and subhalo properties. (i.e., use subhalo distribution instead of HOD)

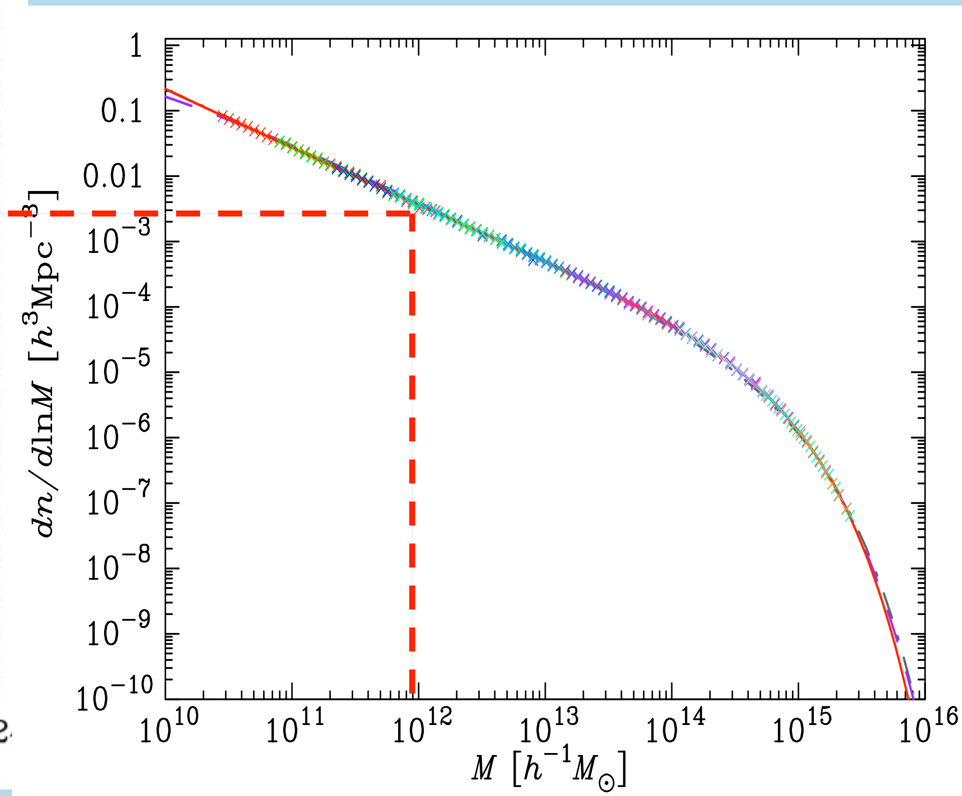


Halo/Subhalo Abundance Matching

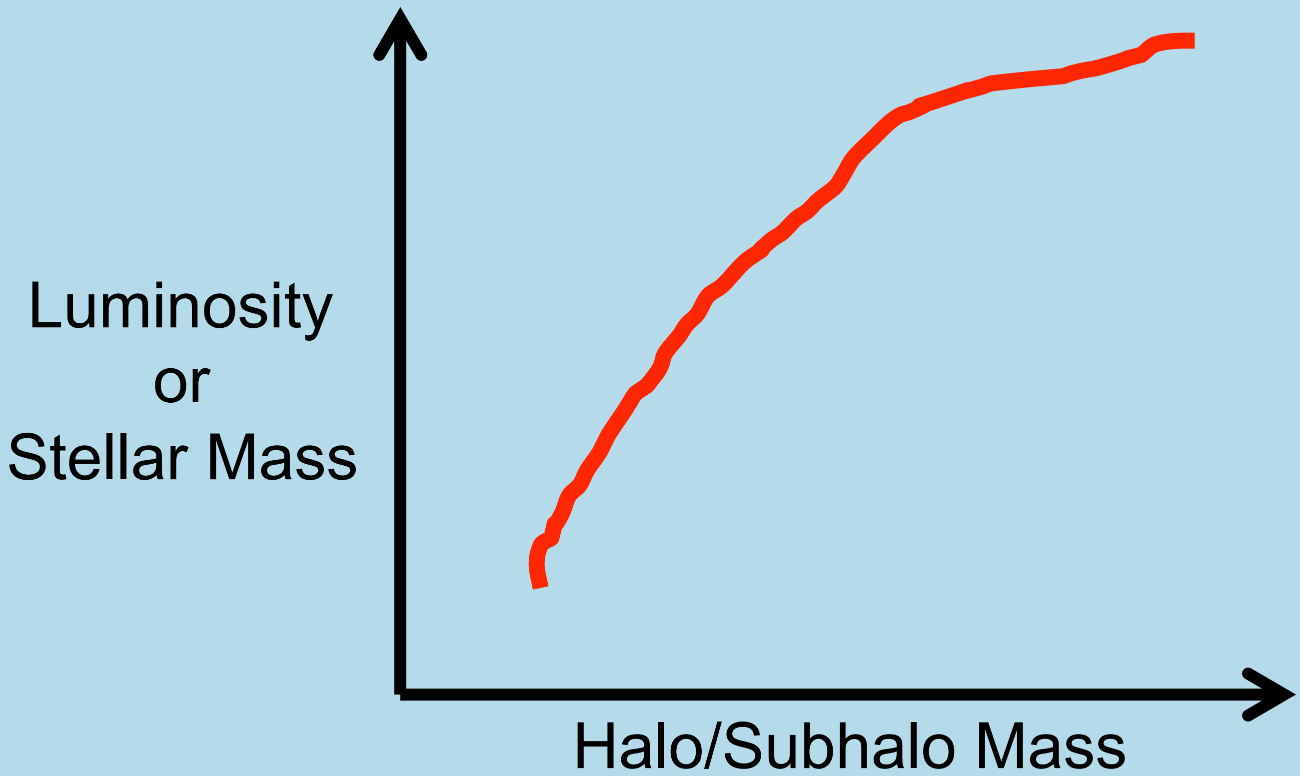
Galaxy luminosity function

Halo mass function
(including subhalos)



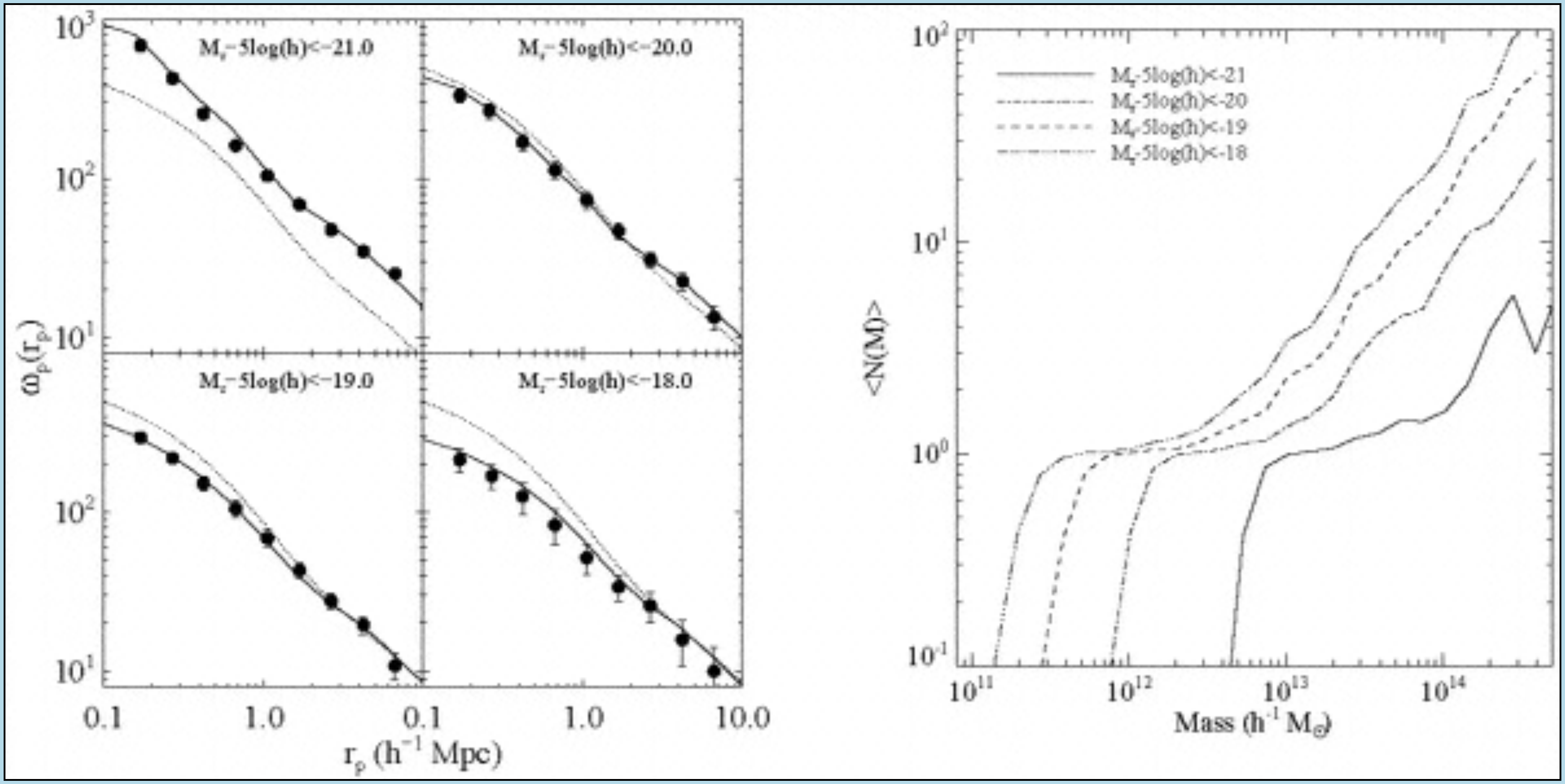


Halo/Subhalo Abundance Matching



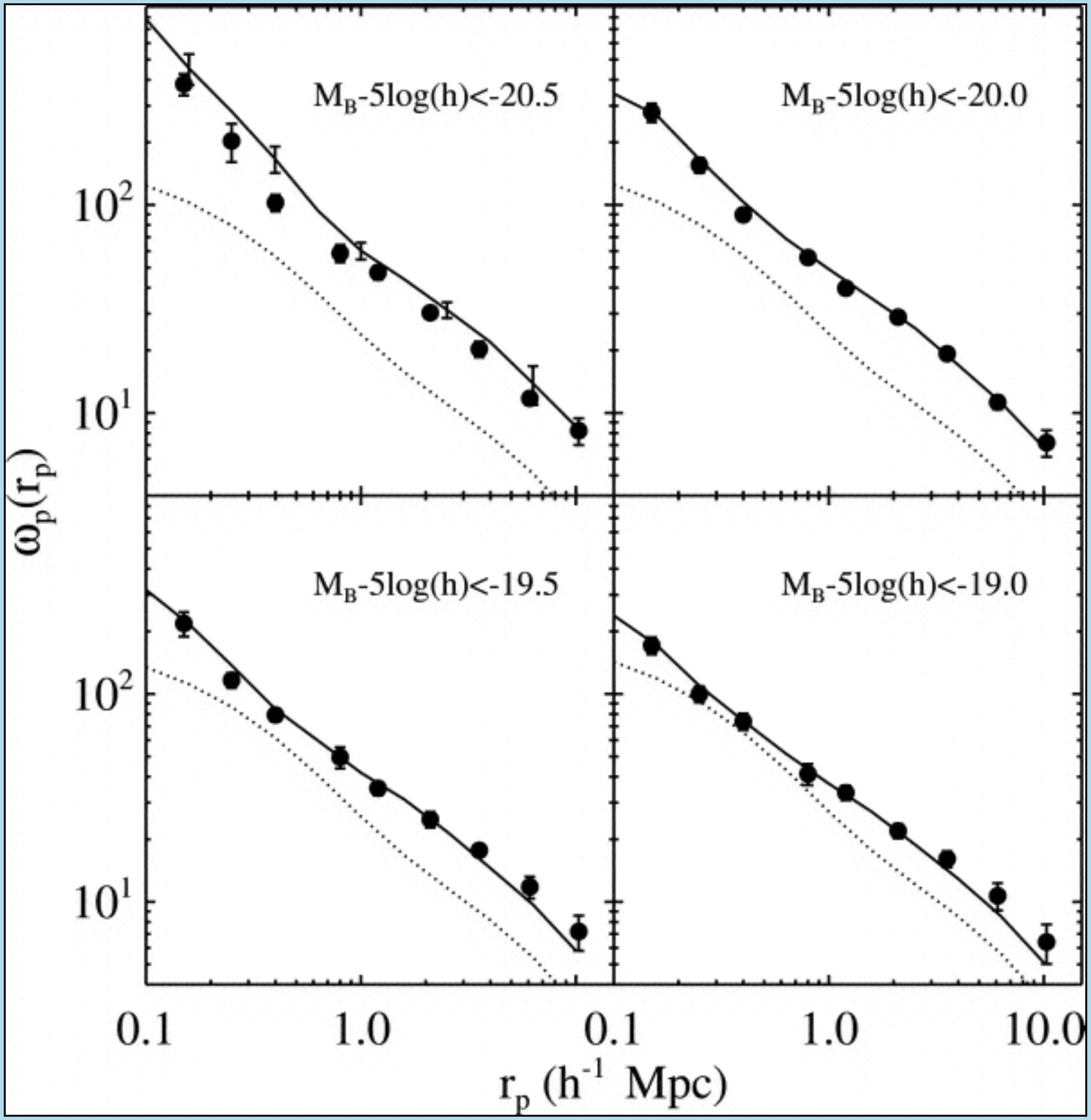
Halo/Subhalo Abundance Matching

SDSS
z~0



Conroy et al. (2006)

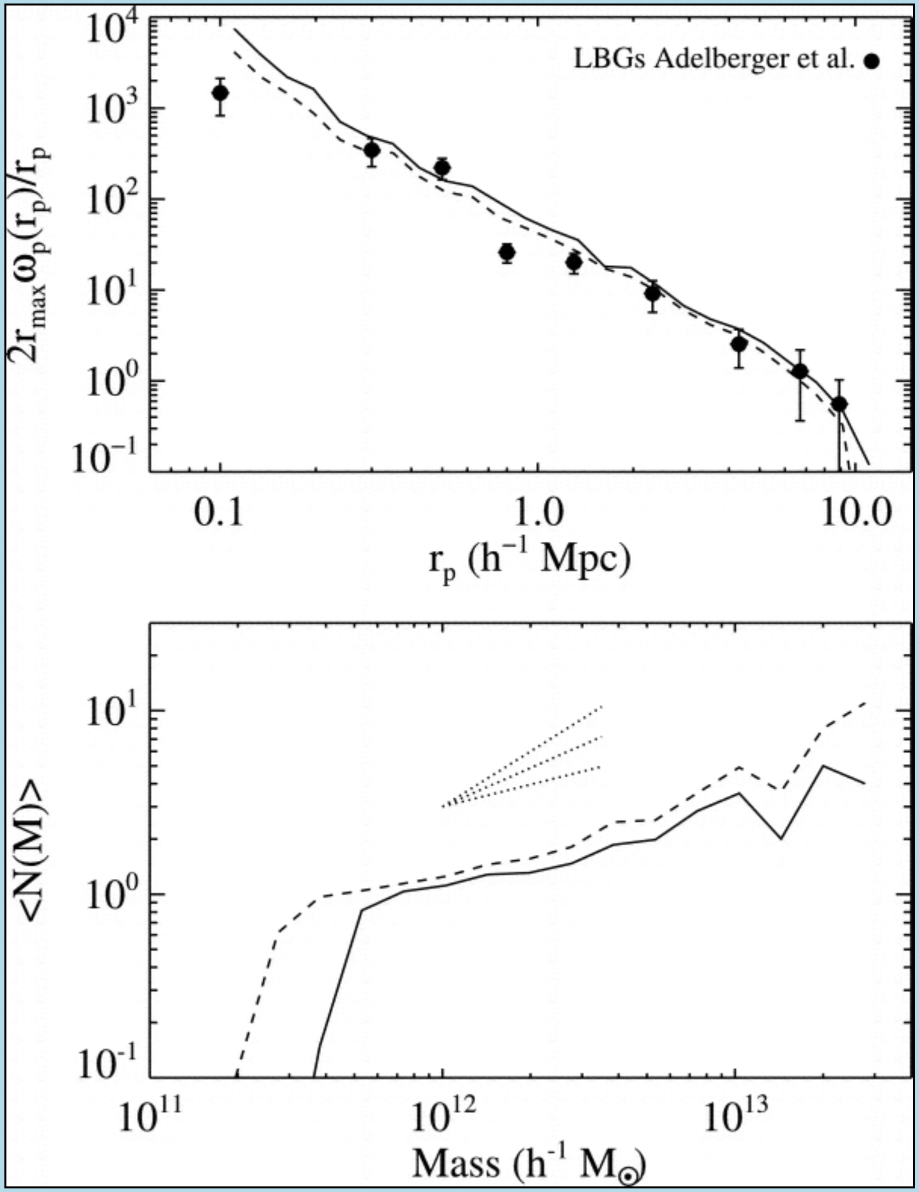
Halo/Subhalo Abundance Matching



Conroy et al. (2006)

DEEP2
z~1

Halo/Subhalo Abundance Matching

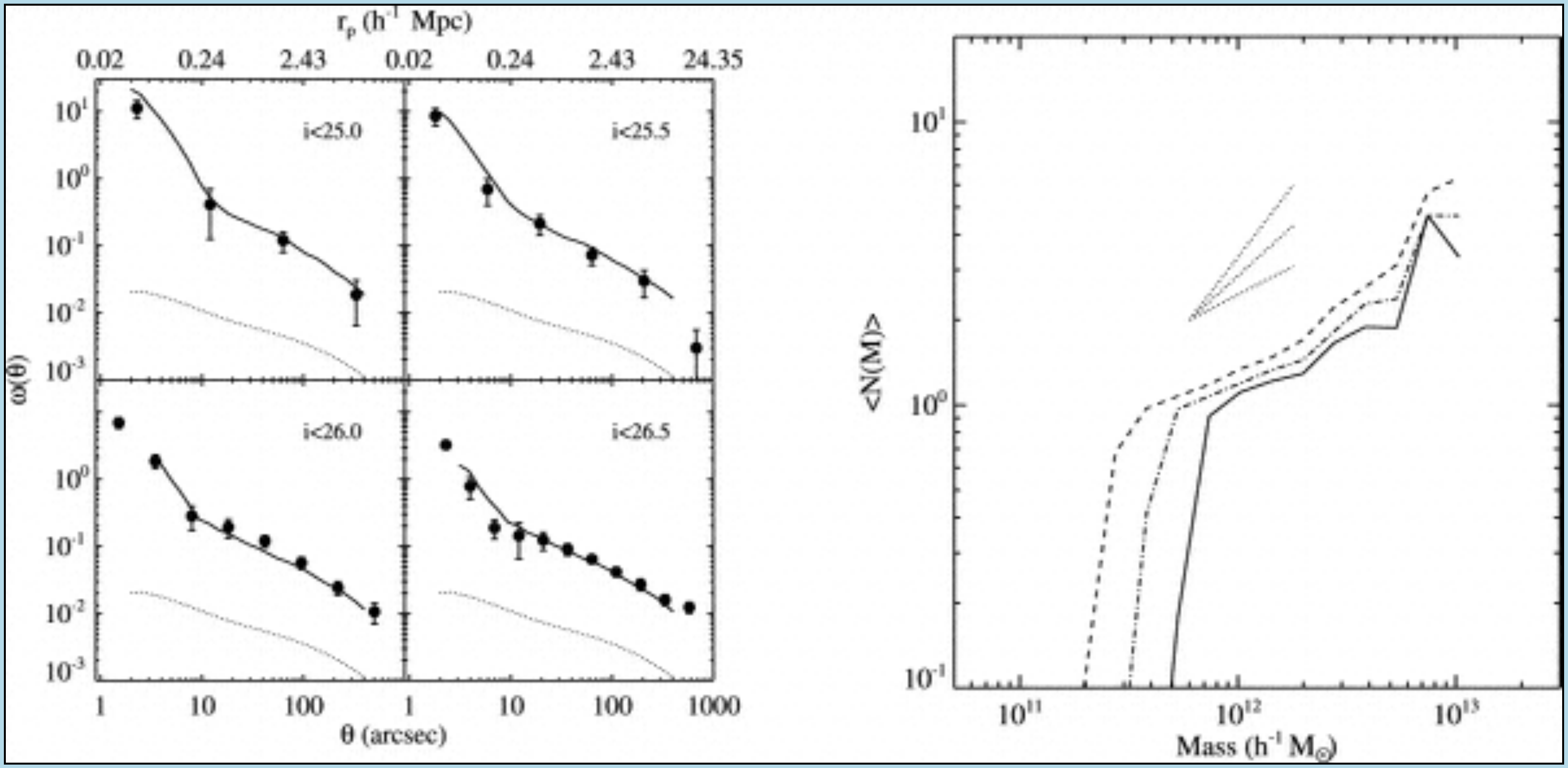


Conroy et al. (2006)

LBGs
z~3

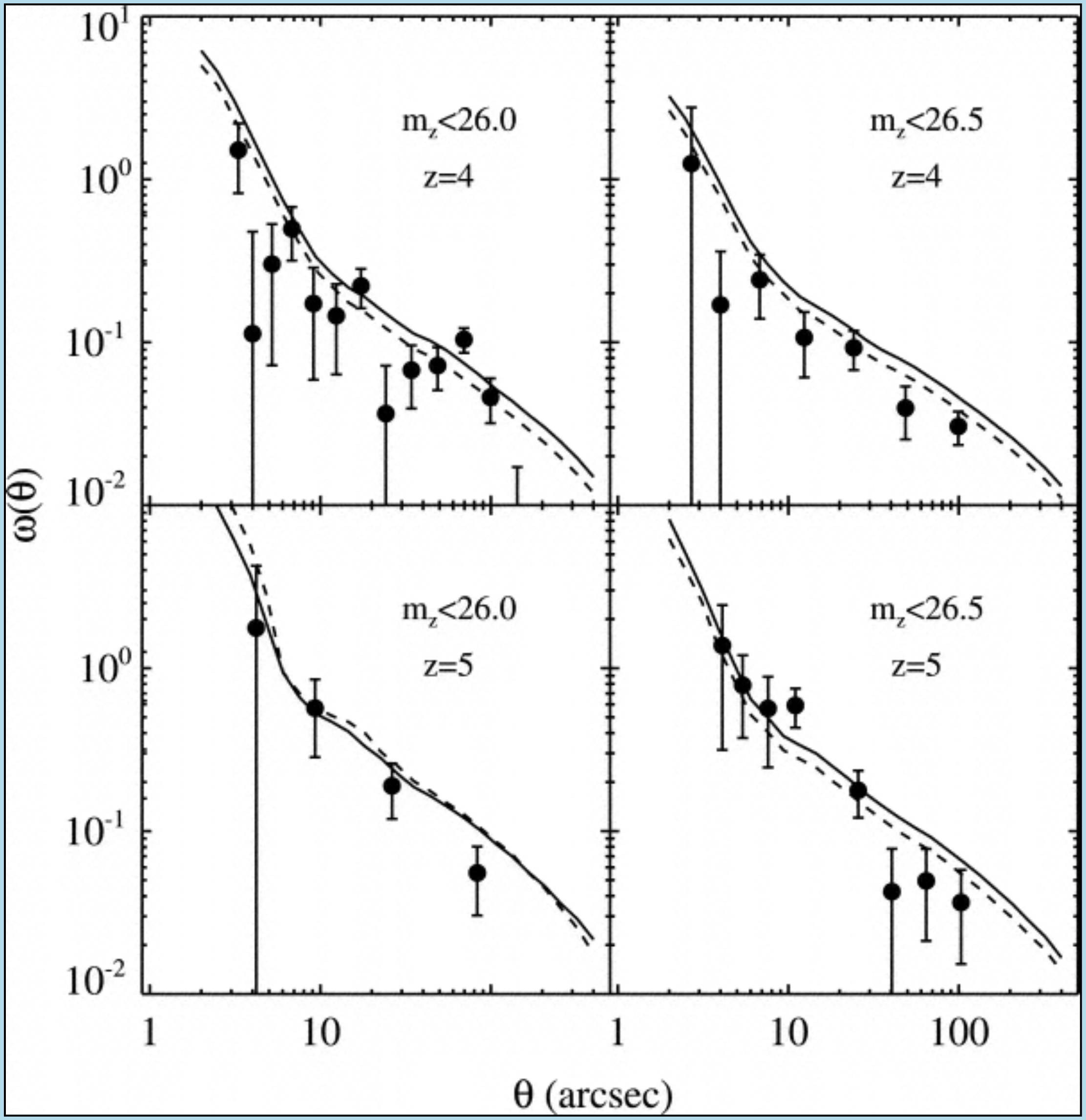
Halo/Subhalo Abundance Matching

Subaru
z~4



Conroy et al. (2006)

Halo/Subhalo Abundance Matching

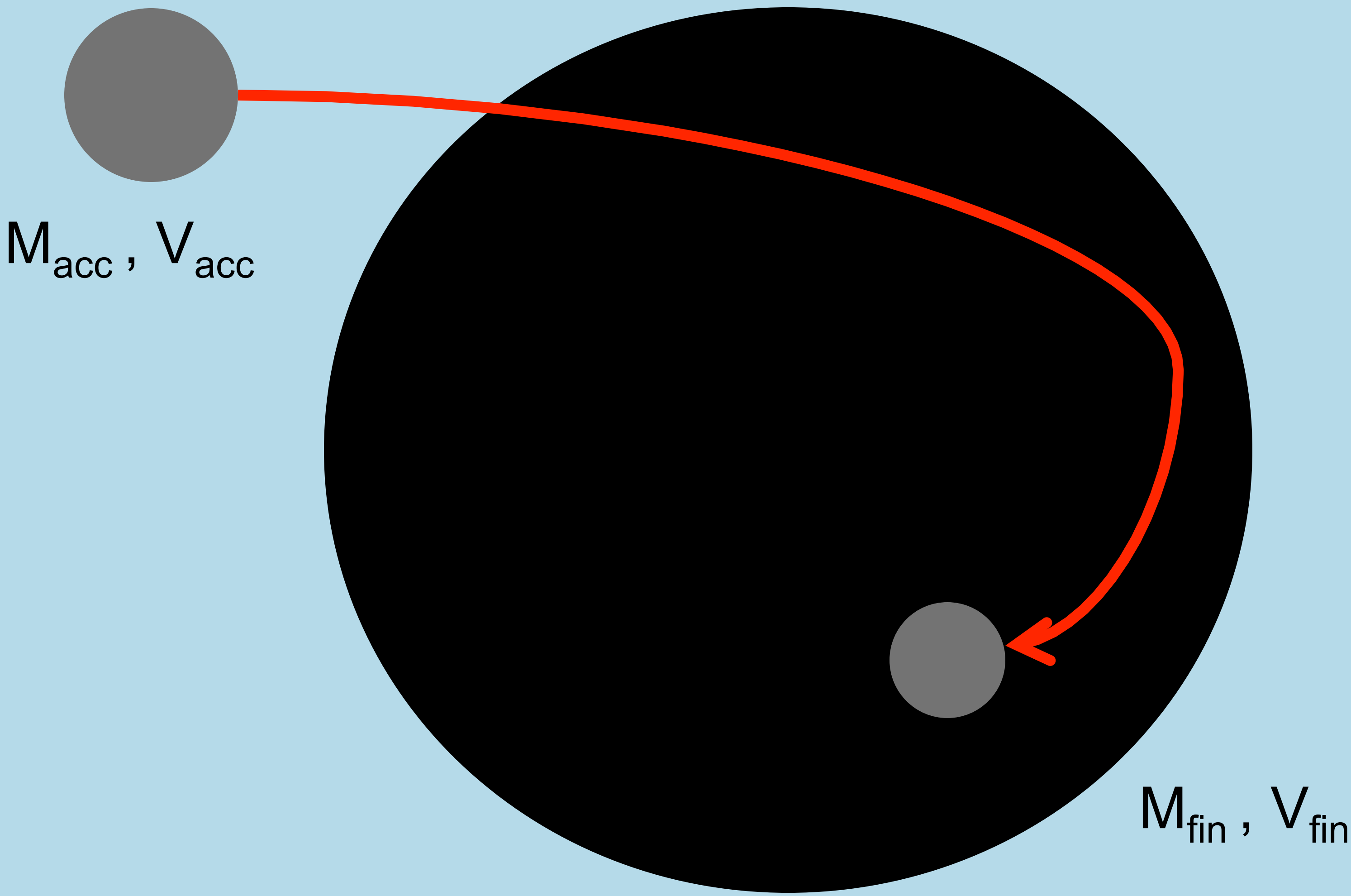


GOODS
z~4-5

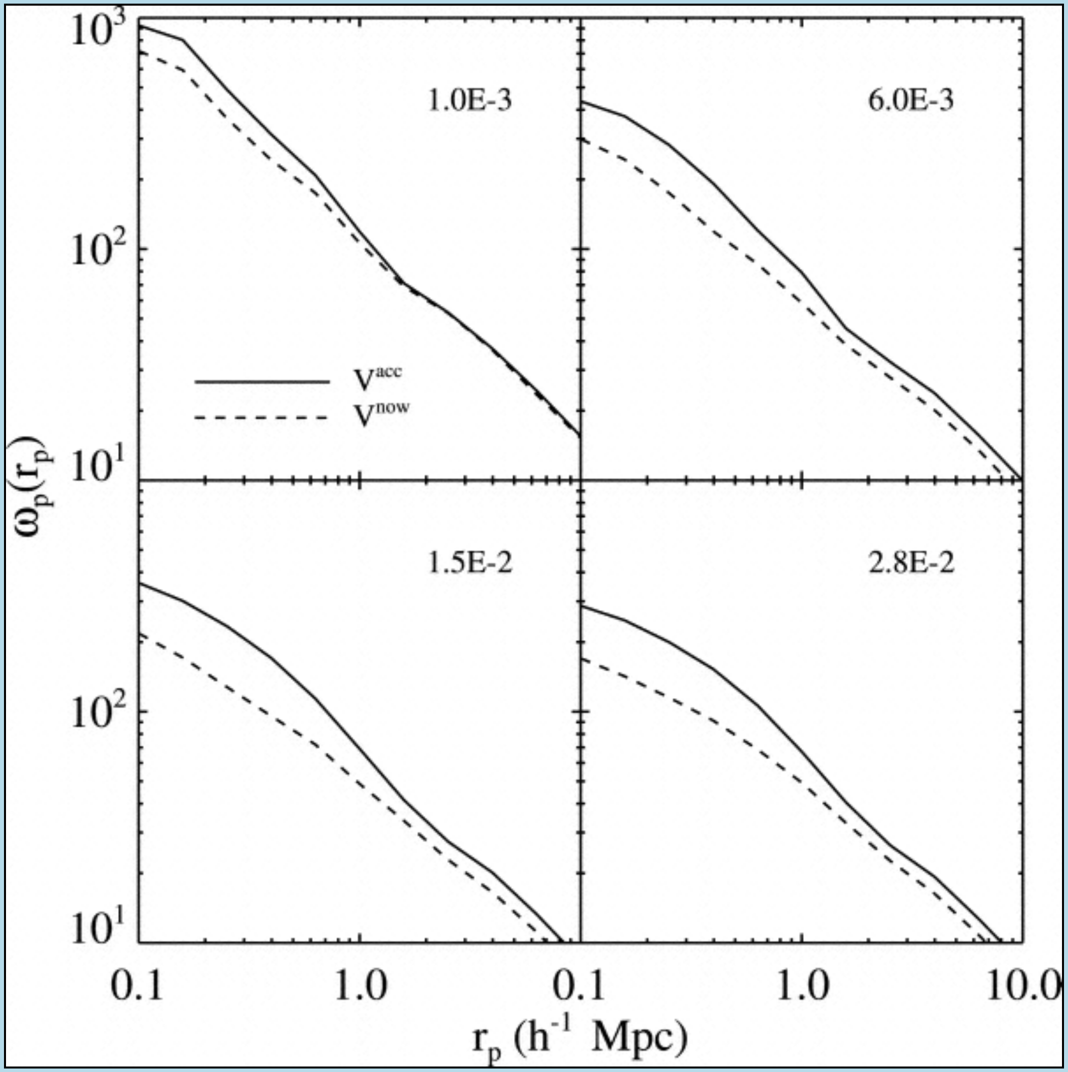
Conroy et al. (2006)

Halo/Subhalo Abundance Matching

What subhalo property should be used?

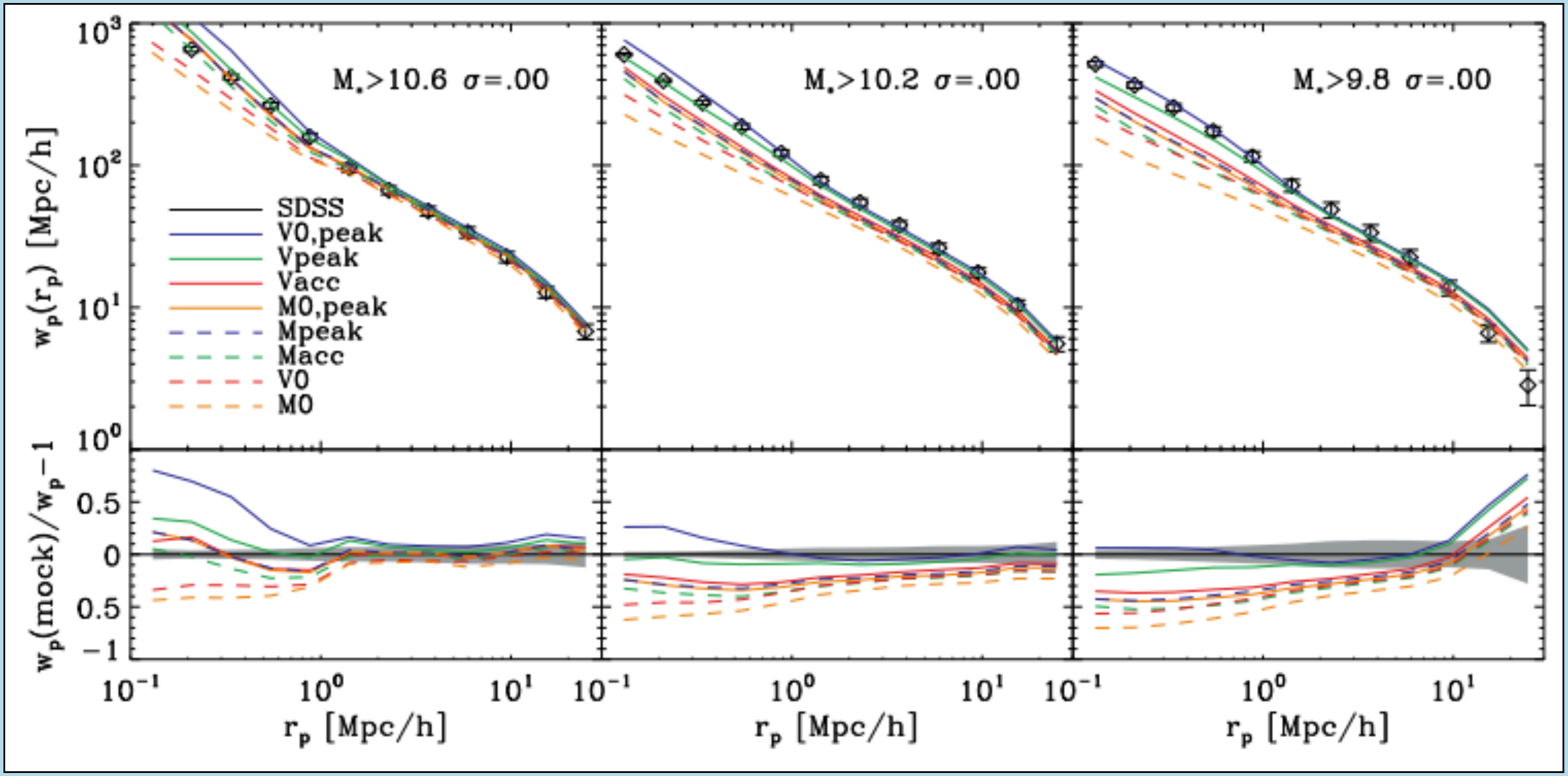


Halo/Subhalo Abundance Matching



Conroy et al. (2006)

Halo/Subhalo Abundance Matching



Reddick et al. (2013)

Alternatives to the halo model / HOD approach

Use a high resolution N-body simulation to place galaxies in halos + subhalos, assuming relations between galaxy and subhalo properties. (i.e., use subhalo distribution instead of HOD)

Advantages:

- Let gravity predict what the spatial and velocity distribution of galaxies is.
- Works fairly well for luminosity threshold samples: $L_{gal} \sim M_{sub}$ (Conroy et al. 2006)

Disadvantages:

- Not clear how to populate subhalos with non-trivial galaxy samples (split by color, type, etc).
- Assumes that subhalo evolution within host halos traces that of galaxies.
- Much too slow to constrain cosmology.

Alternatives to the halo model / HOD approach

Use a Conditional Luminosity Function (CLF) to model the luminosity dependence of clustering.

$$\Phi(L) = \int_0^\infty dM \frac{dn}{dm} \Phi(L|M) \qquad \langle N \rangle_M = \int_{L_{\min}}^\infty dL \Phi(L|M)$$

Advantages:

- Don't have to assume a form for <N(M)>
- More ambitious: model the luminosity dependence explicitly

Disadvantages:

- Have to assume a form for $\phi(L|M)$
- More ambitious: luminosity dependence is model dependent

Methods are very similar and complementary.

Describing vs. Understanding

HOD/HAM/CLF are excellent statistical tools for *describing* the wealthof galaxy clustering data: for translating complicated statistics into a more physically informative language.

It is still essential that we *understand* the physics behind the data: gas cooling, star formation, feedback, etc. For this we need ab-initio models such as hydrodynamic simulations and semi-analytic models.

The methods are highly complementary.

SYNOPSIS

- Galaxy properties
- Stellar populations
- Distance measures
- Hubble expansion and z-space distortions
- Redshift surveys
- Galaxy environments
- Galaxy groups and clusters
- Galaxy clustering statistics
- Cosmological parameters
- Expansion history of the universe
- Growth of perturbations
- N-body simulations
- Dark matter halos
- Galaxy formation
- The halo model