Describing Large Scale Structure
Millennium Simulation: Springel et al. (2005)
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Series of correlation functions is needed to completely describe arbitrary distribution.

- two-point correlation function (2PCF), three-point (3PCF), four-point, etc.

- 2PCF alone can fully describe a Gaussian universe.
- Correlation function formalism is Poisson model.
- Formalism developed in late 1970s and early 1980s.
- “easy” to calculate for distribution of points

The Purpose

A physically meaningful parameterization of large scale structure.
2PCF: A simple example
Cosmic Cotton Balls

- Isothermal spheres:
  - higher density center
  - truncated at 1 Mpc radius
- Distributed uniformly in volume
- Note: rapid reduction in correlation
- Note: $\xi(r)$ less than zero at 2 radii
Defining the 2pt correlation function

- Correlation of points with respect to Poisson:
  \[ \delta P = n^2 \delta V_1 \delta V_2 [1 + \xi(r_{12})] \]
- \( \xi(r) \): connected spatial two point correlation function
- Local density fluctuation about the mean:
  \[ \delta(\vec{x}) = \frac{\rho(\vec{x}) - \langle \rho \rangle}{\langle \rho \rangle} \]
  \[ \xi(r_{12}) = \langle \delta(\vec{x}_1) \delta(\vec{x}_2) \rangle \]
- Simple model:
  \[ \xi(r) = \left( \frac{r}{r_o} \right)^{-\gamma} \]
2PCF: Power Law Model Fit

Fit $r_o$ and $\gamma$ to SDSS galaxies: Zehavi et al. 2005
Redshift Space Distortions

\[ cz \approx H_0 d + v_{\text{los}} \]

Complex effects based on velocities and dynamics of region.
MOVIE: SDSS GALAXIES
Decompose the separation vector between points into components:

\[ r = \sqrt{r_p^2 + \pi^2} \]

- \( r_p \): perpendicular to line of sight
- \( \pi \): path along line of sight

\[ \pi \] now contains the redshift distortion

\[ \xi(r) \rightarrow \xi(r_p, \pi) \]

Projected 2PCF

\[ w_p(r_p) = 2 \int_0^{\pi_{max}} \xi(r_p, \pi) d\pi \]
Projected 2PCF: $r_p - \pi$ diagram
Galaxy Redshift Surveys:
- finite depth
- complicated footprint
- different samples:
  1. Flux-limited
  2. Volume-limited

\[ m - M = 5 \log(D_{\text{max}}) + 25 \]
2PCF: Magnitude Dependence

Zehavi et al. 2005, Fig 8, Panel 1
2PCF: Color Dependence

Zehavi et al. 2005, Fig 13
Higher order functions are important!

Three Point Correlation Function

- phase information randomized for image on right
- same 2PCF but very different distributions
Definition of 3PCF
Three Point Correlation Function

The probability for 3 points joined by three separations \( r_a, r_b \) and \( r_c \)

\[
\delta P = n^3 \delta V_1 \delta V_2 \delta V_3 \left[ 1 + \xi_a + \xi_b + \xi_c + \zeta_{abc} \right]
\]

Connected Three Point:

\[
\zeta(r_a, r_b, r_c) = \zeta_{abc}
\]

Q: Normalized 3PCF

\[
Q(r_a, r_b, r_c) = \frac{\zeta_{abc}}{\xi_a \xi_b + \xi_b \xi_c + \xi_c \xi_a}
\]

\[
Q(r_a, r_b, \theta) \quad \text{where} \quad \cos \theta = \frac{r_a^2 + r_b^2 - r_c^2}{2r_ar_b}
\]

- \( Q \) initially believed to be roughly constant
- measurements find \( Q \) not quite constant for any scales
- Useful normalization: varies much less than \( \zeta \) or \( \xi \)
- Insensitive to many cosmological parameters (leading order)
Both *scale* and *configuration* dependence

Characterize dependence on configurations as:

- "weak dependence" when very little change (hierarchical)
- "strong dependence" significant difference.
3pt Measurements on the Main Galaxy Sample

$-21.5 < M_r < -20.5$
Luminosity dependence?
3pt Measurements on the Main Galaxy Sample
Redshift vs Projected Measurements
3pt Measurements on the Main Galaxy Sample

DR6: $r_1/r_2 = 3:6$ Mpc/h

DR6: $r_1/r_2 = 6:12$ Mpc/h

DR6: $r_1/r_2 = 9:18$ Mpc/h