N-body Simulations

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ON THE CLUSTERING TENDENCIES AMONG THE NEBULAE

II. A STUDY OF ENCOUNTERS BETWEEN LABORATORY MODELS OF STELLAR SYSTEMS BY A NEW INTEGRATION PROCEDURE

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ABSTRACT

In a previous paper[1] the writer discussed the possibility of explaining the observed clustering effects among extragalactic nebulae as a result of captures. The present investigation deals with the important problem of whether the loss of energy resulting from the tidal disturbances at a close encounter between two nebulae is large enough to effect a capture. The tidal deformations of two models of stellar systems, passing each other at a small distance, are studied by reconstructing, piece by piece, the orbits described by the individual mass elements. The difficulty of integrating the total gravitational force acting upon a certain element at a certain point of time is solved by replacing gravitation by light. The mass elements are represented by light-bulbs, the candle power being proportional to mass, and the total light is measured by a photocell (Fig. 1). The nebulae are assumed to have a flattened shape, and each is represented by 37 light-bulbs. It is found that the tidal deformations cause an increase in the attraction between the two objects, the increase reaching its maximum value when the nebulae are separating, i.e., after the passage. The resulting loss of energy (Fig. 6) is comparatively large and may, in favorable cases, effect a capture. The spiral arms developing during the encounter (Figs. 4) represent an interesting by-product of the investigation. The direction of the arms depends on the direction of rotation of the nebulae with respect to the direction of their space motions.

N-body Simulations

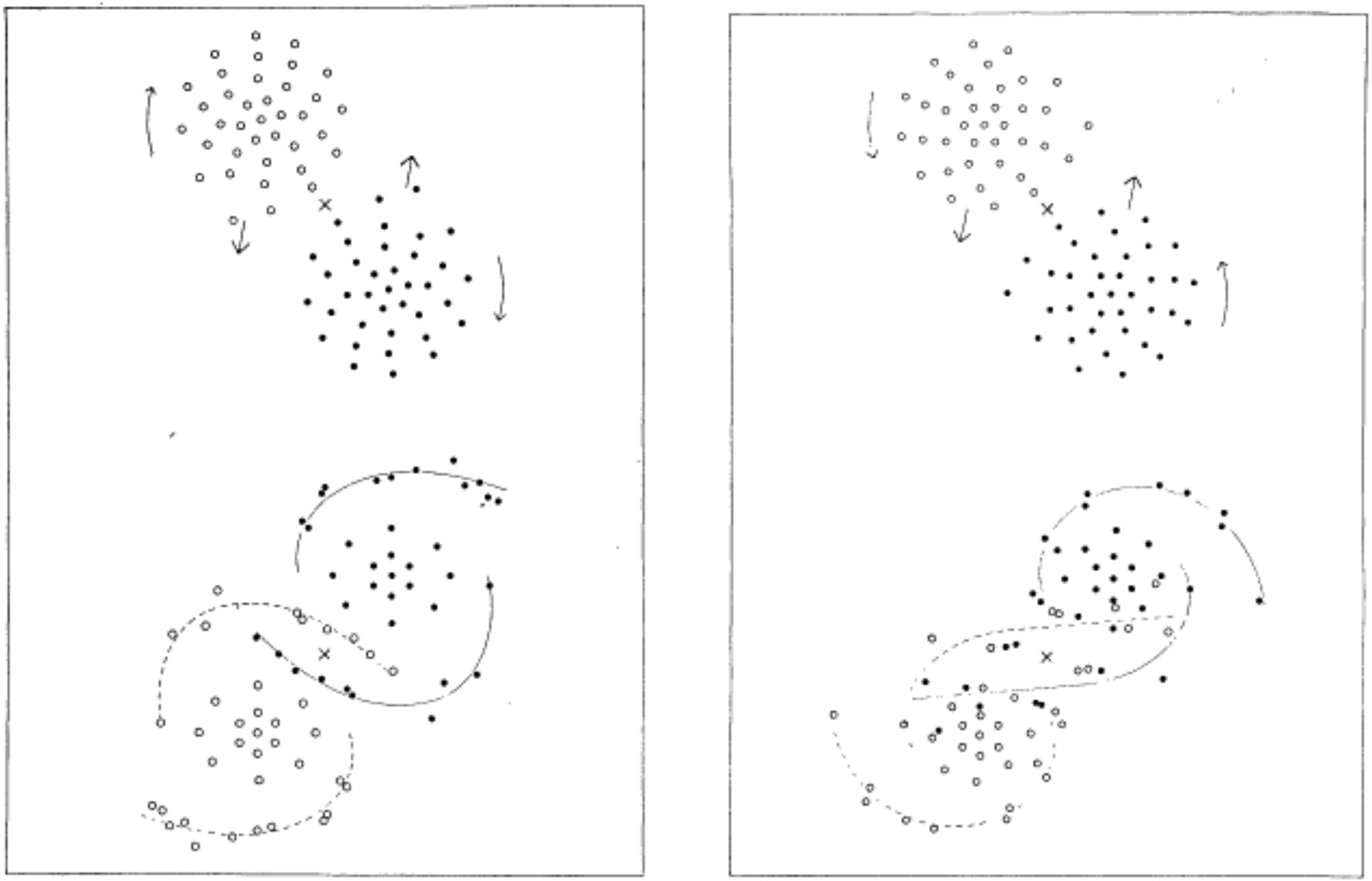


Fig. 4a

Fig. 4b

Fig. 4a.—Tidal deformations corresponding to parabolic motions, clockwise rotations, and a distance of closest approach equal to the diameters of the nebulae. The spiral arms point in the direction of the rotation.

Fig. 4b.—Same as above, with the exception of counterclockwise rotations. The spiral arms point in the direction opposite to the rotation.

N-body Simulations

Initial conditions:

- What kind of Dark Matter?
- How much Dark Matter?
- Initial density fluctuations P(k)

↓

GRAVITY

↓

Final distribution of dark matter.

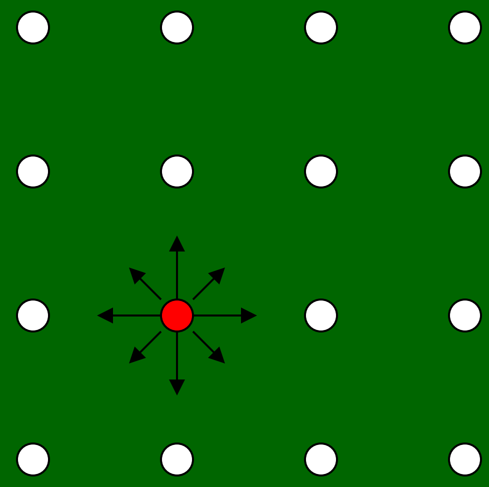
N-body Simulations

• Start with a grid of particles representing dark matter.

• Give them initial smooth density fluctuations.

• Compute the force of gravity on every particle from every other particle in a series of time steps.

A very computationally expensive technique! Only made possible recently with fast computers.



N-body Simulations: Initial Conditions

Cosmological model

$$h, \Omega_m, \Omega_b, \Omega_\nu, \sigma_8, n_s \rightarrow P(k)$$

Phases for modes

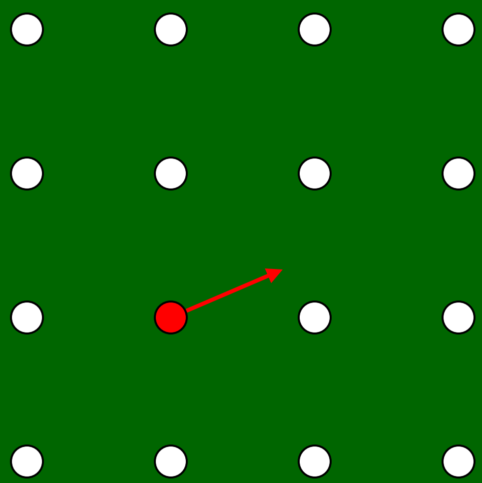
- Gaussian random phase δ(r)
- Non-Gaussian?

Evolve to starting redshift

- z_{init}=30-200
- Zel'dovich approximation
- 2LPT

N-body Simulations: Initial Conditions

Assign initial positions and velocities using Zel' dovich approximation



$$\vec{x} = \vec{q} + D(t)\vec{\psi}(\vec{q})$$

$$\vec{v} = a\frac{dD}{dt}\vec{\psi}(\vec{q})$$

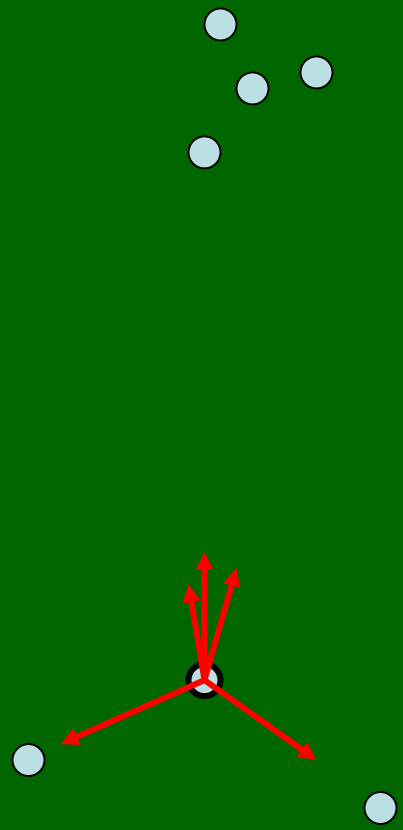
$$\vec{\nabla}\cdot\vec{\psi} = -\frac{\delta(\vec{q})}{D(t)}$$

q : initial position
Ψ : dispacement field
D : growth function
δ : initial density field

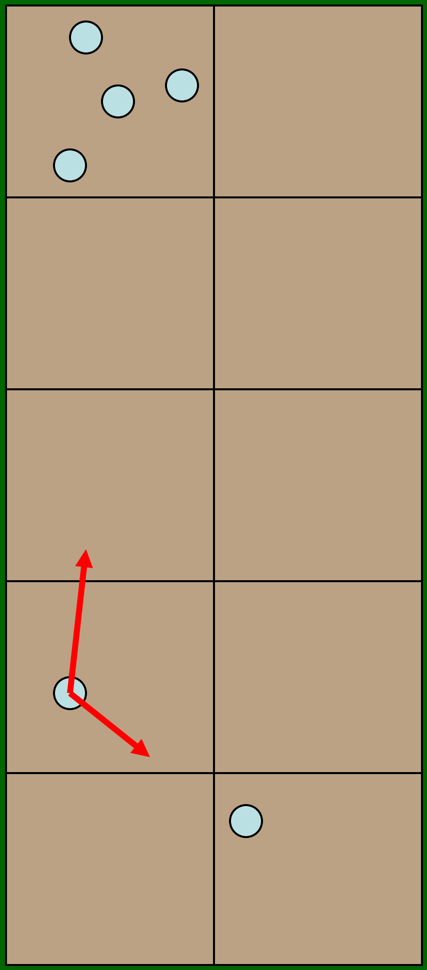
N-body Simulations: Force calculations

- Direct particle-particle (N^2)
- Particle-Mesh (PM) $(N_g \log N_g)$
- Particle-particle particle-mesh (P^3M) $(N^2 \; / \; N_g \log N_g)$
- Tree $(N \log N)$
- Tree-PM $(N \log N \; / \; N_g \log N_g)$
- Adaptive mesh refinement (AMR)
- Adaptive refinement tree (ART)
- Moving mesh (AREPO)

N-body Simulations: Direct N-body



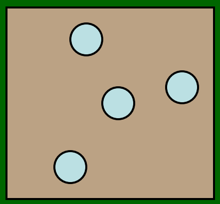
N-body Simulations: Particle-Mesh

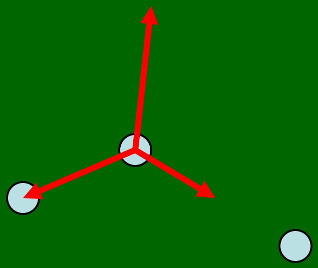


$$\nabla^2 \Phi = 4\pi G \rho$$

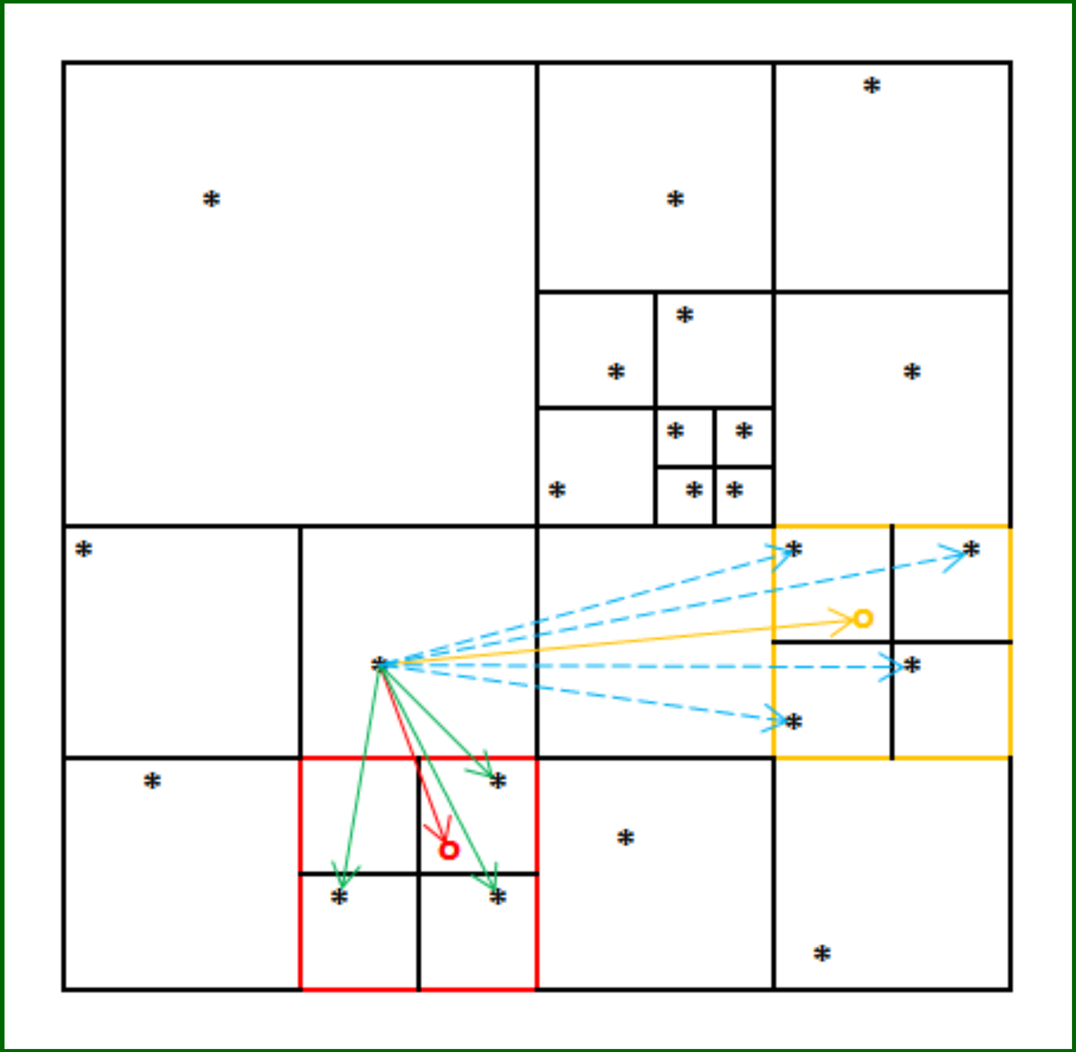
$$\hat{\Phi} = -4\pi G \frac{\hat{\rho}}{k^2}$$

N-body Simulations: Particle-Particle, Particle-Mesh

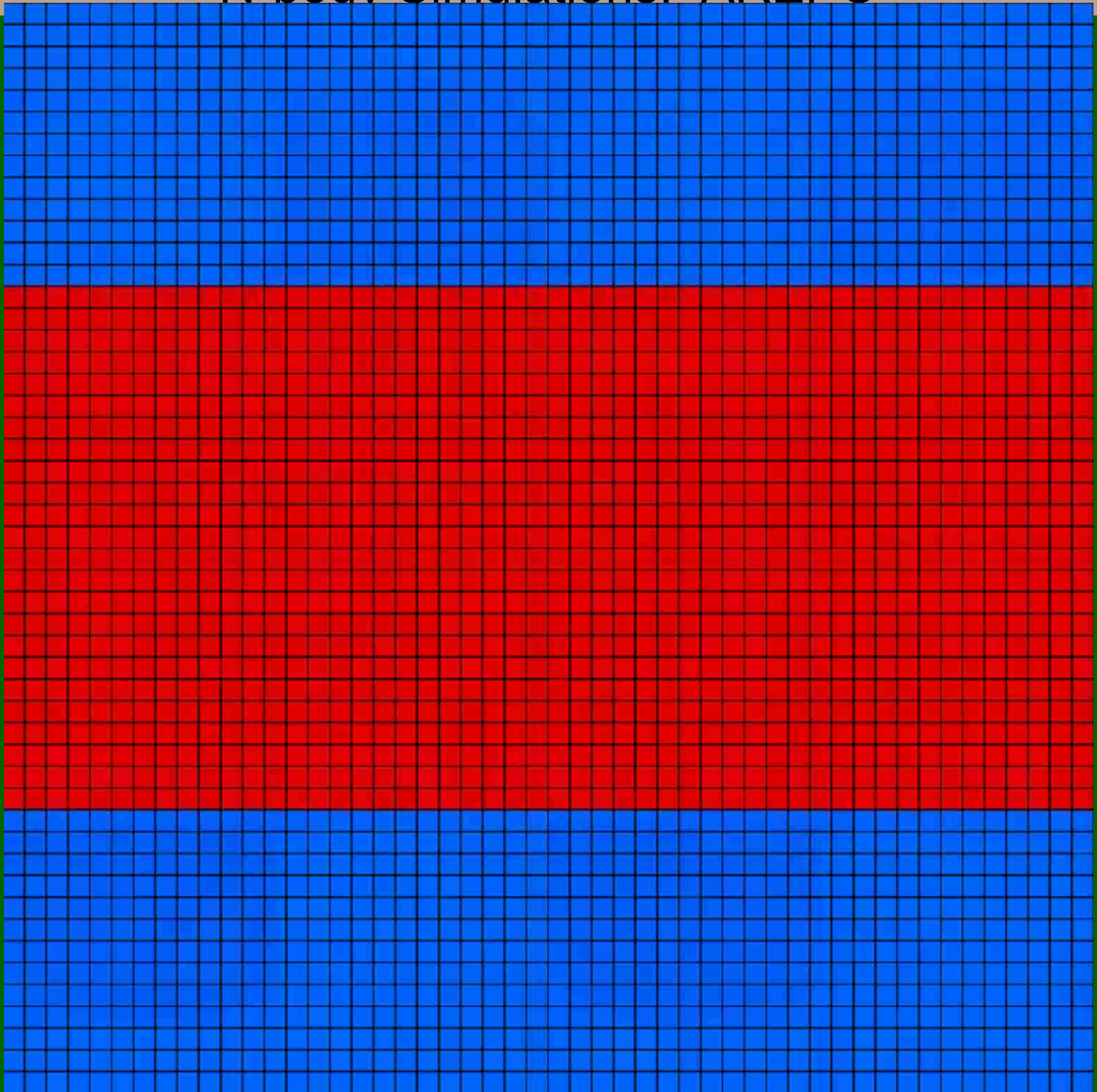




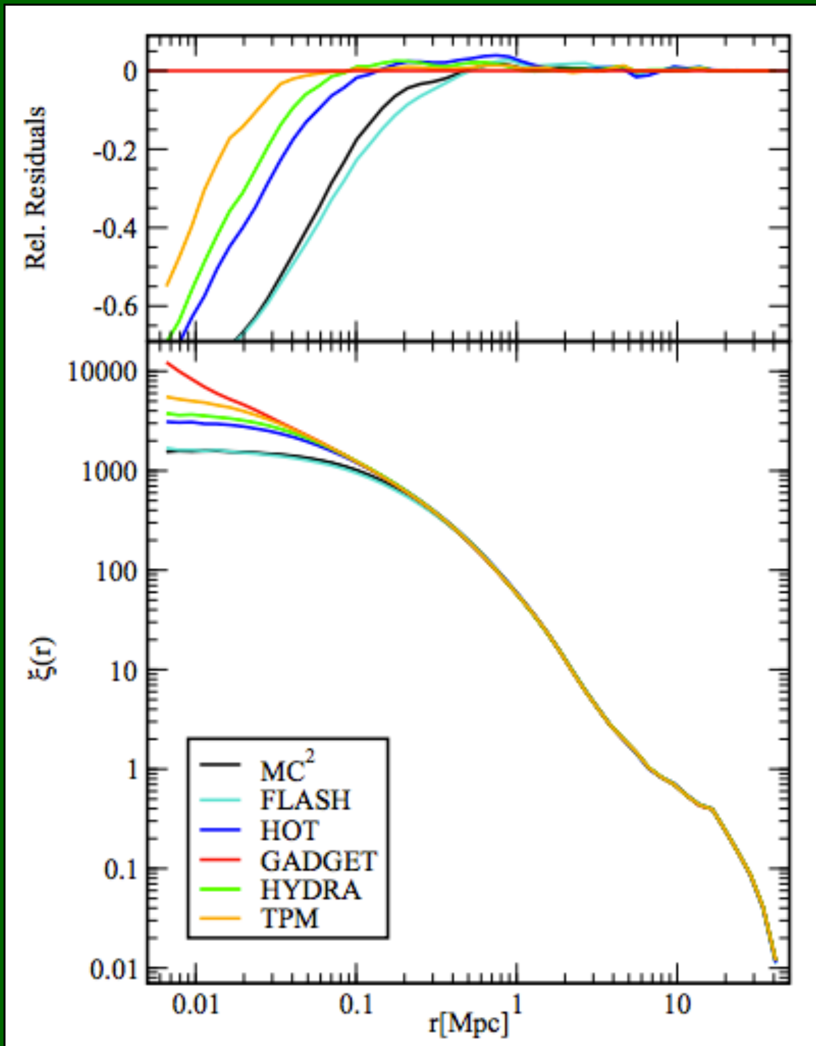
N-body Simulations: Tree Code

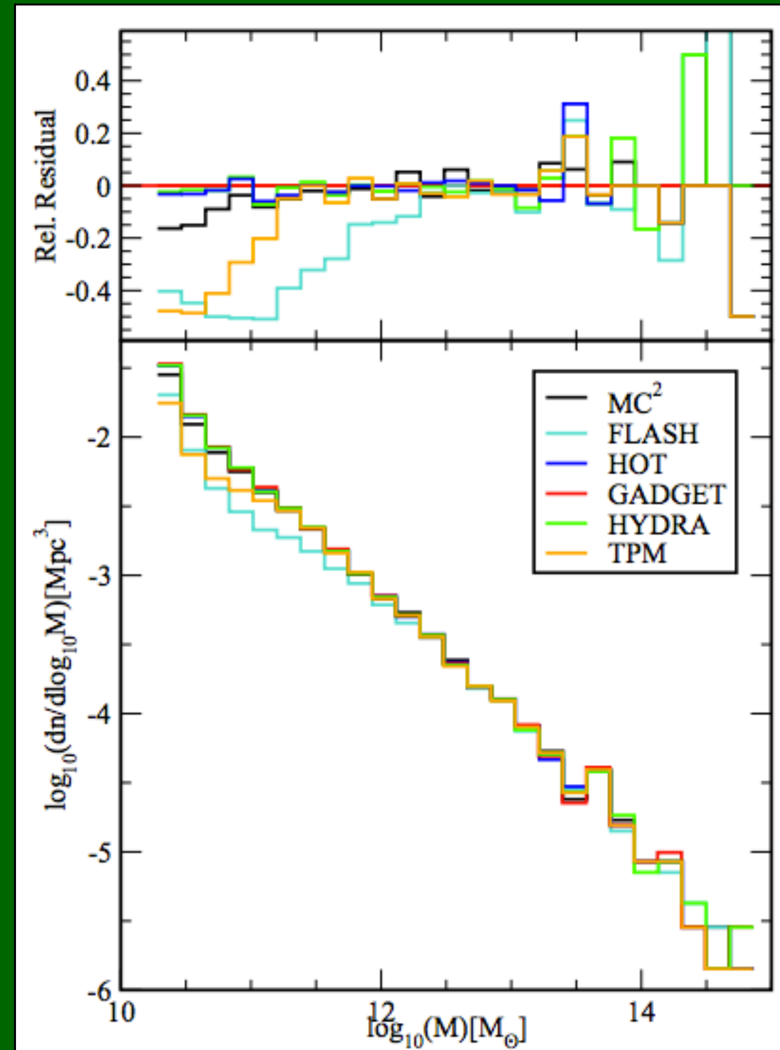


N-body Simulations: AREPO



N-body Simulations: Code comparisons

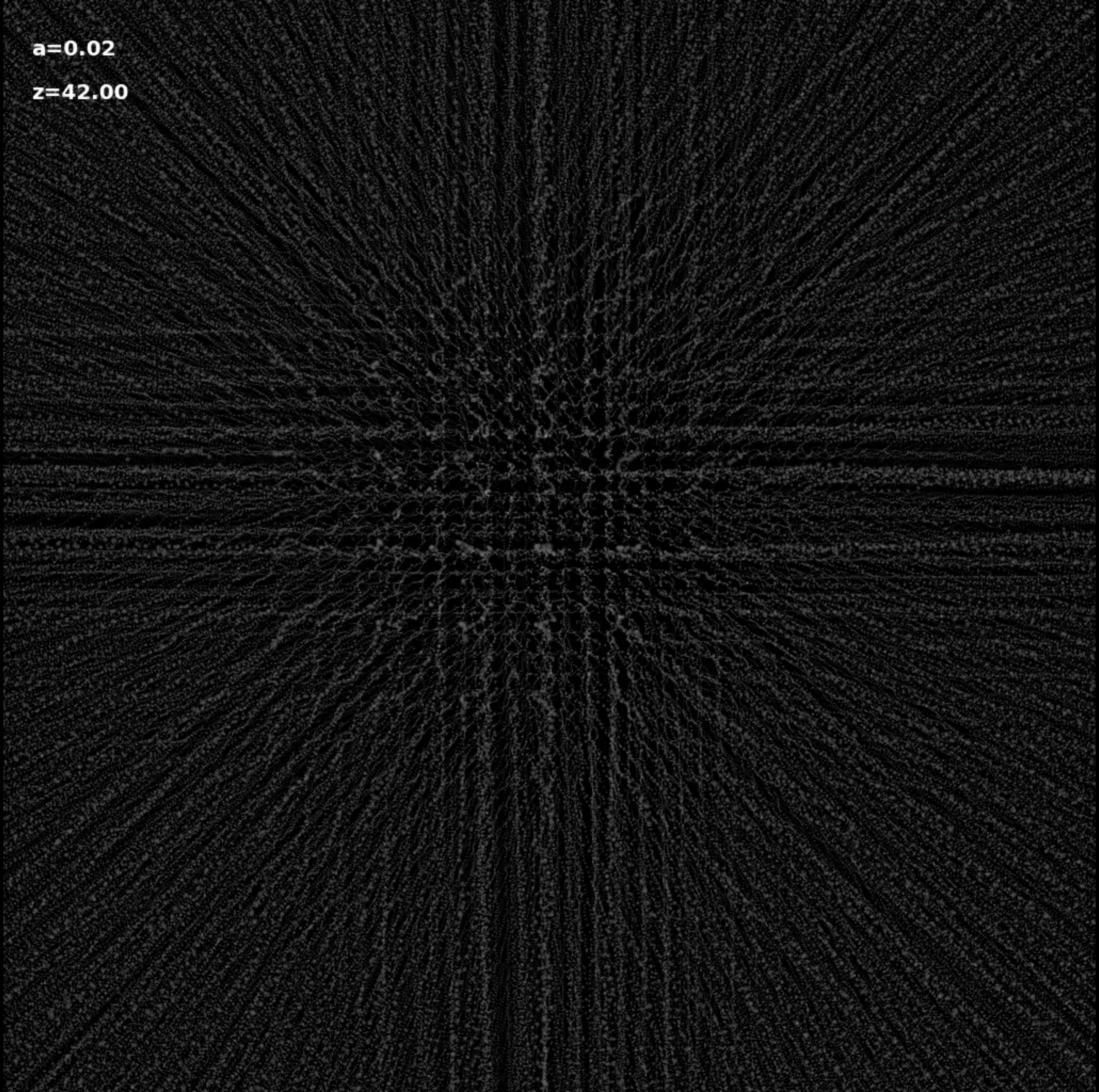




Heitmann et al. (2005)

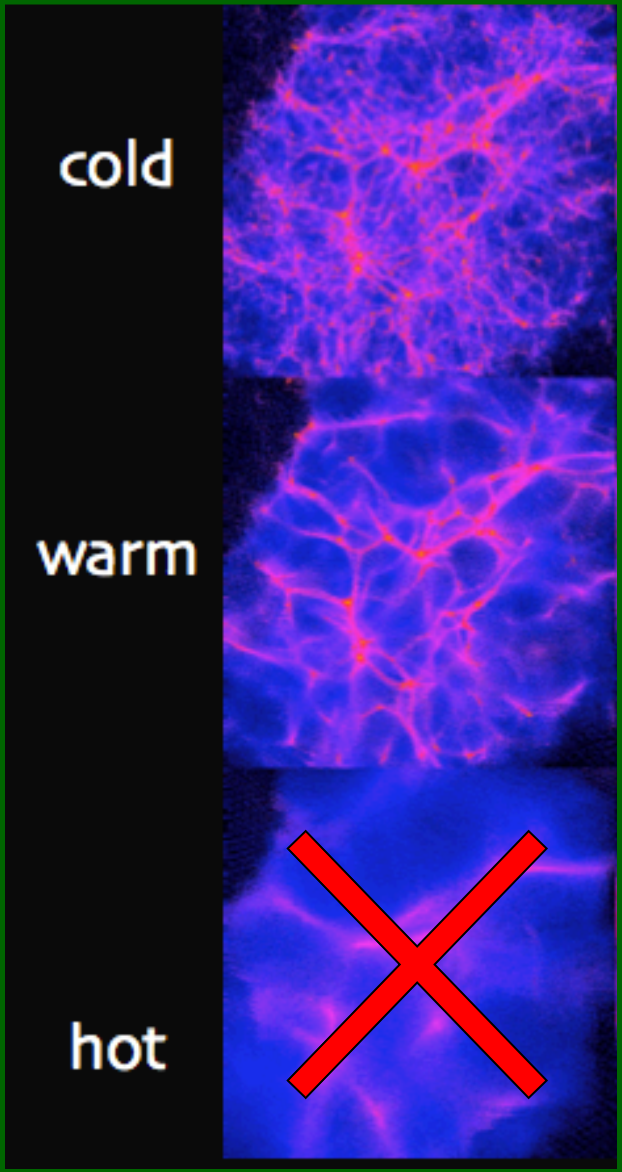
a=0.02

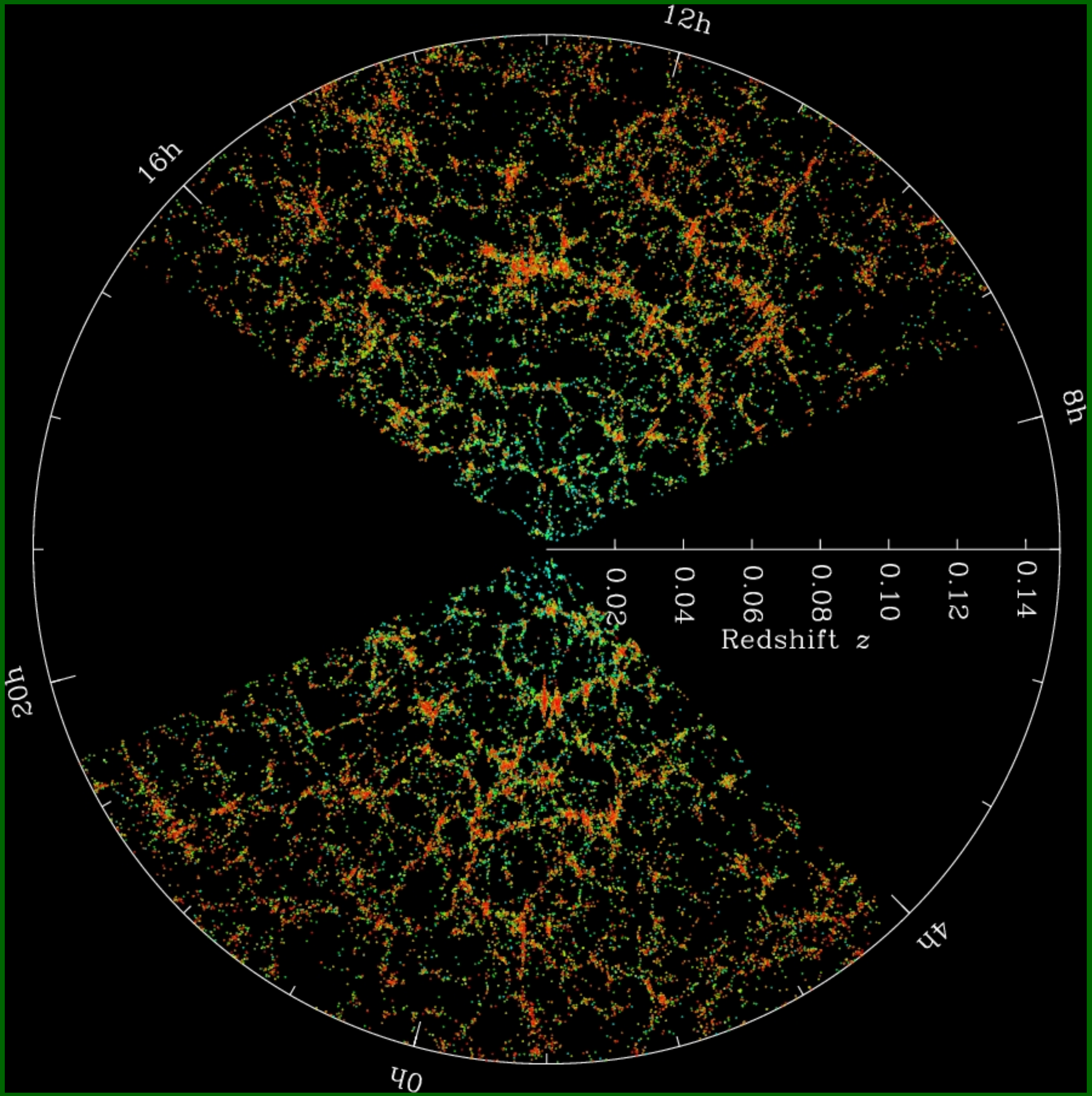
z=42.00


a=0.02
z=42.00

We can constrain cosmological models.

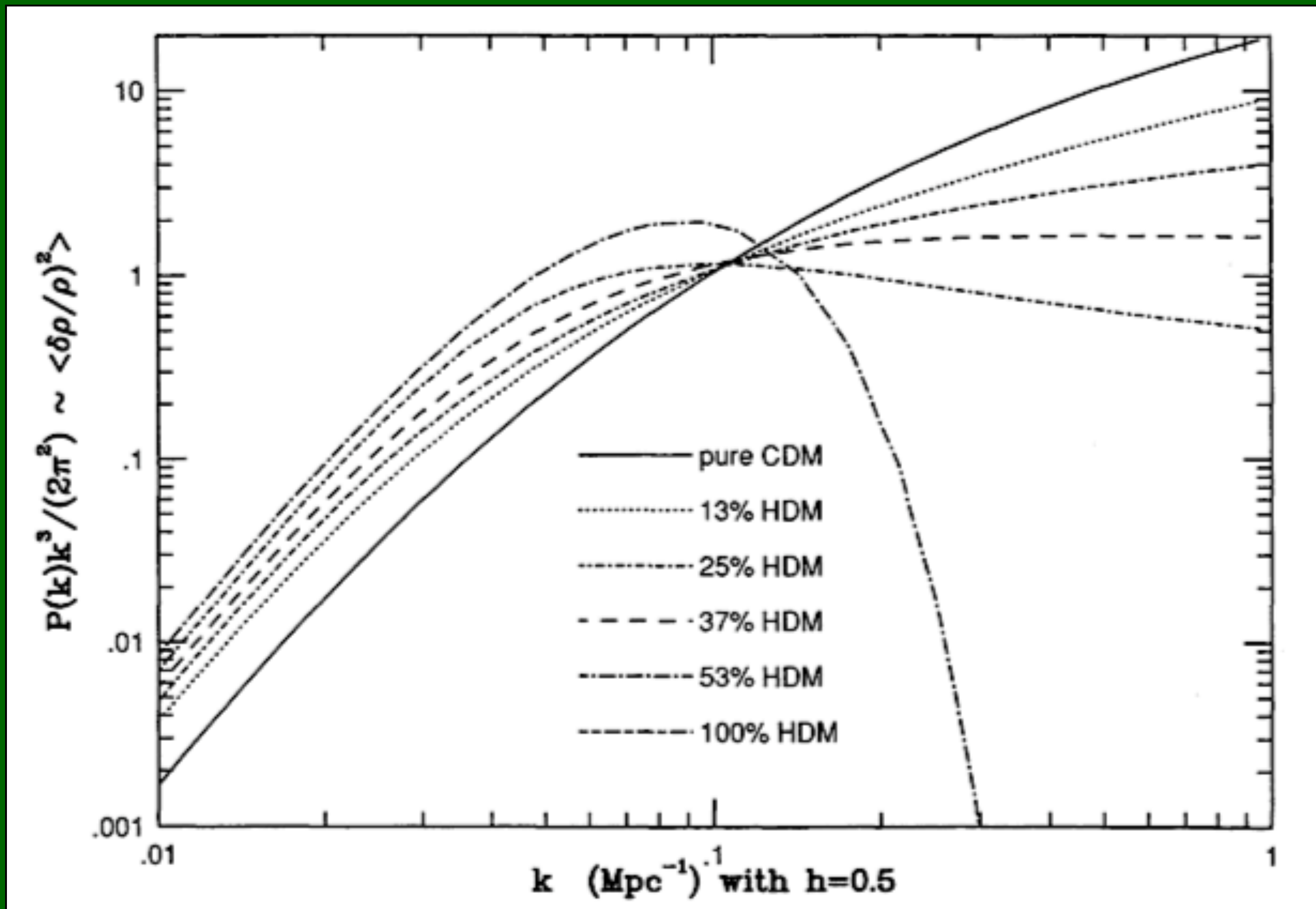
e.g., dark matter models





SDSS observations

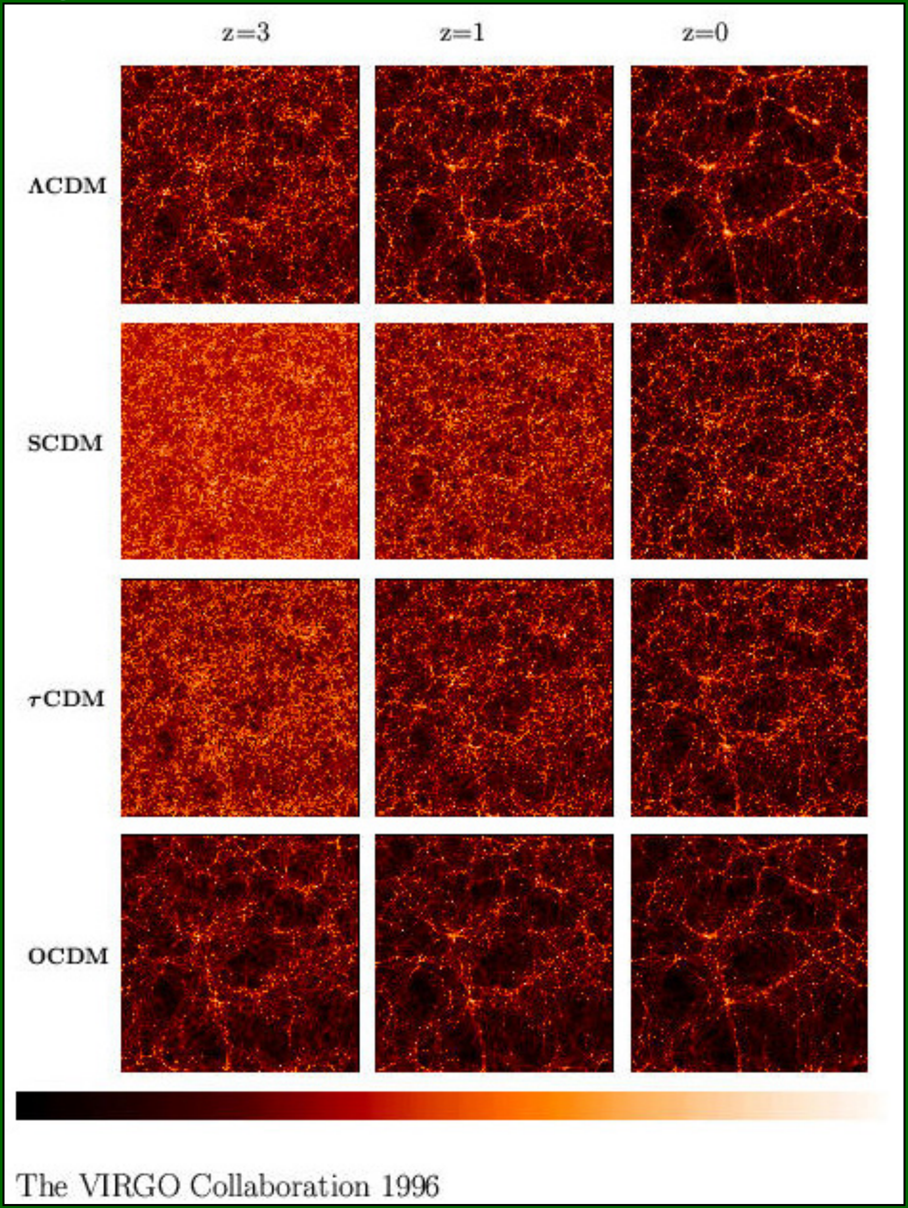
We can constrain cosmological models.



van Dalen & Schaefer (1992)

We can constrain cosmological models.

Fraction of present age: 1/10 1/2 1



Flat universe with dark energy

$\Omega_m = 0.3 \quad \Omega_\Lambda = 0.7$

Flat universe with high DM

$\Omega_m = 1$

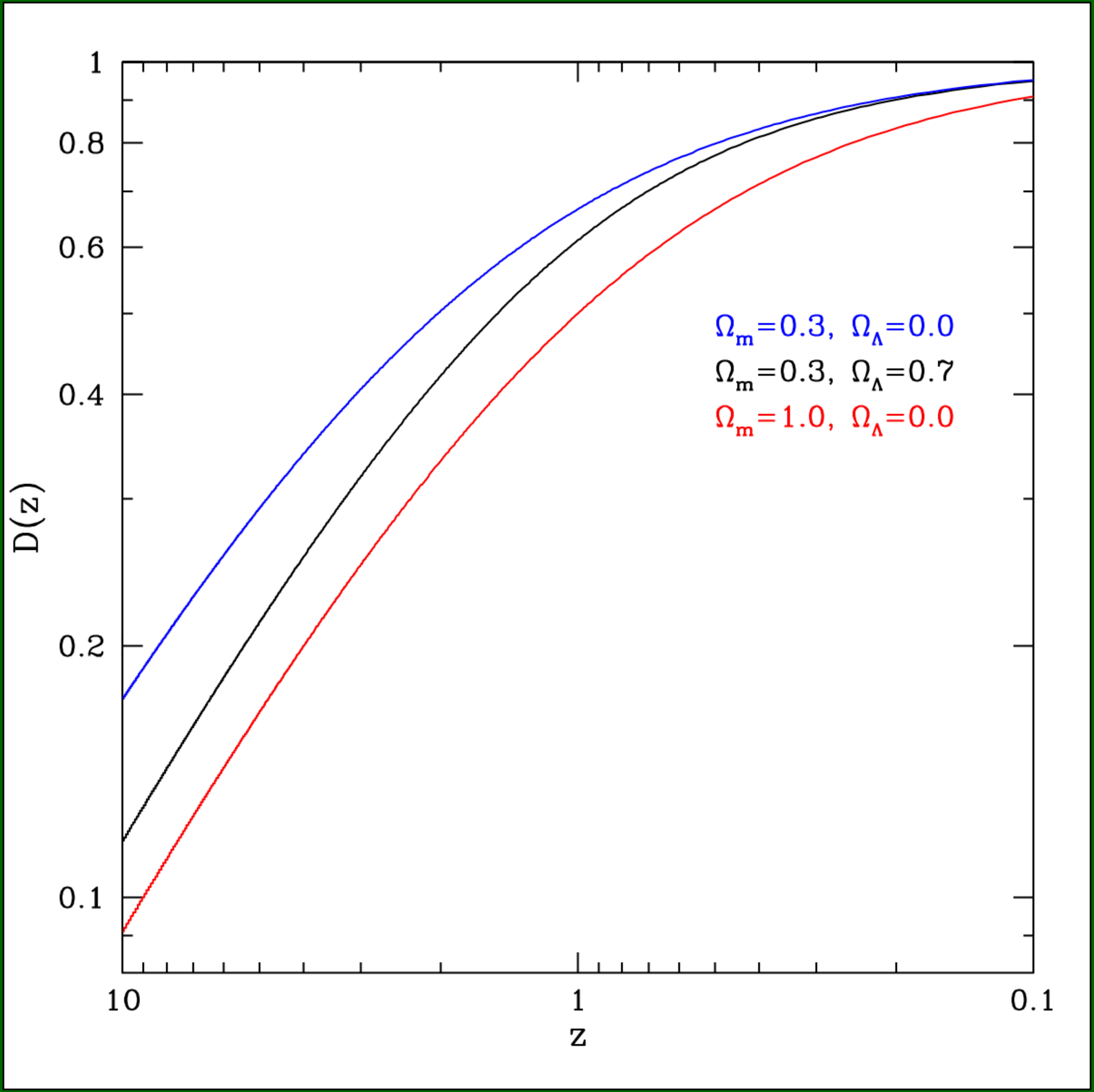
Different initial P(k)

$\Omega_m = 1$

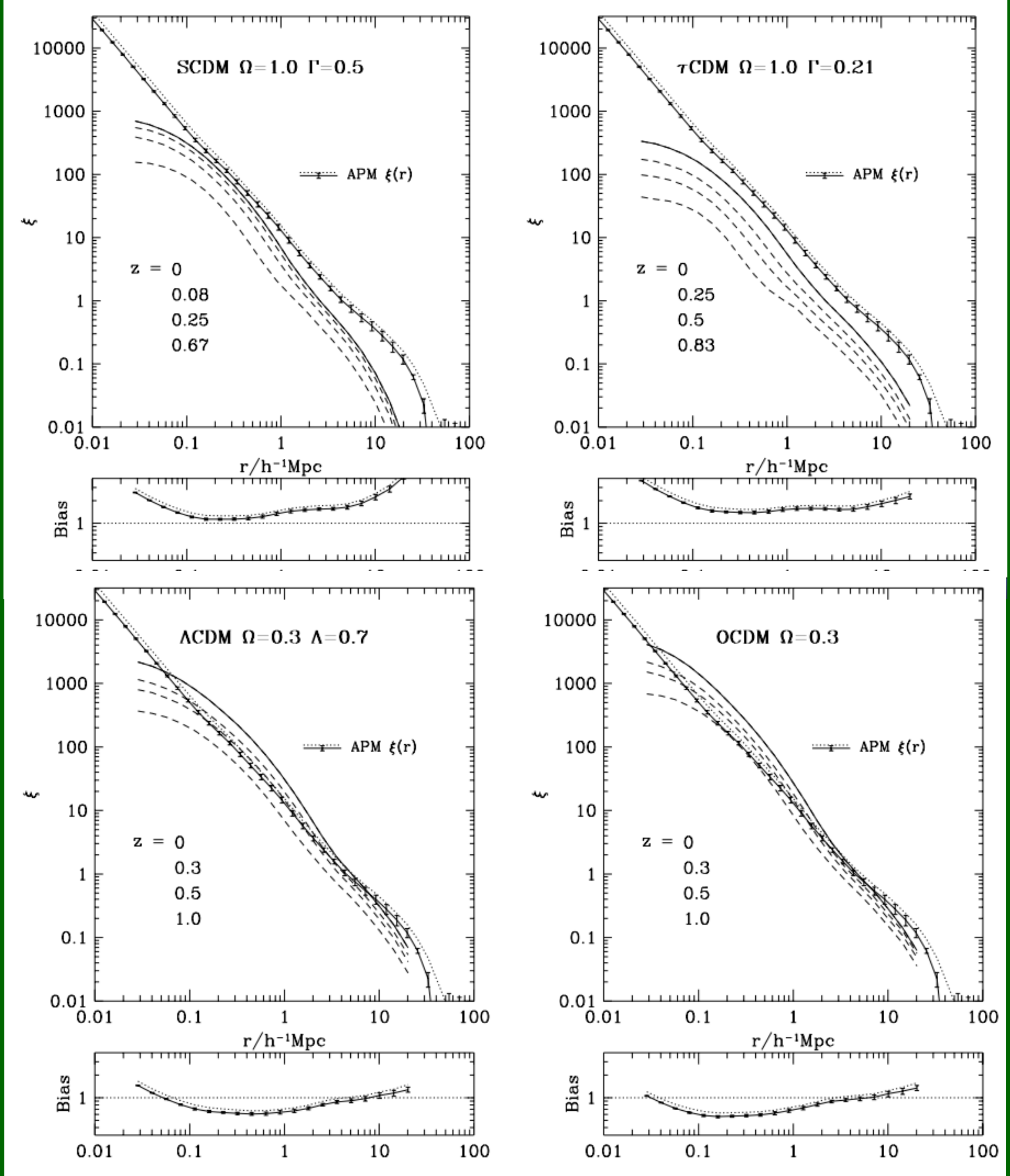
Open universe with low DM

$\Omega_m = 0.3$

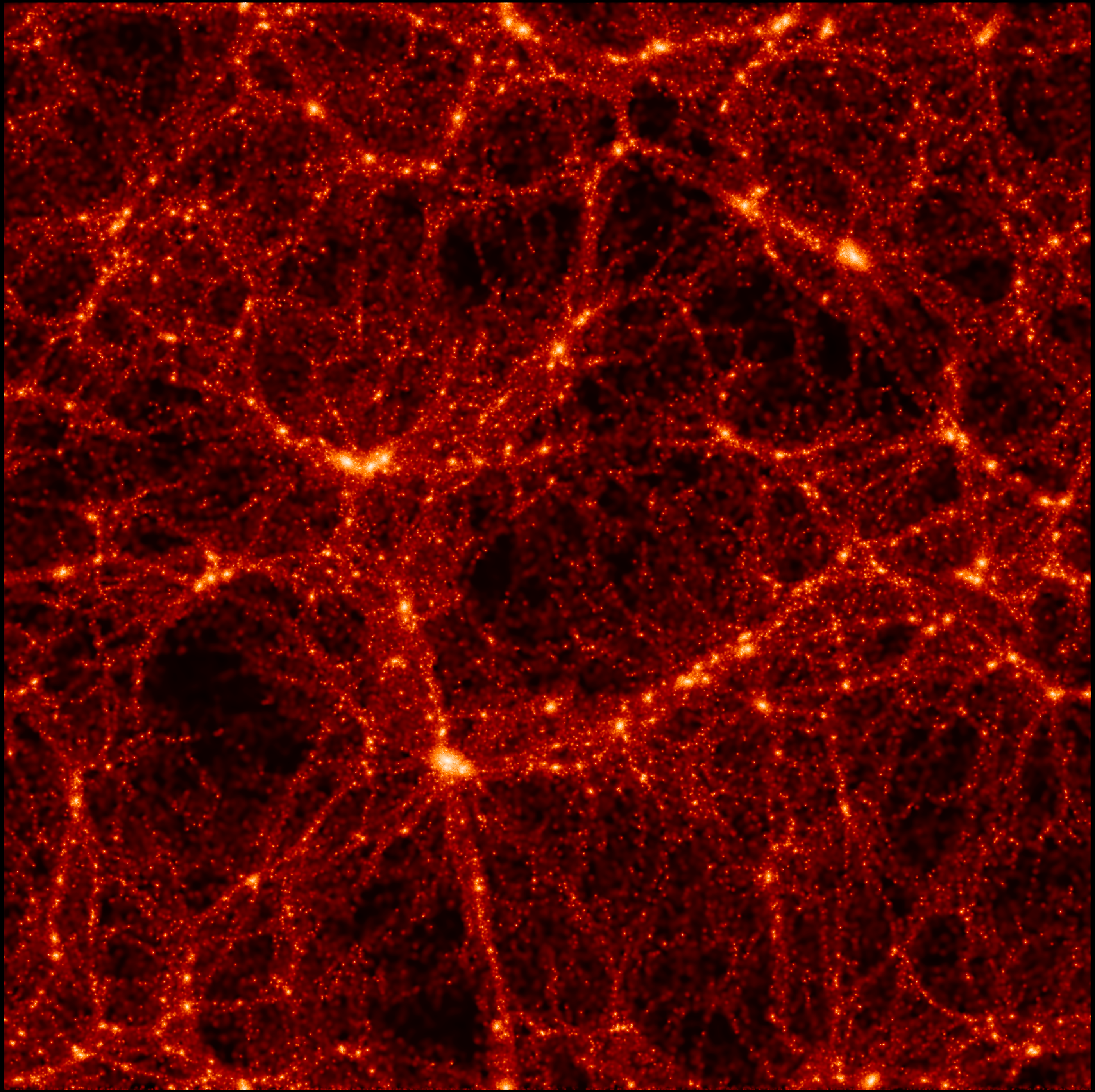
We can constrain cosmological models.



We can constrain cosmological models.

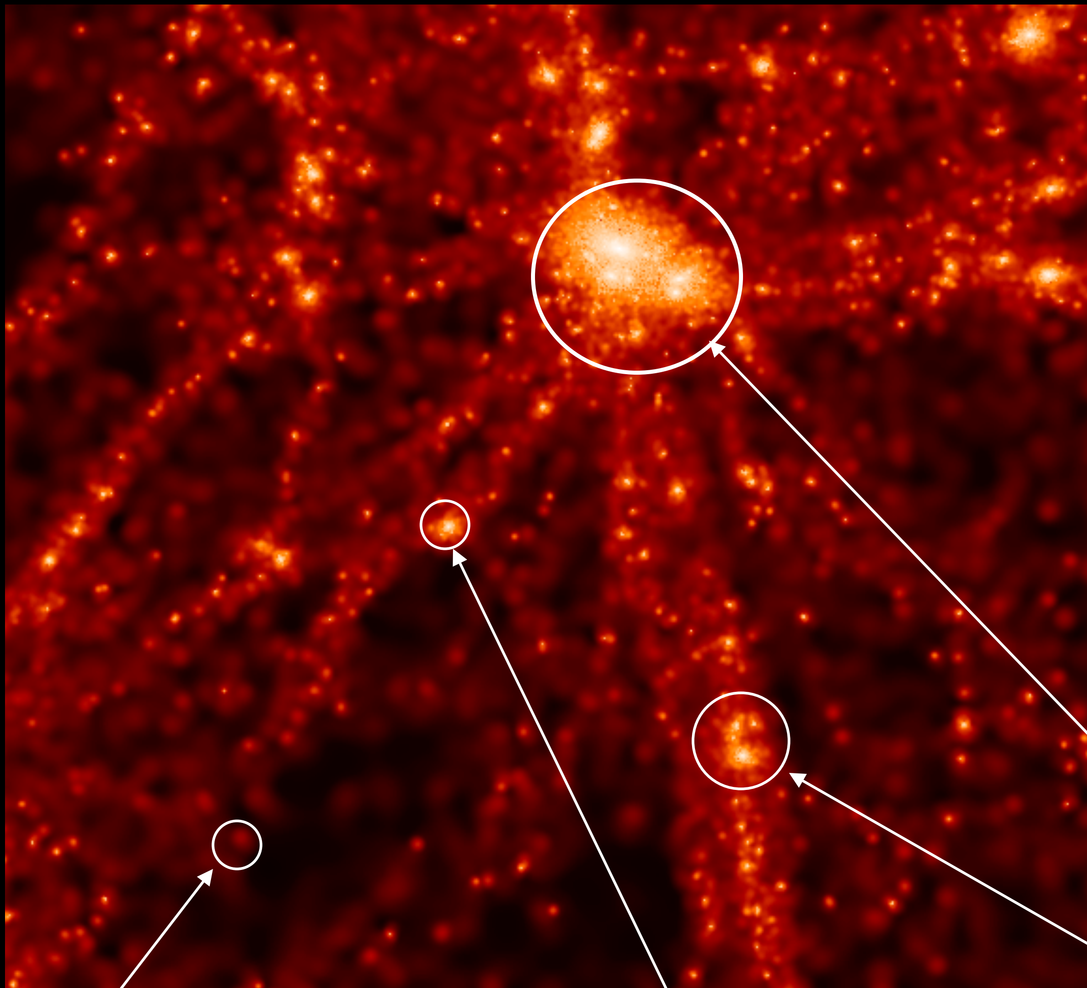


Jenkins et al. (1998)



Virgo Collaboration

What is a dark matter halo?



Dark Matter collapses under its own self-gravity into “virialized” regions, or “halos”.

Halos are typically defined as regions with density of ~200 times the mean density, but there are several halo-finding algorithms.

Halos come in different sizes, masses, and shapes.

High mass halo (the kind that would host a galaxy cluster)

Intermediate mass halo (the kind that would host a galaxy group)

Very low mass halo (the kind that would host no galaxy at all)

Low mass halo (the kind that would host a single galaxy)

What is a dark matter halo?

- Friends-of-Friends (FoF)

  linking length *b*

- Spherical Overdensity (SO)

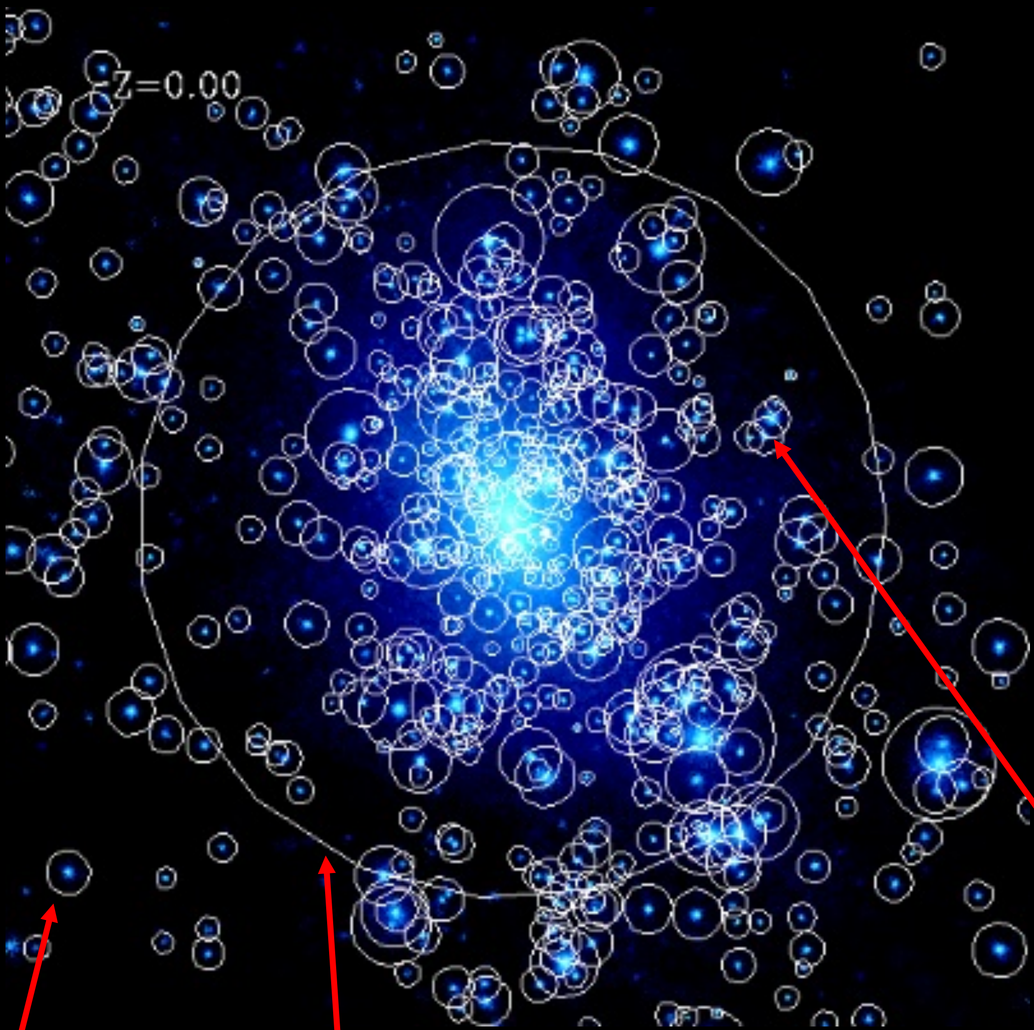
  choice of center, density threshold Δ_{vir}

- Density Maxima (DENMAX, BDM)

  choice of center, density threshold Δ_{vir} , criteria for unbinding

- Other (e.g., Voronoi tesselation)

What is a dark matter halo?



Kravtsov et al.

Halo

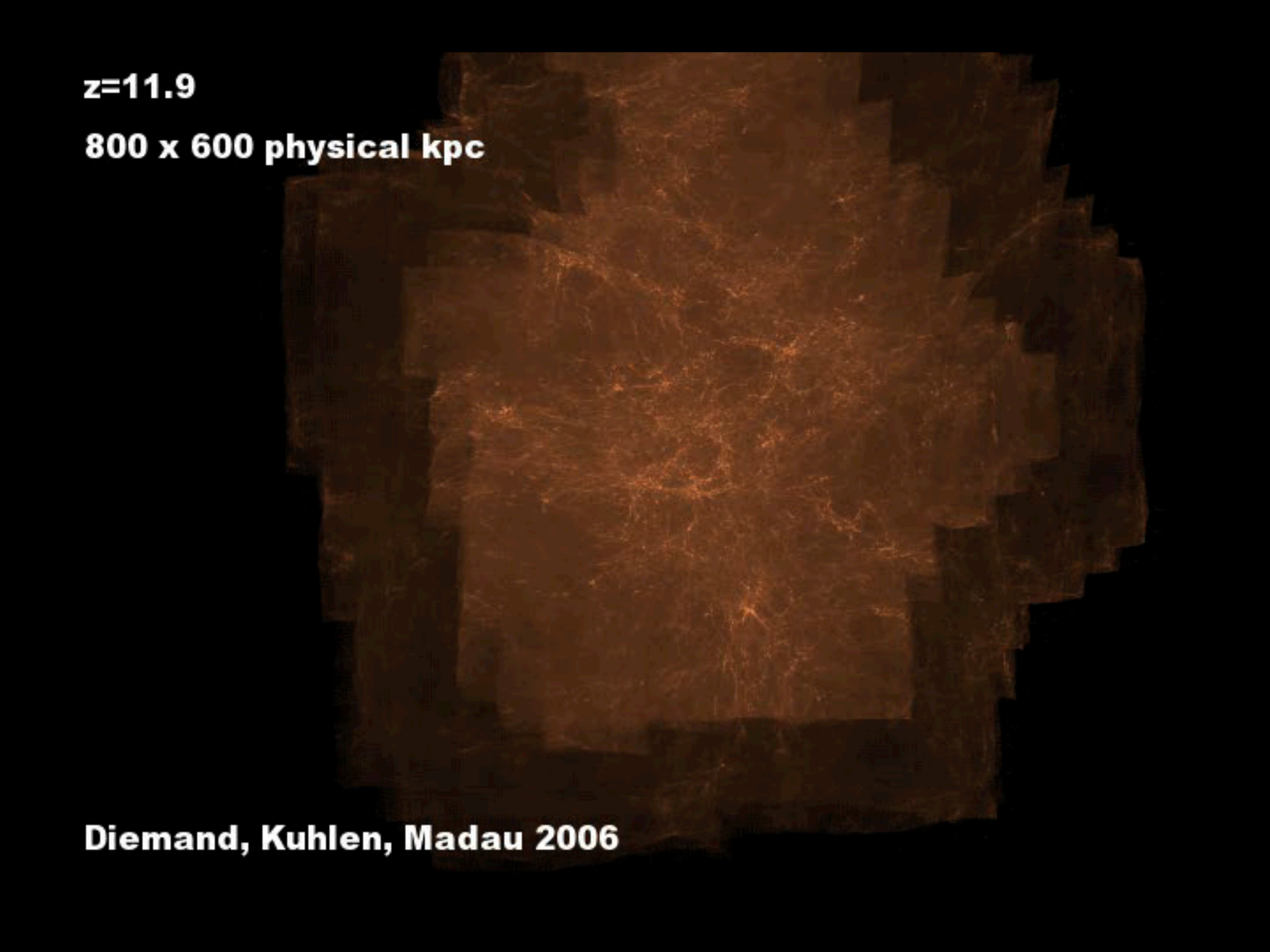
Host/Parent/Central halo

Subhalo / Satellite halo

Bound dense regions within a larger halo are referred to as subhalos, substructure, or satellite halos.

Subhalos have a density higher than ~200 times the mean.

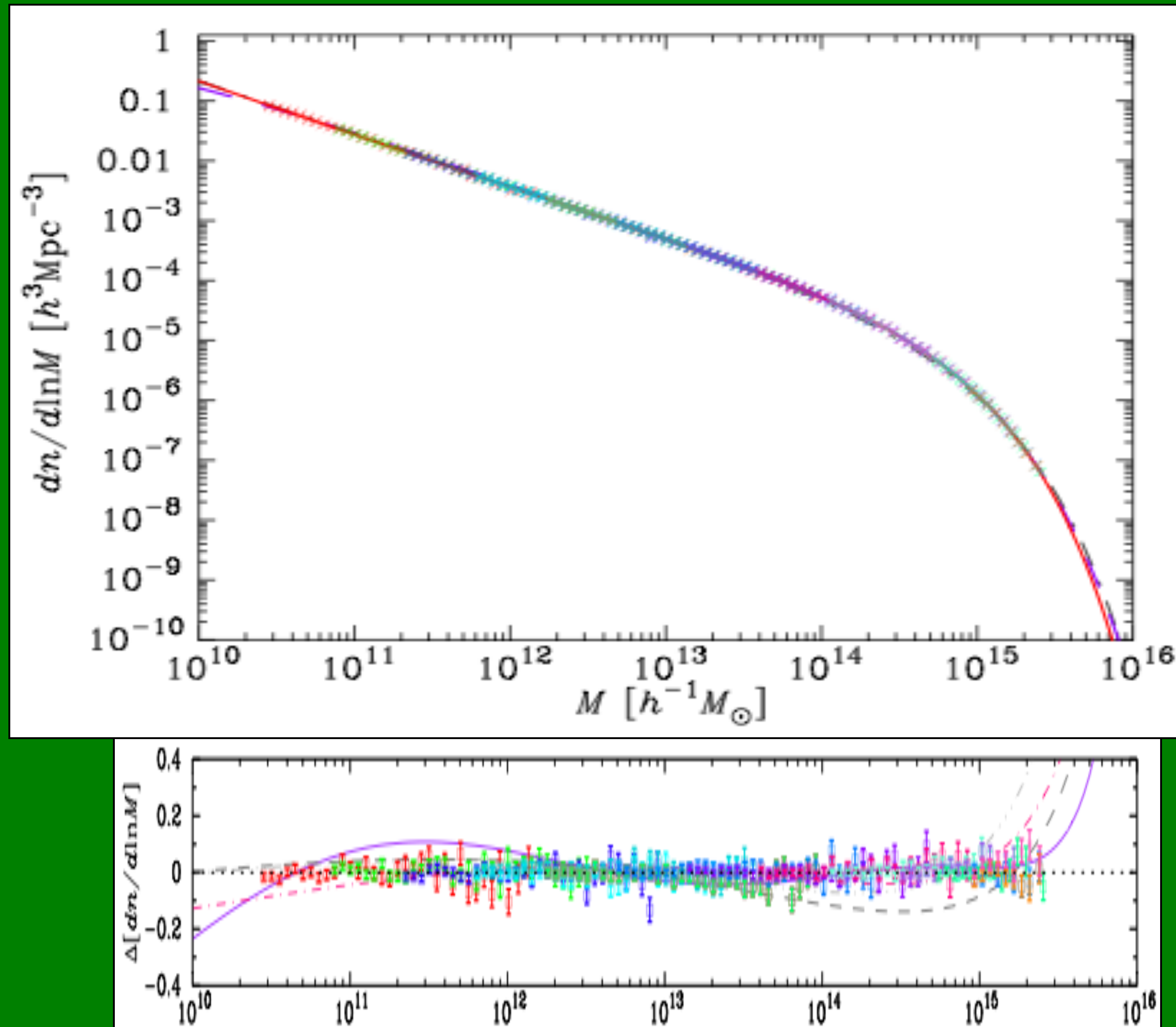
A halo can host a single galaxy, or a cluster of galaxies. Within a cluster, individual galaxies would sit inside subhalos.


z=11.9
800 x 600 physical kpc
Diemand, Kuhlen, Madau 2006

Halo properties: Abundance (mass function)

We now know the z=0 mass function to ~5% for reasonable choices of cosmological parameters. (*For one N-body code and one halo-finder*)

There may be larger uncertainties for higher redshifts or more exotic cosmological models.



M

Warren et al. (2006)

Halo properties: Abundance (mass function)

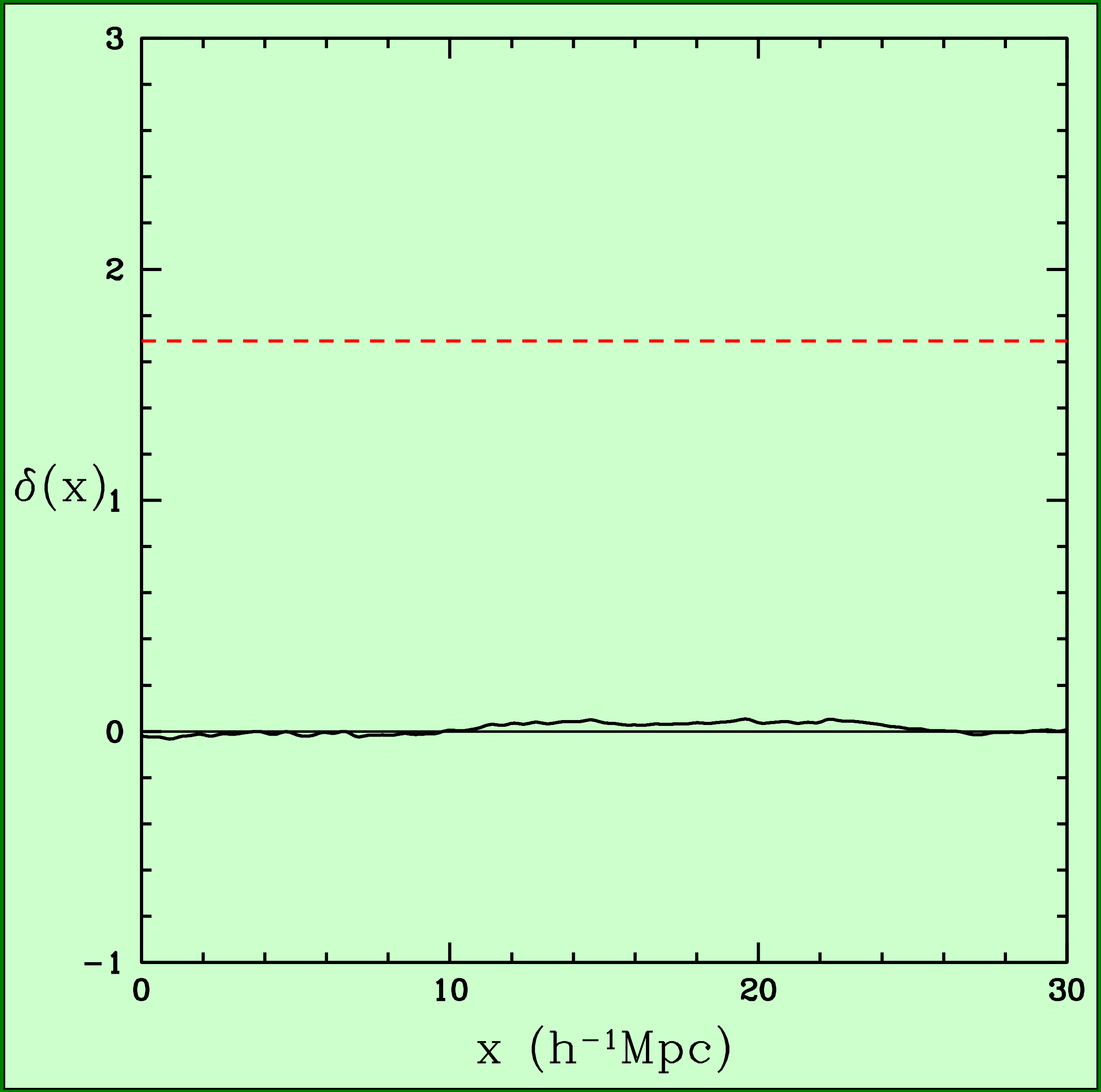
Press-Schechter (1974) theory

Halos collapse from regions in the primordial density field that exceed a threshold density. One halo forming does not influence the likelihood of other halos forming nearby.

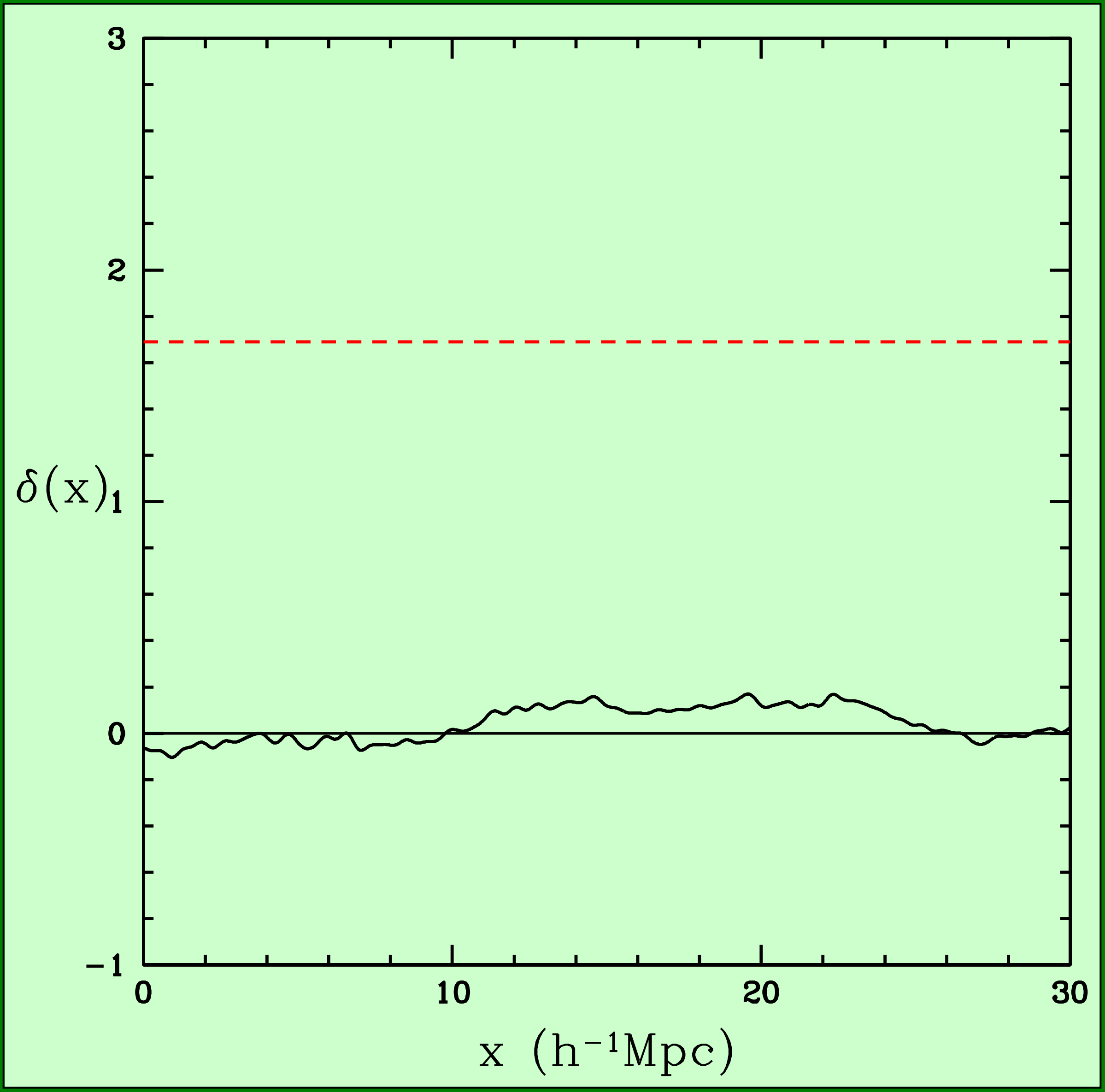
The halo mass function thus depends on:

- The distribution of initial densities $P(k)$
- The linear growth of fluctuations $D(z)$
- The density threshold for collapse δ_{crit}

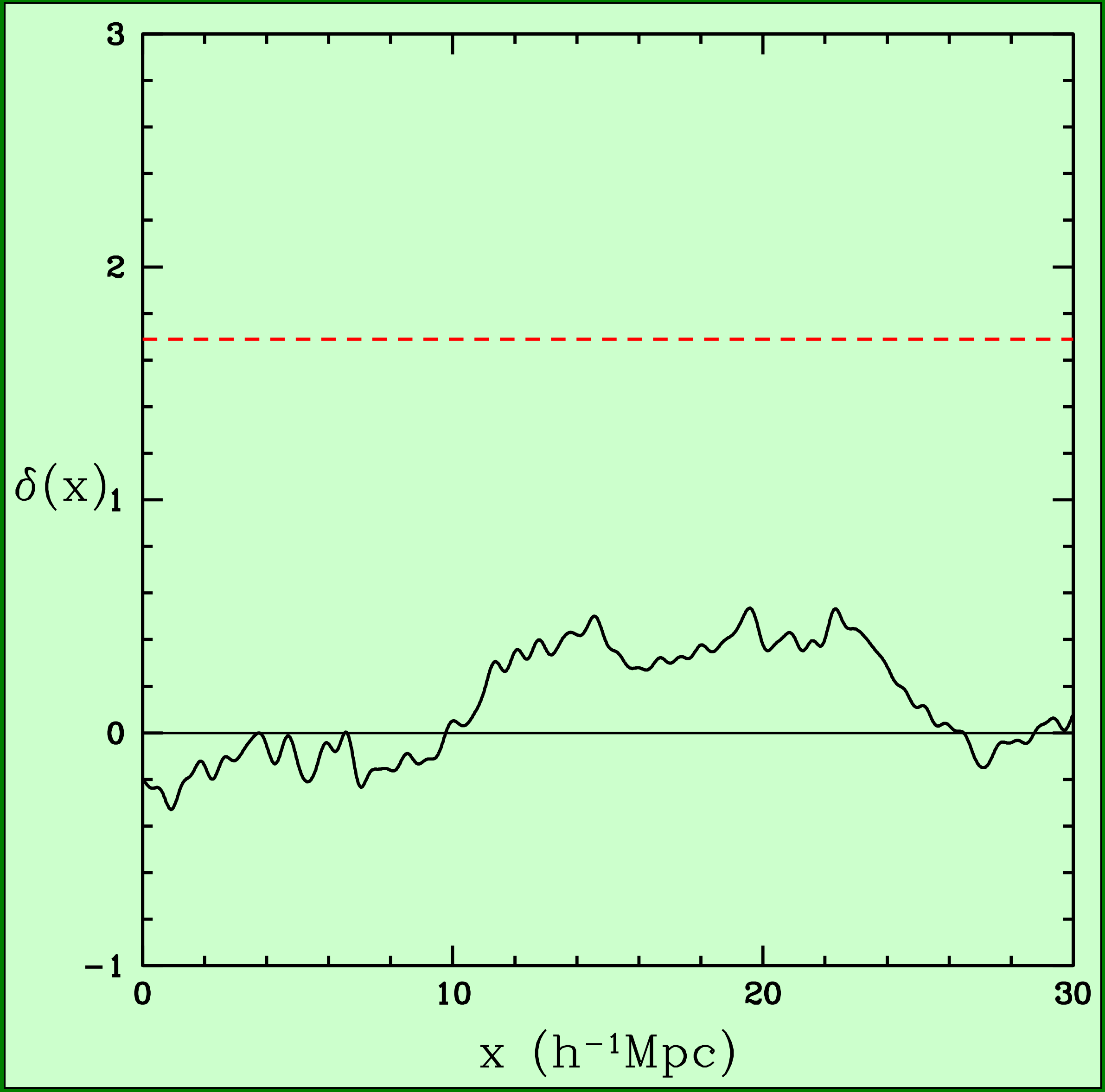
Halo properties: Abundance (mass function)



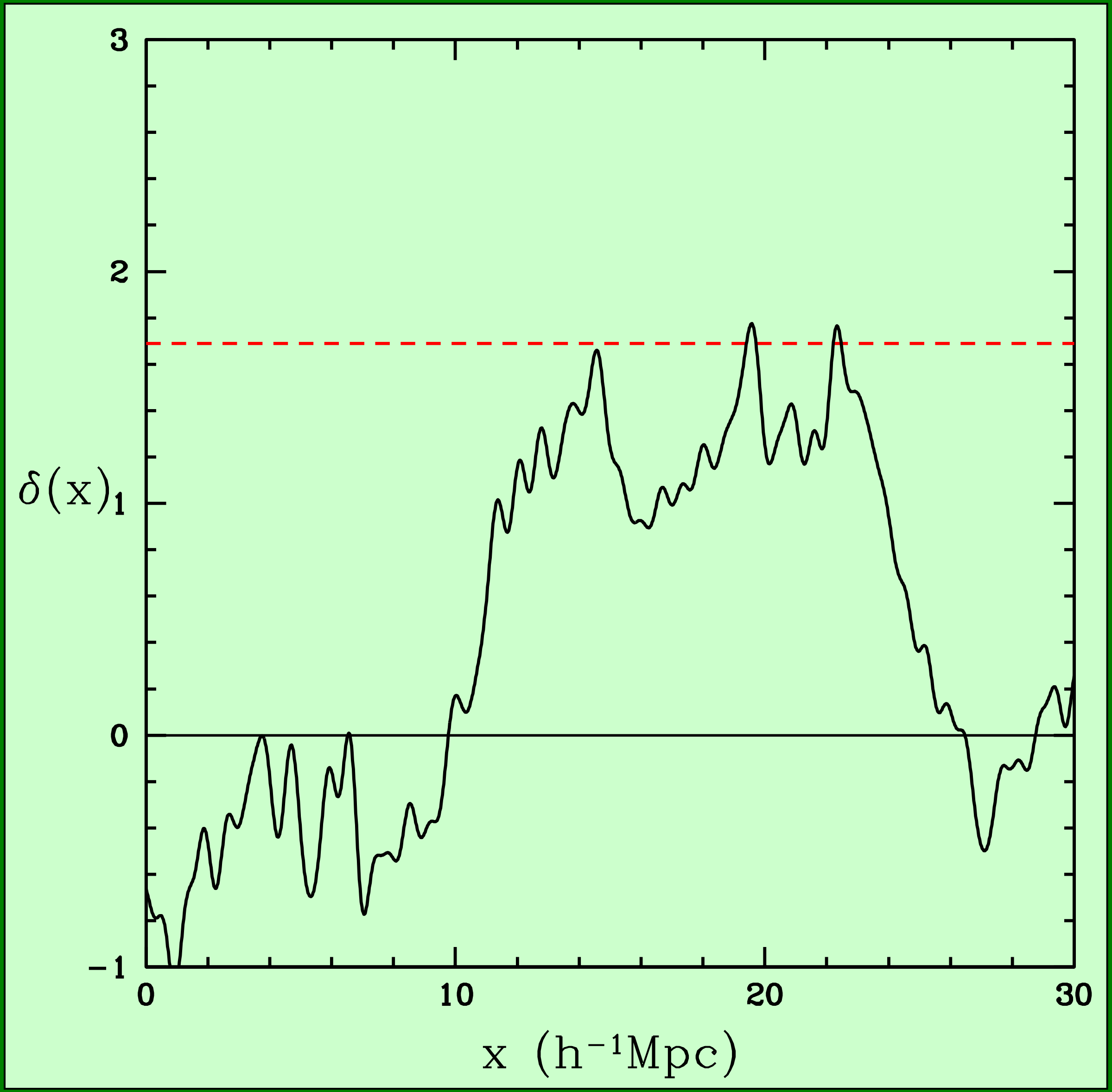
Halo properties: Abundance (mass function)



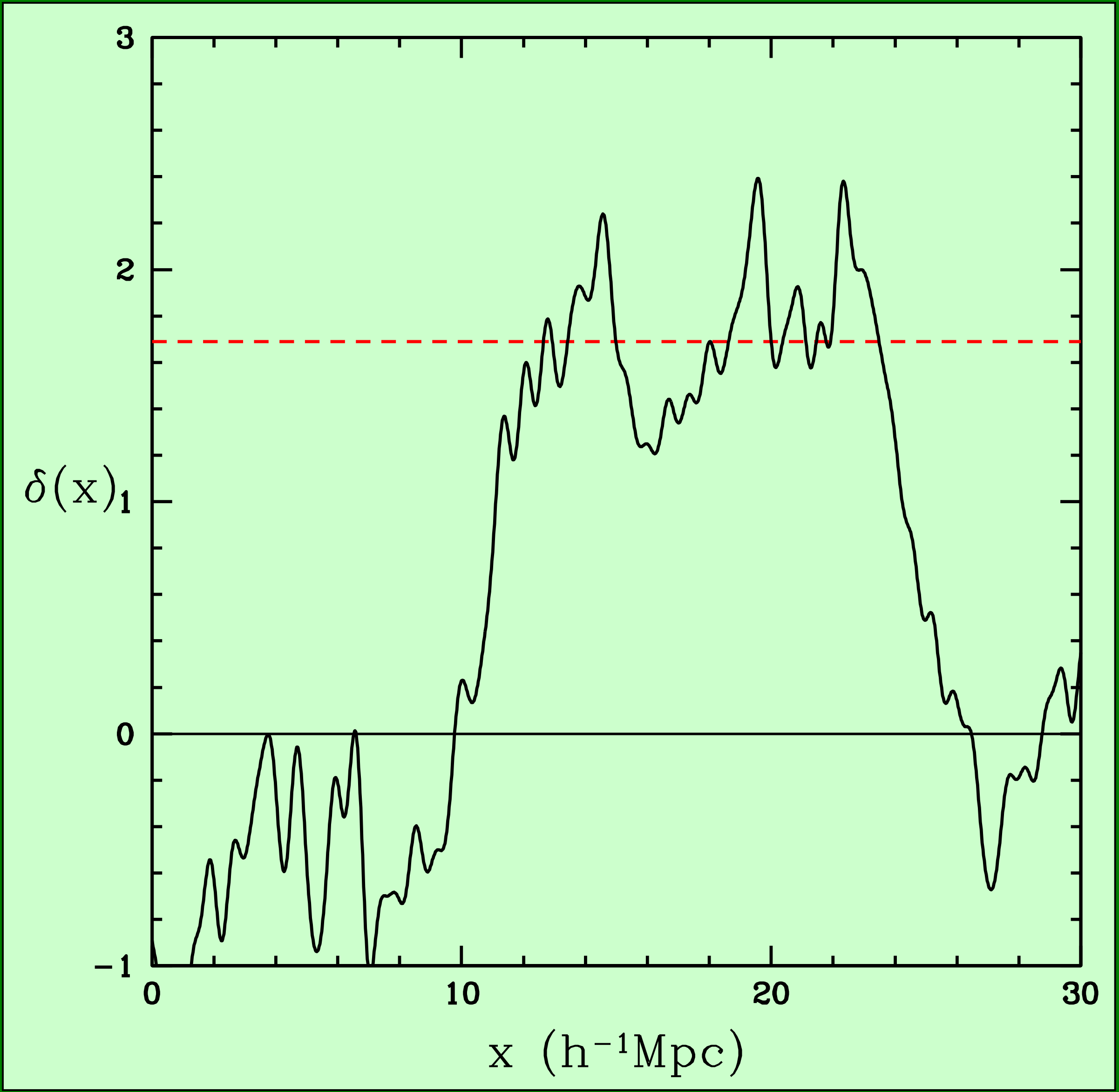
Halo properties: Abundance (mass function)



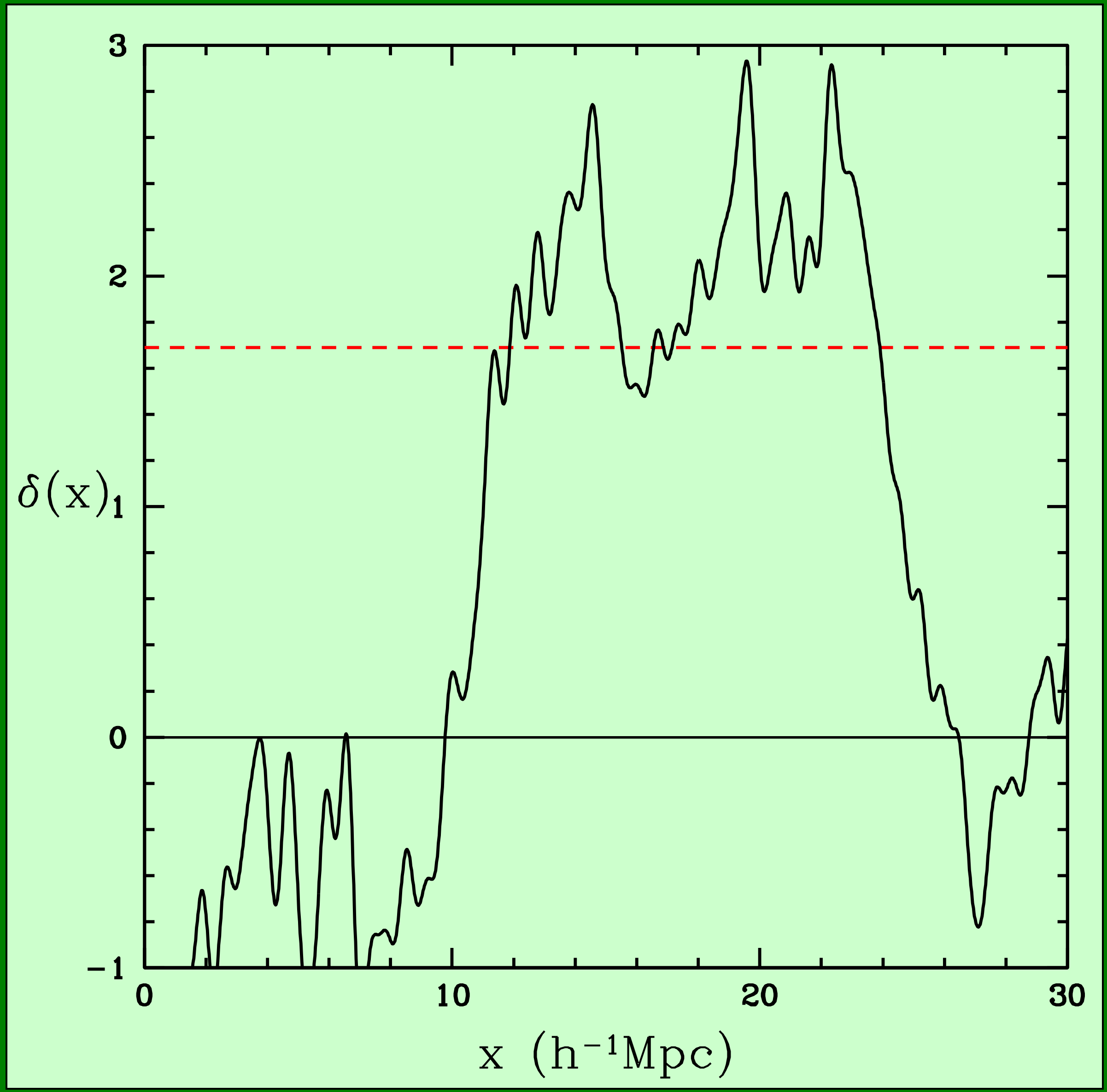
Halo properties: Abundance (mass function)



Halo properties: Abundance (mass function)



Halo properties: Abundance (mass function)



Halo properties: Abundance (mass function)

Consider a spherical region of mass *M*. This region corresponds to a scale:

$$R = \left(\frac{3M}{4\pi\bar{\rho}} \right)^{1/3}$$

The density field smoothed on this scale has a variance of:

$$\sigma_R^2 = \int P(k) \tilde{W}_R(k)^2 d^3k \quad \rightarrow \quad \sigma(M)$$

The probability of the density having a value between δ and δ+dδ is:

$$P(\delta \mid M) d\delta = \frac{1}{\sqrt{2\pi}\sigma(M)} \exp\left[-\frac{\delta^2}{2\sigma(M)^2} \right] d\delta$$

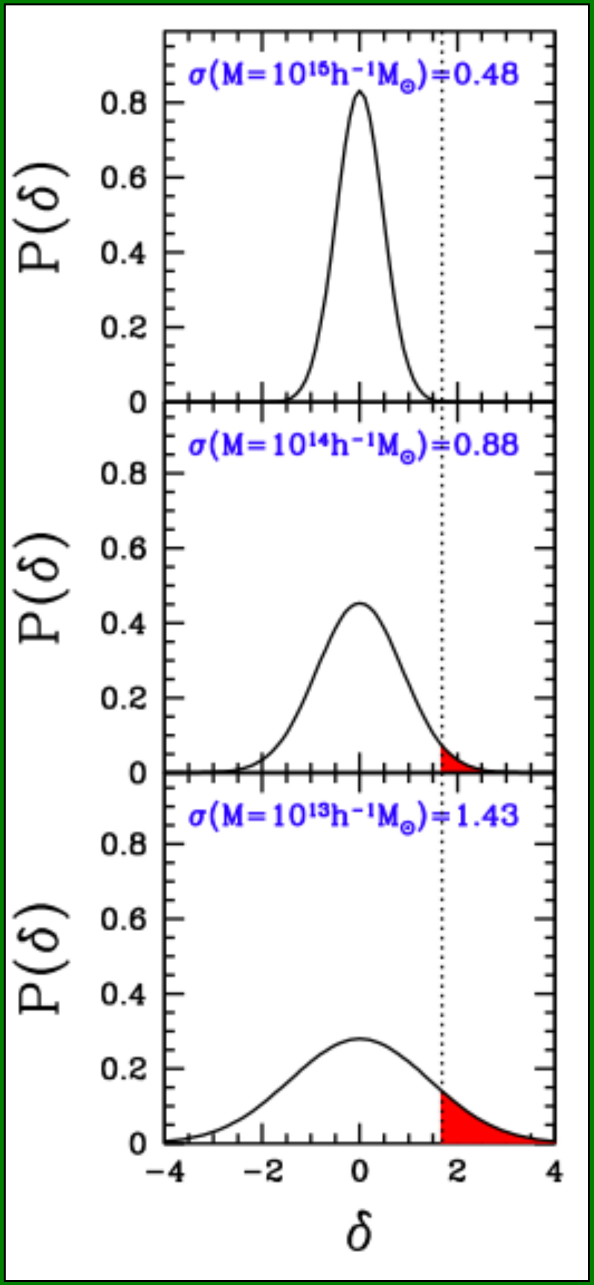
Halo properties: Abundance (mass function)

The fractional volume in this smoothed density field with $\delta > \delta_c$ is:

$$F(>M) = \int_{\delta_c}^{\infty} P(\delta \mid M)\, d\delta$$

The fractional volume corresponding to masses in the range *M* to *M+dM* is:

$$\frac{dF(>M)}{dM}\, dM$$



Halo properties: Abundance (mass function)

The volume of a region that will make a single halo of mass *M* is:

$$\frac{M}{\bar{\rho}}$$

The number of halos of mass in the range *M* to *M*+*dM* is:

$$\frac{\text{fraction of volume} \times V_{\text{tot}}}{\text{volume of 1 halo}} = \frac{\bar{\rho}}{M}\frac{dF(>M)}{dM}dM \times V_{\text{tot}}$$

The number density of halos of mass in the range *M* to *M*+*dM* is:

$$\frac{dn}{dM}dM = \frac{\bar{\rho}}{M}\frac{dF(>M)}{dM}dM$$

Halo properties: Abundance (mass function)

$$\frac{dn}{dM} = \sqrt{\frac{2}{\pi}} \frac{\bar{\rho}}{M^2} \frac{\delta_c}{\sigma} \frac{d \ln \sigma}{d \ln M} \exp\left[-\frac{\delta_c^{\ 2}}{2\sigma^2} \right]$$

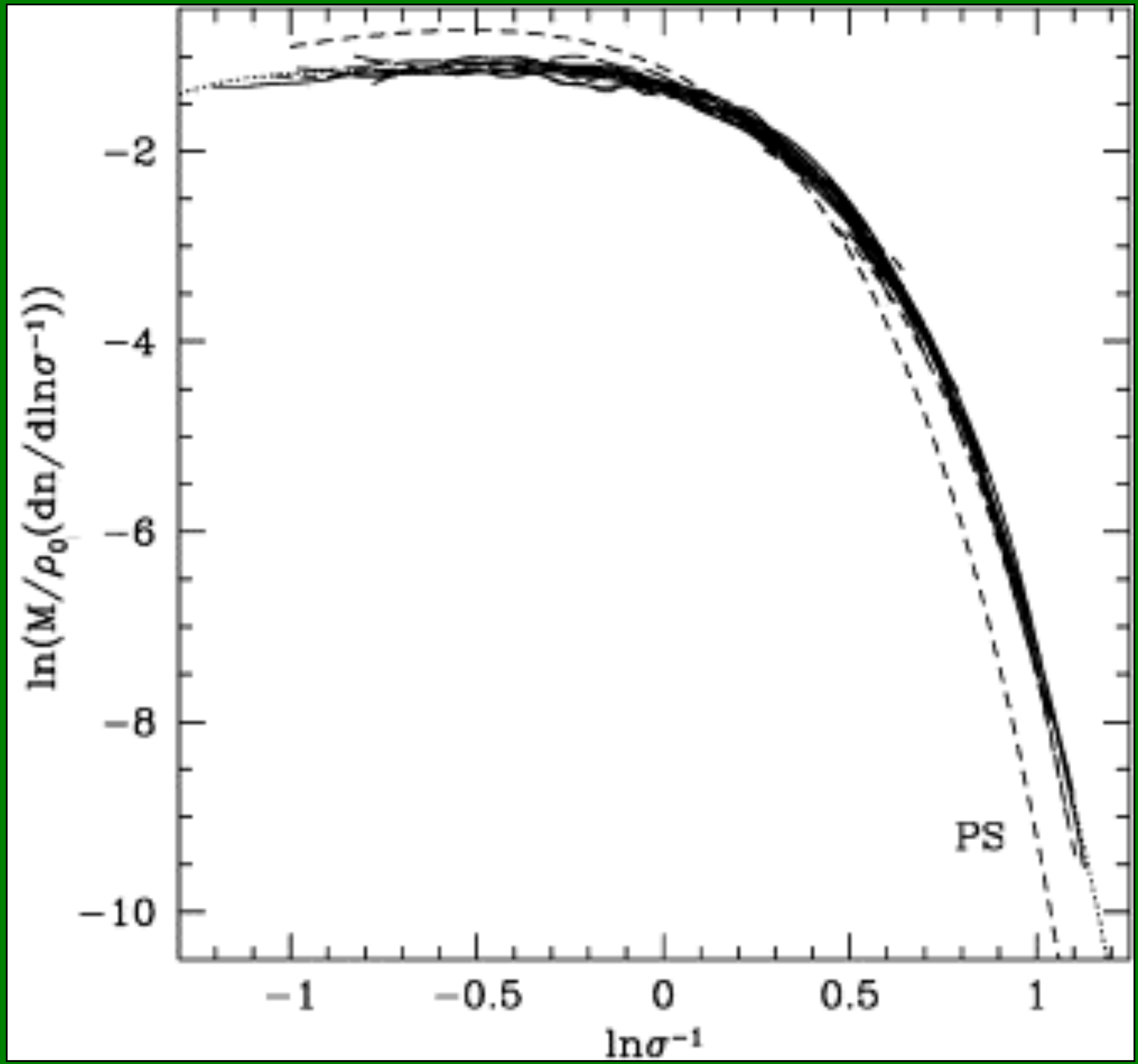
Spherical collapse model

Power spectrum

Linear theory growth rate $\sigma(M, z) = \sigma(M, z = 0) D(z)$

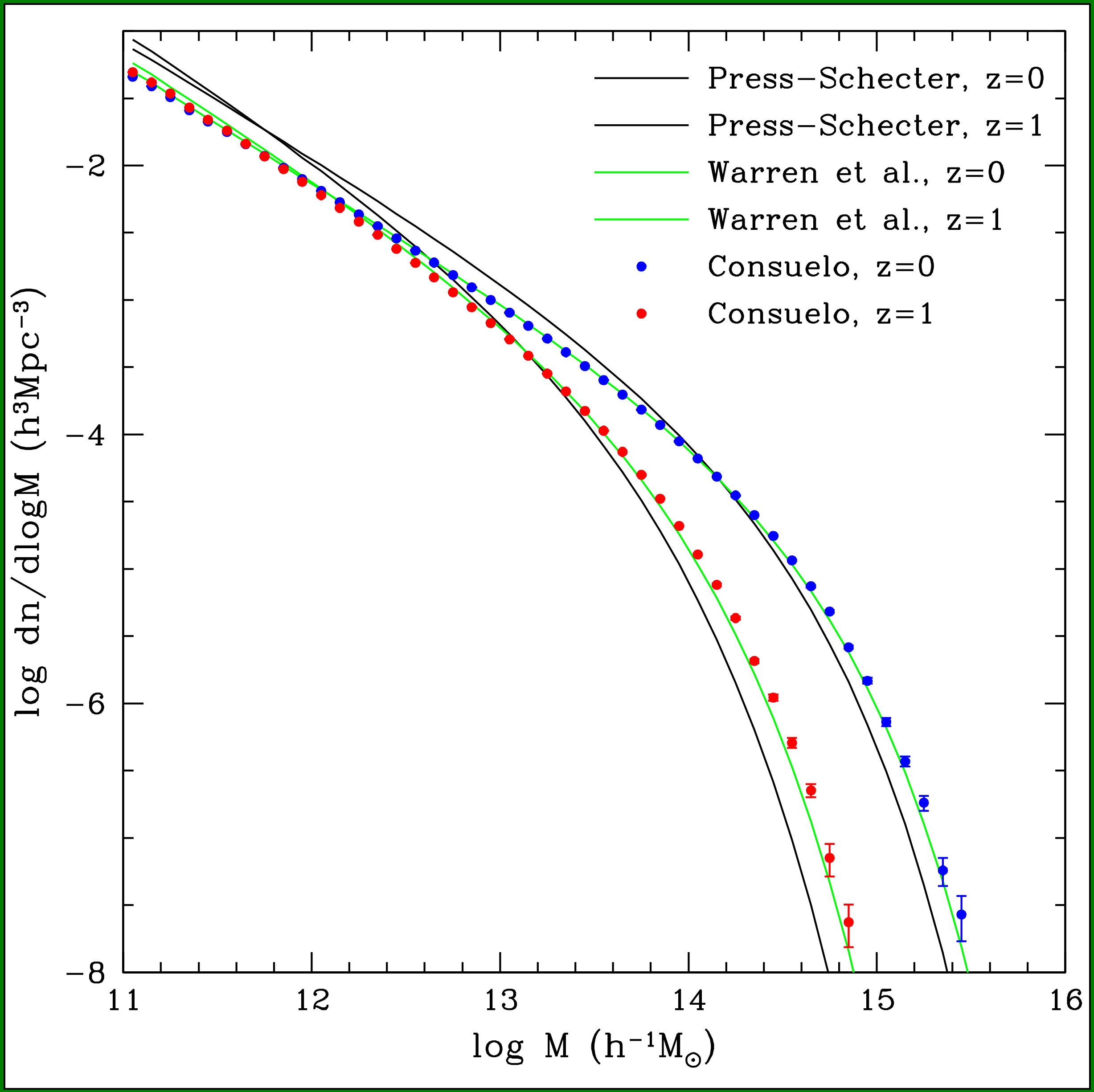
There are numerous improvements to the Press-Schechter mass function

Halo properties: Abundance (mass function)

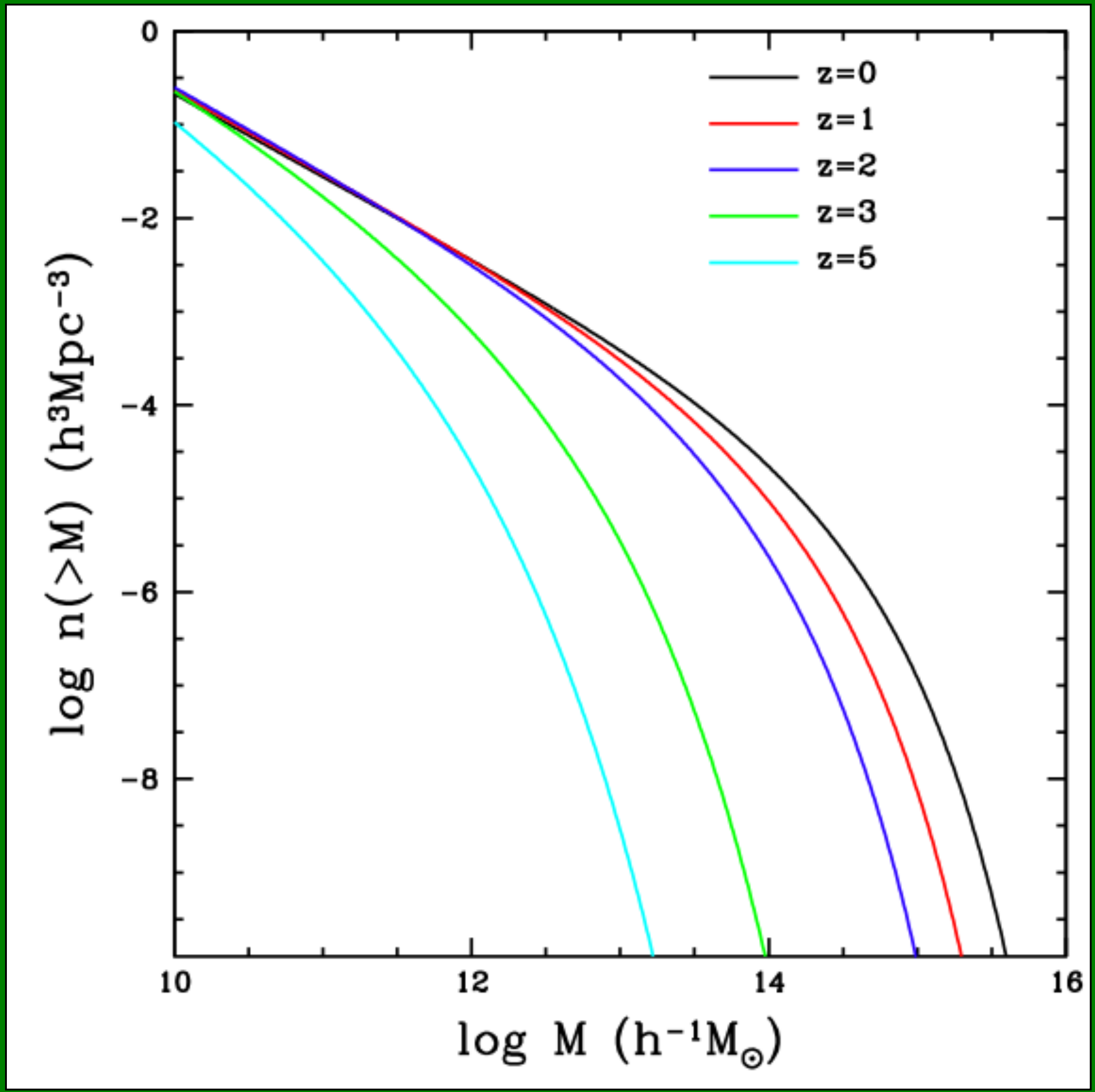


Cooray & Sheth (2002)

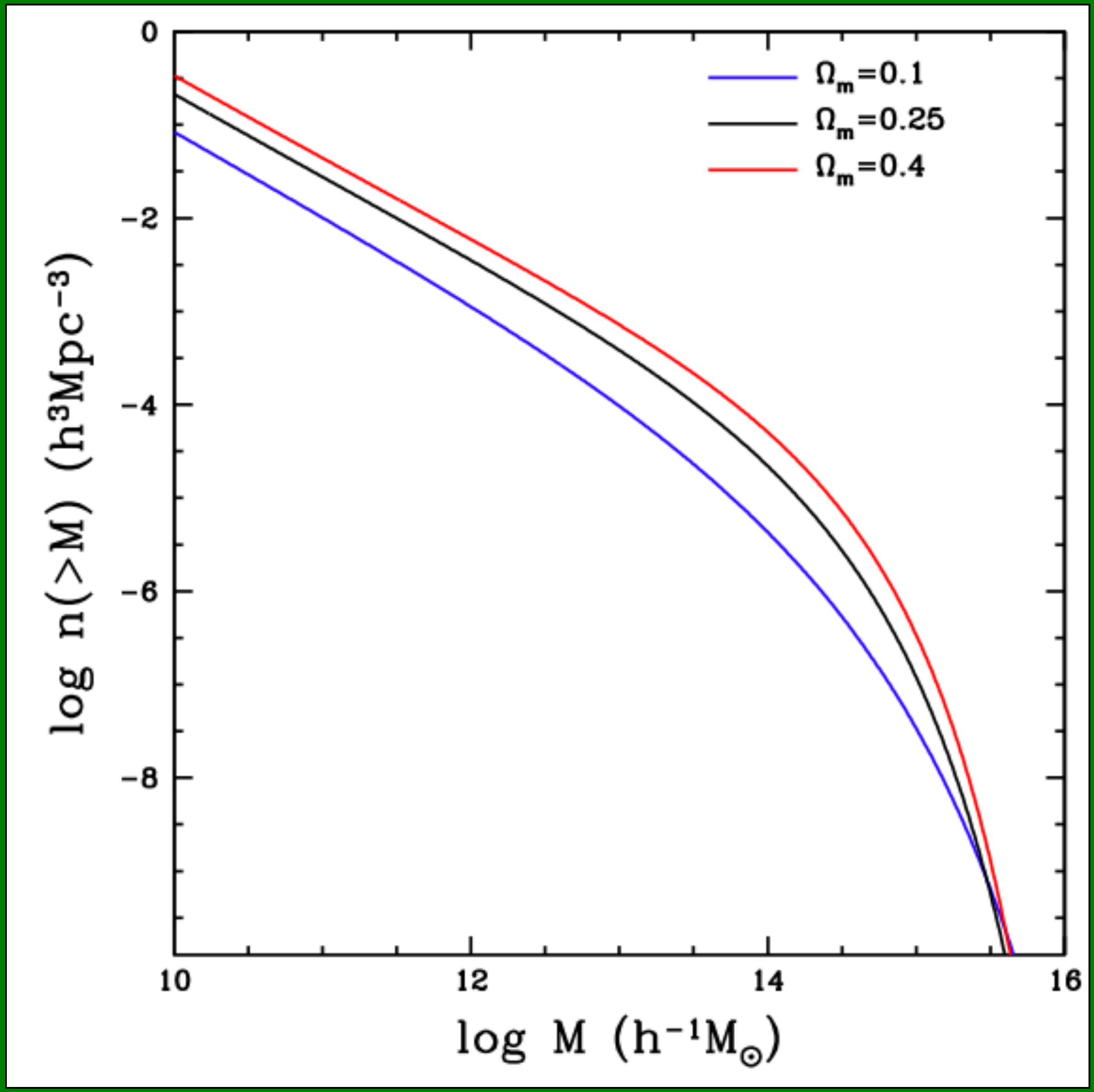
Halo properties: Abundance (mass function)



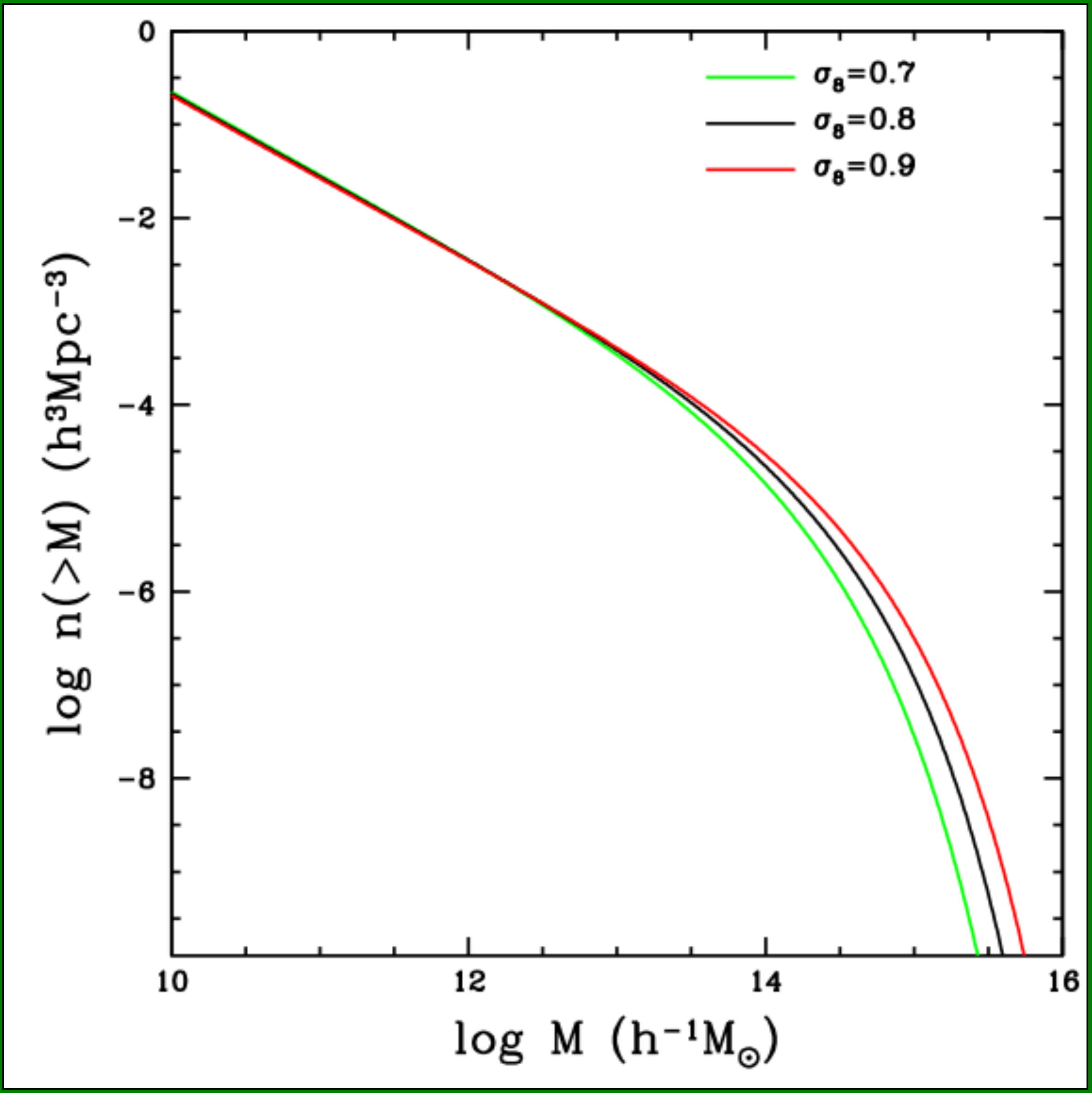
Halo properties: Abundance (mass function)



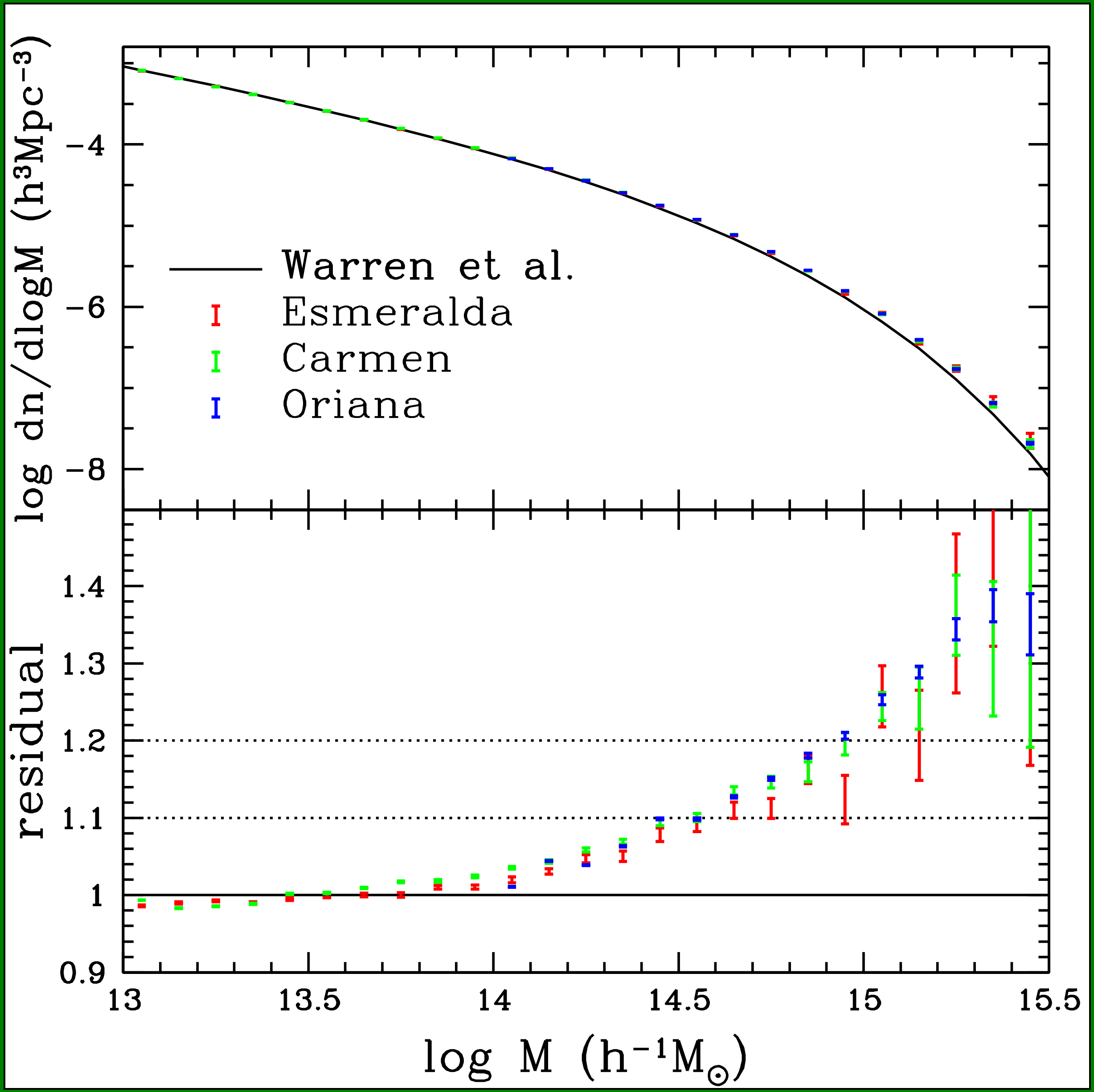
Halo properties: Abundance (mass function)



Halo properties: Abundance (mass function)

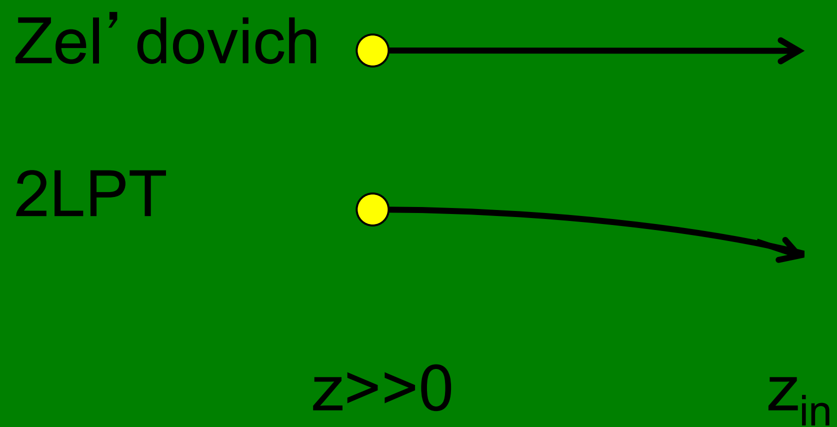


Halo properties: Abundance (mass function)

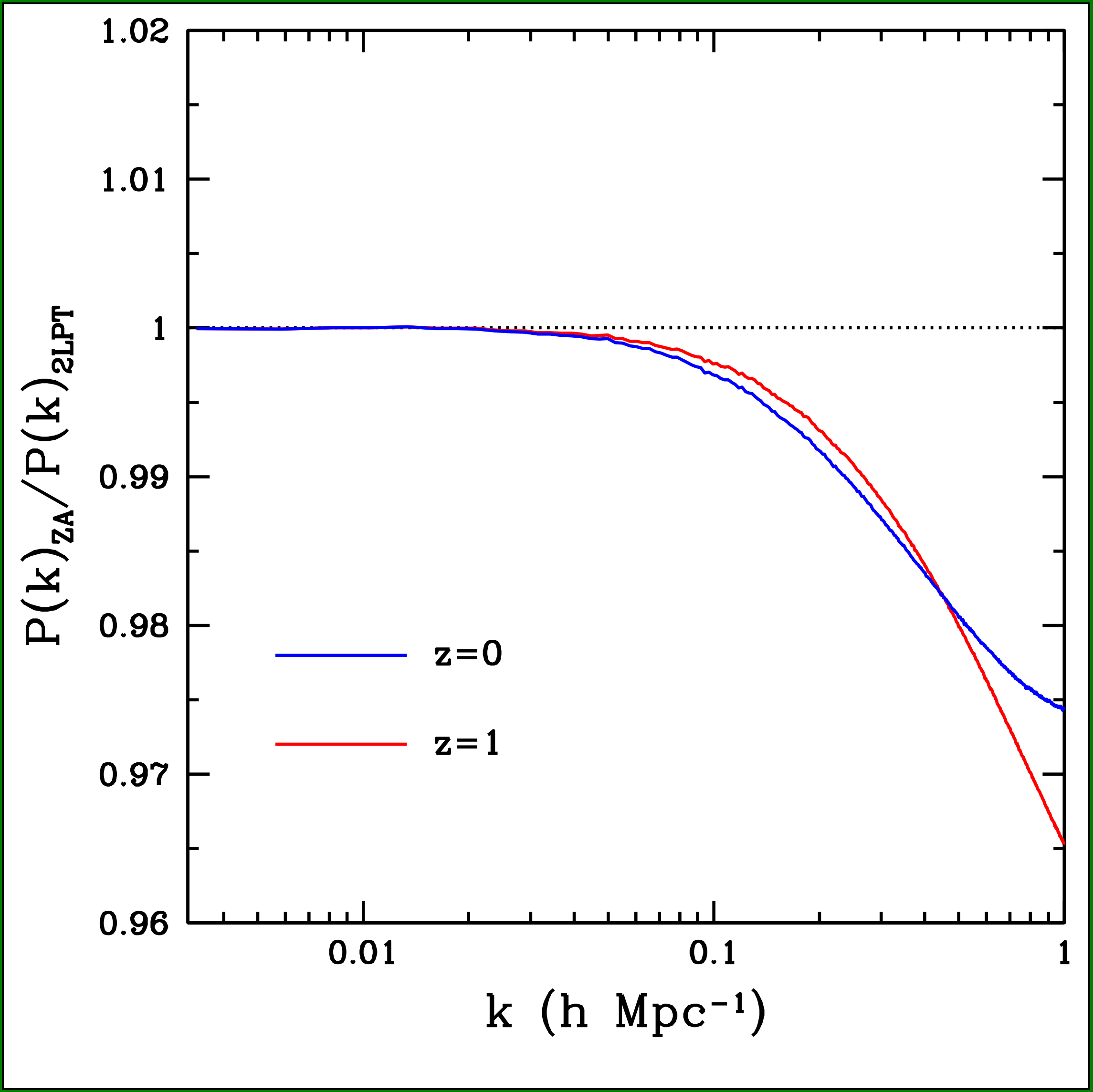


LasDamas

Halo properties: Abundance (mass function)

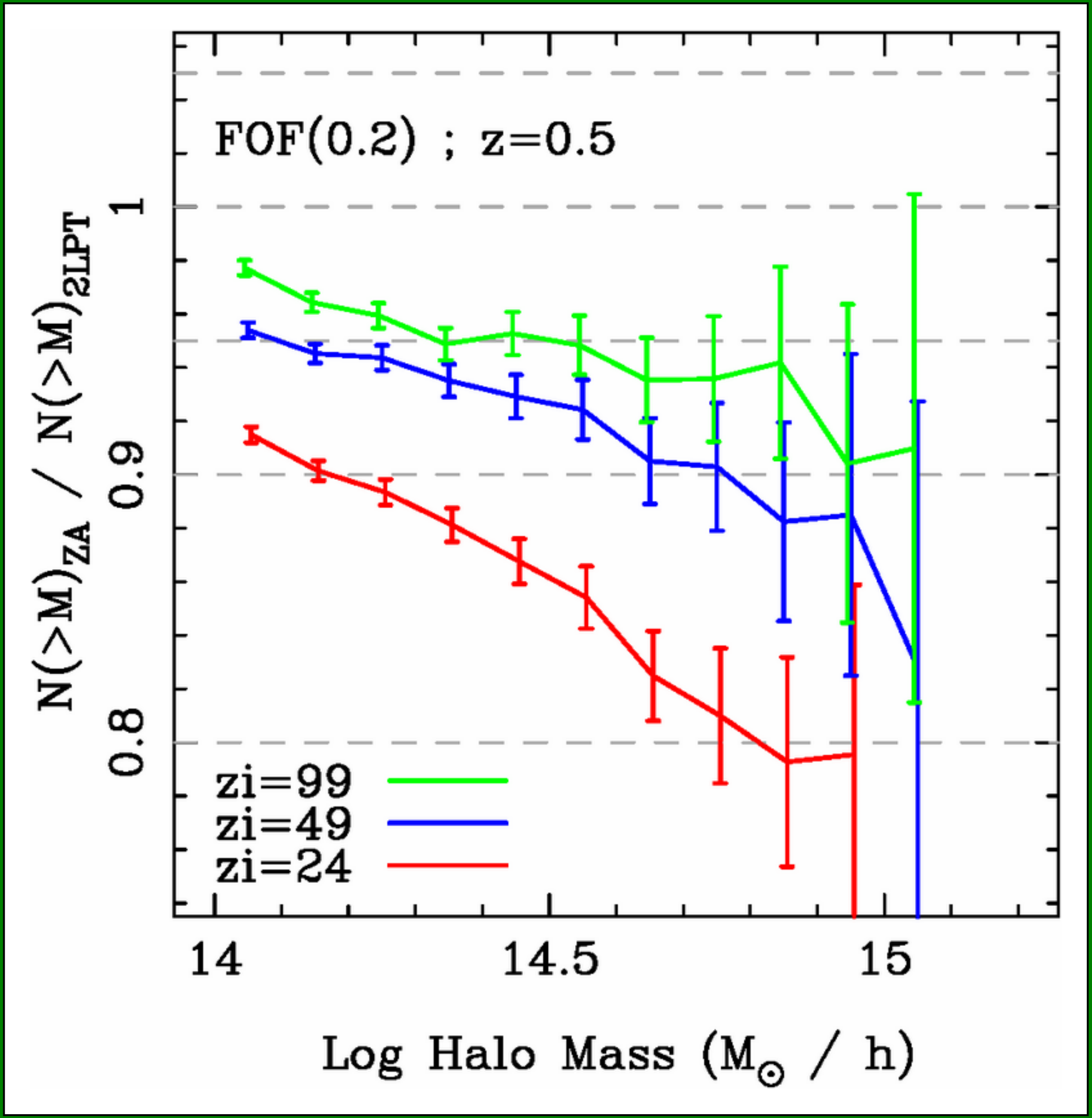


Halo properties: Abundance (mass function)



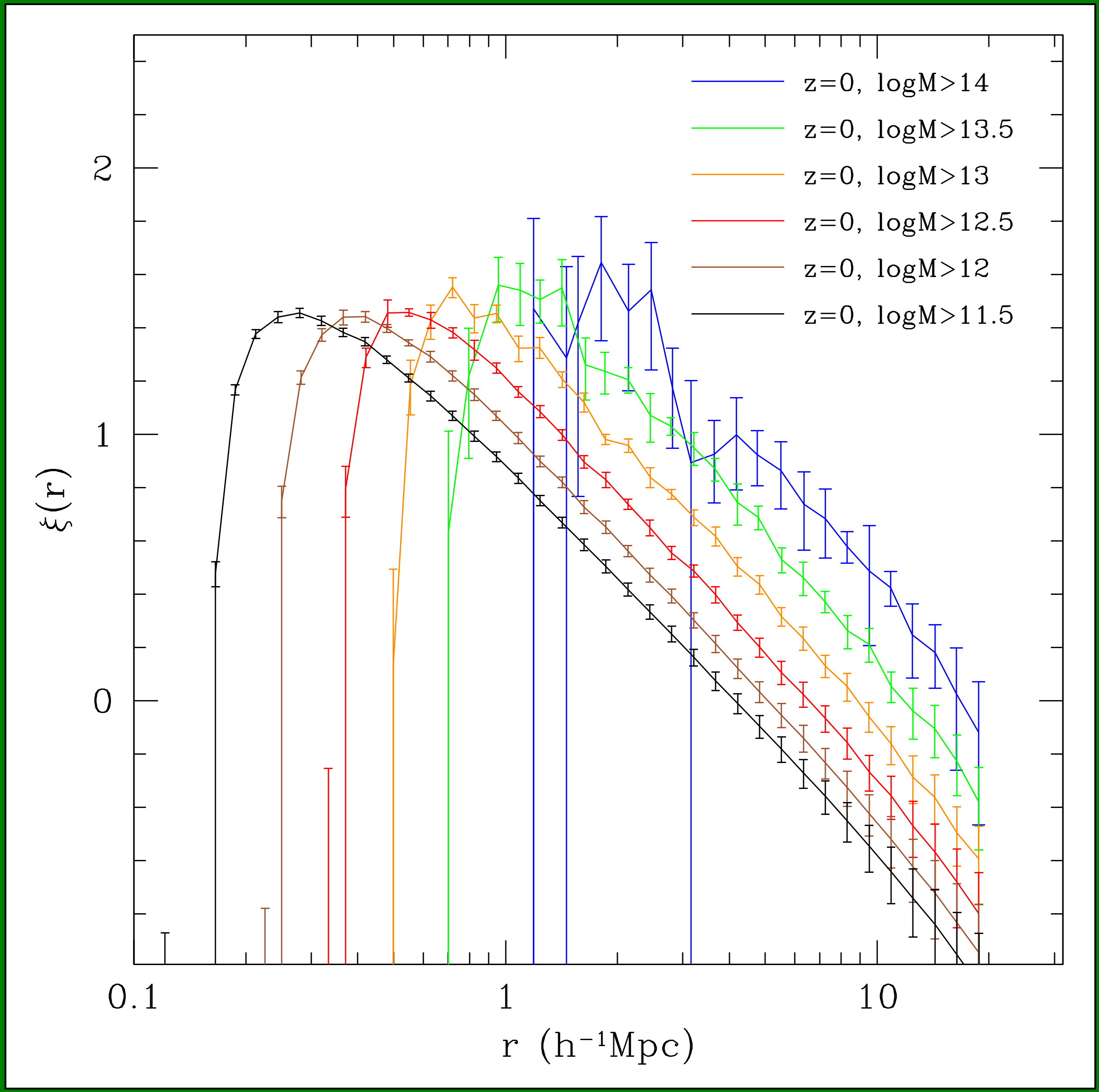
LasDamas

Halo properties: Abundance (mass function)



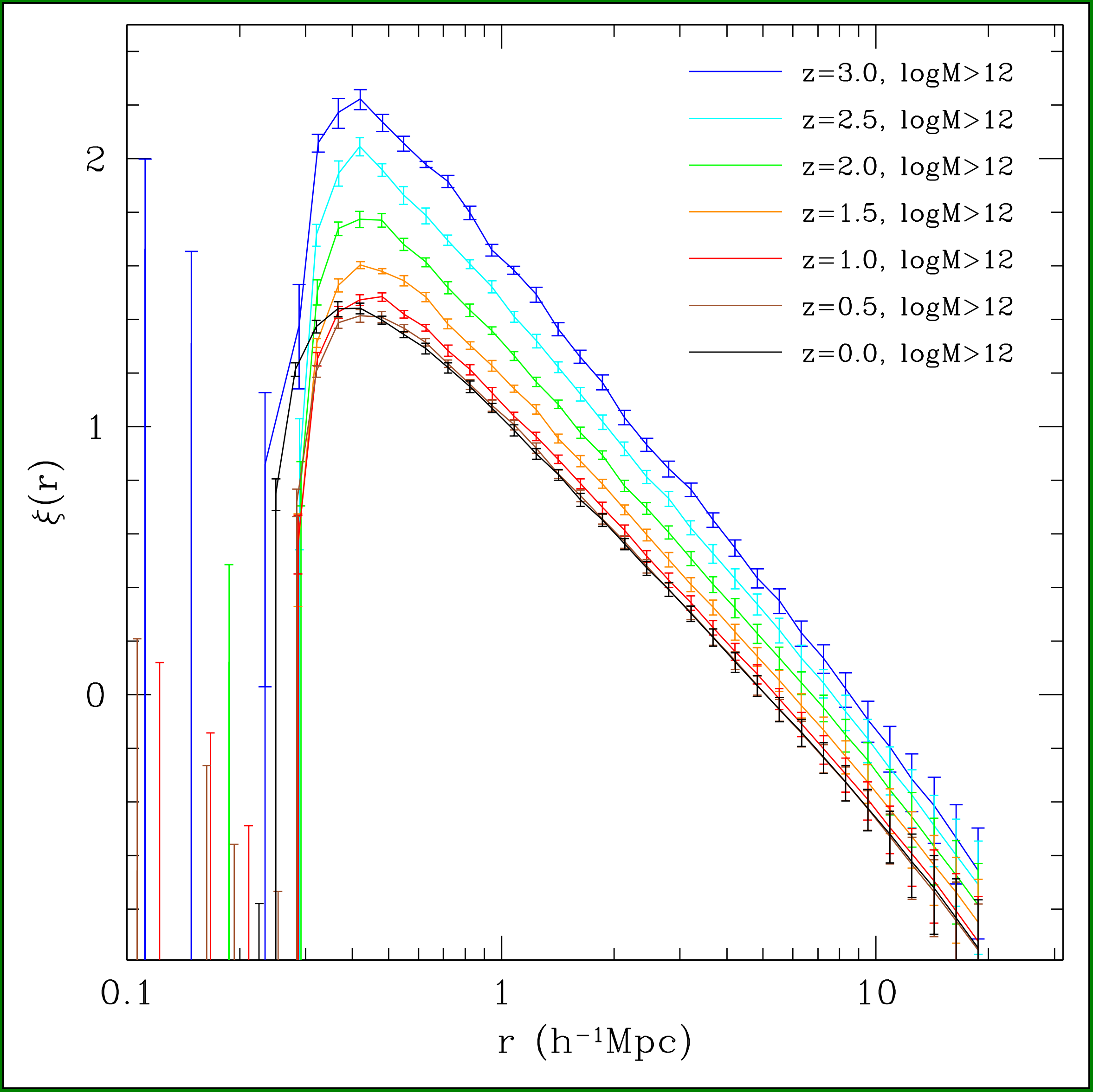
LasDamas

Halo properties: Clustering (halo bias)



LasDamas

Halo properties: Clustering (halo bias)

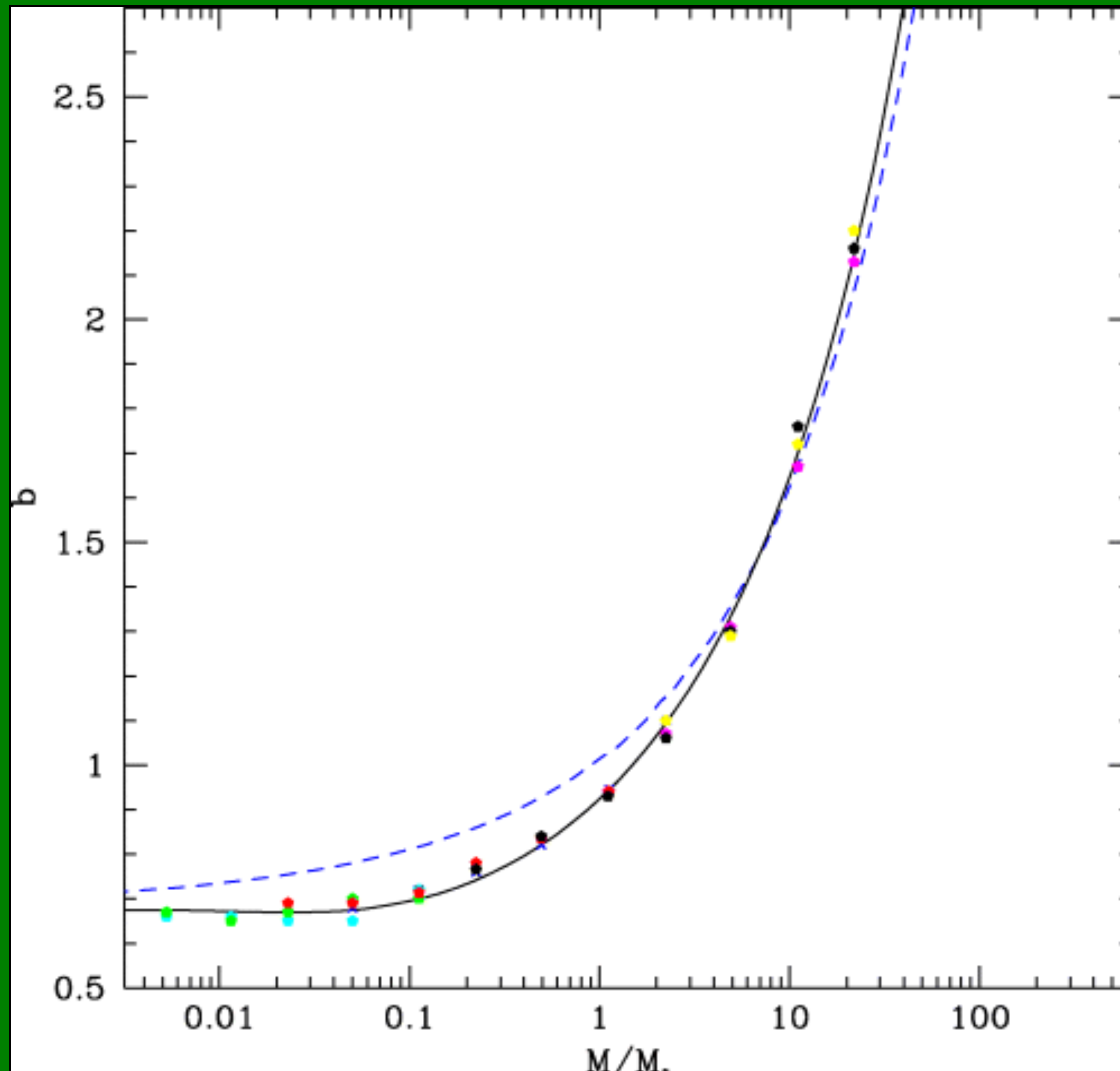


LasDamas

Halo properties: Clustering (halo bias)

Good to ~3% for standard cosmology.

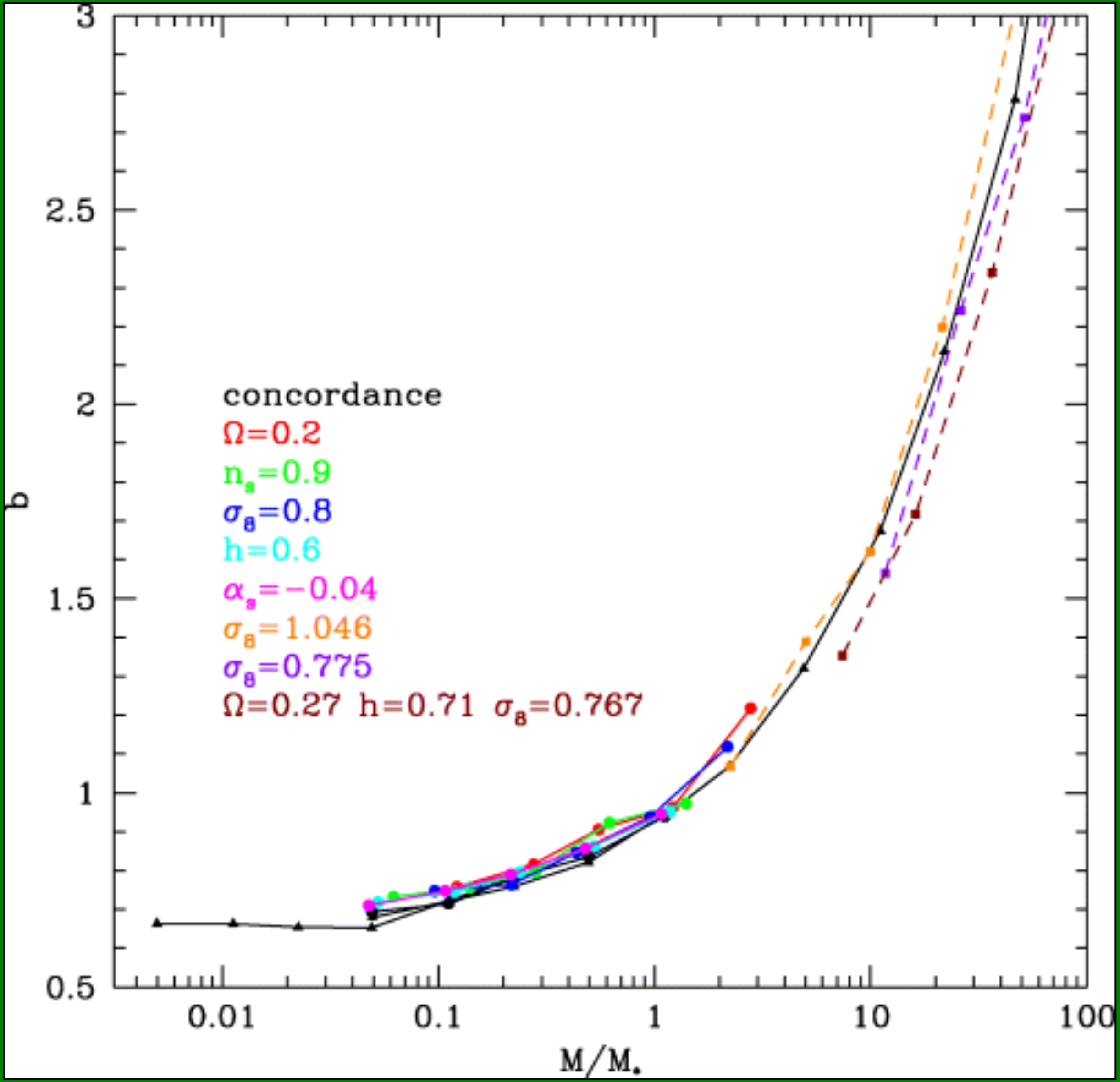
$$b_h = \sqrt{\frac{P_{hh}(k<0.1)}{P_{mm}(k<0.1)}}$$



Seljak & Warren (2005)

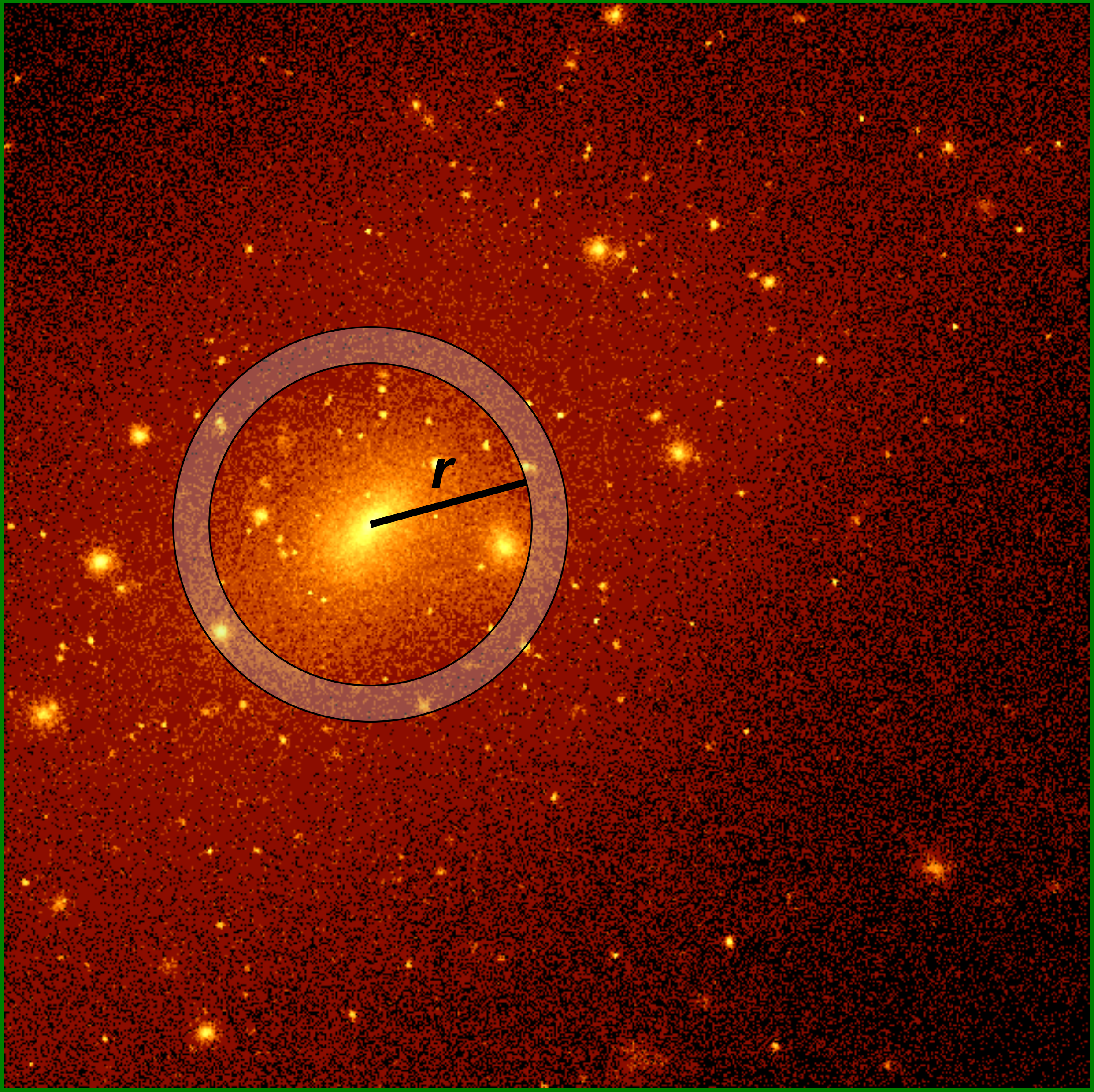
Halo properties: Clustering (halo bias)

Good to ~10% across different cosmologies.

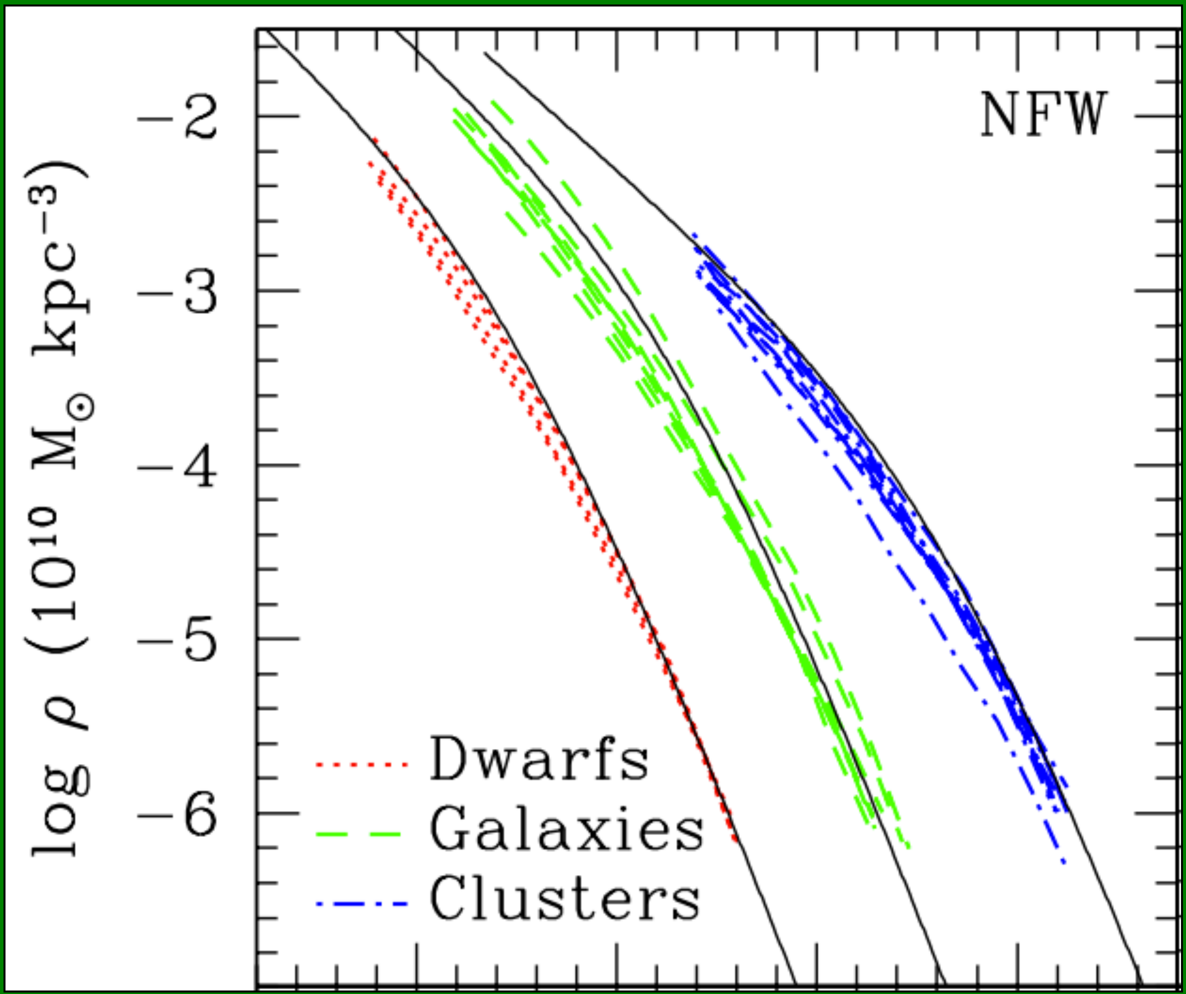


Seljak & Warren (2005)

Halo properties: Structure (density profile)



Halo properties: Structure (density profile)



log*r*

Navarro et al. (2004)

Navarro, Frenk & White (NFW)

$$\rho(r) = \frac{\rho_s}{(1 + r/r_s)^2 (r/r_s)}$$

Halo properties: Structure (density profile)

Navarro, Frenk & White (NFW)

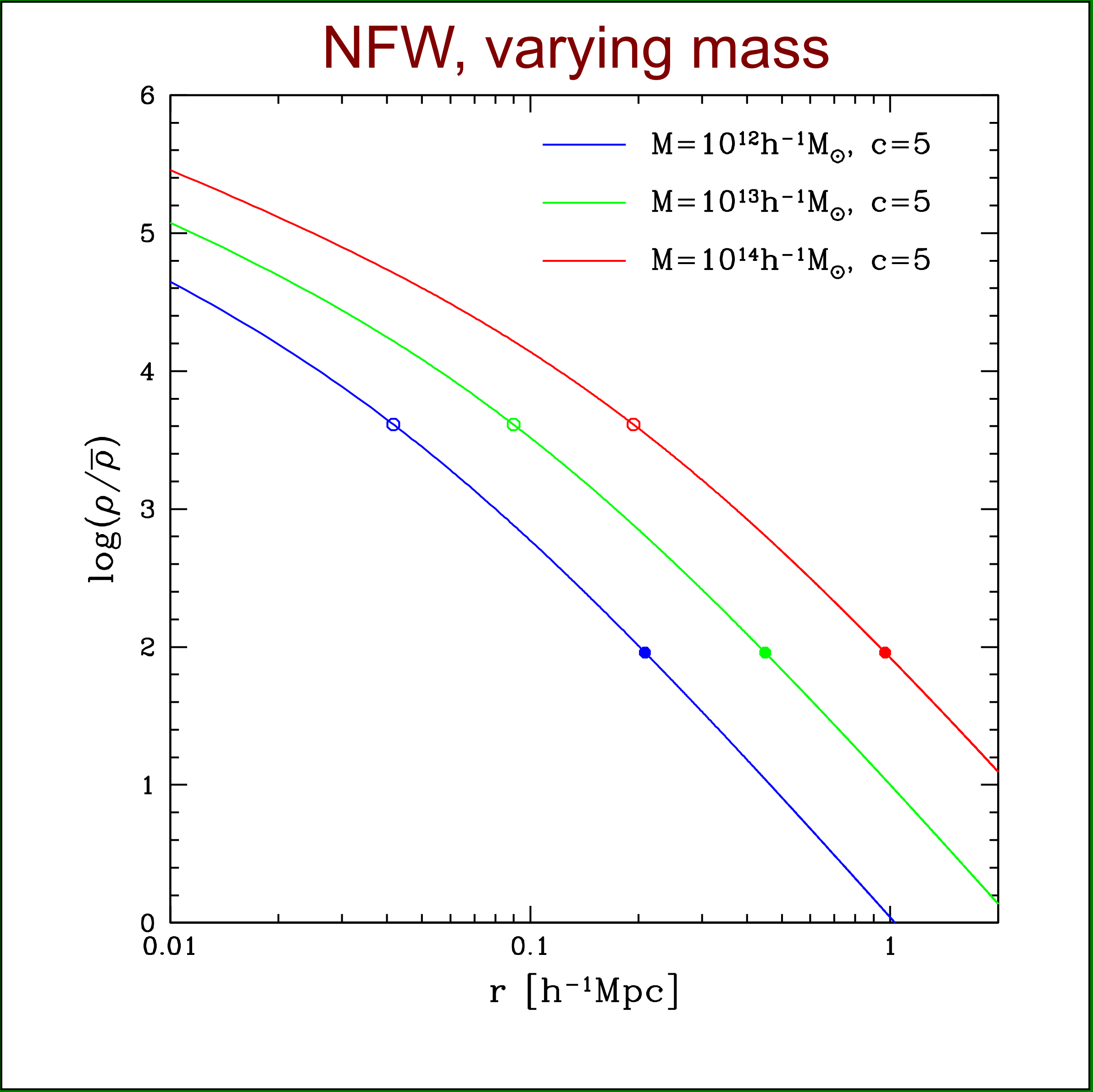
$$\rho(r) = \frac{\rho_s}{\left(1 + r/r_s\right)^2 \left(r/r_s\right)}$$

$$M_{vir} = \frac{4}{3}\pi R_{vir}^3 \Delta_{vir} \bar{\rho}$$

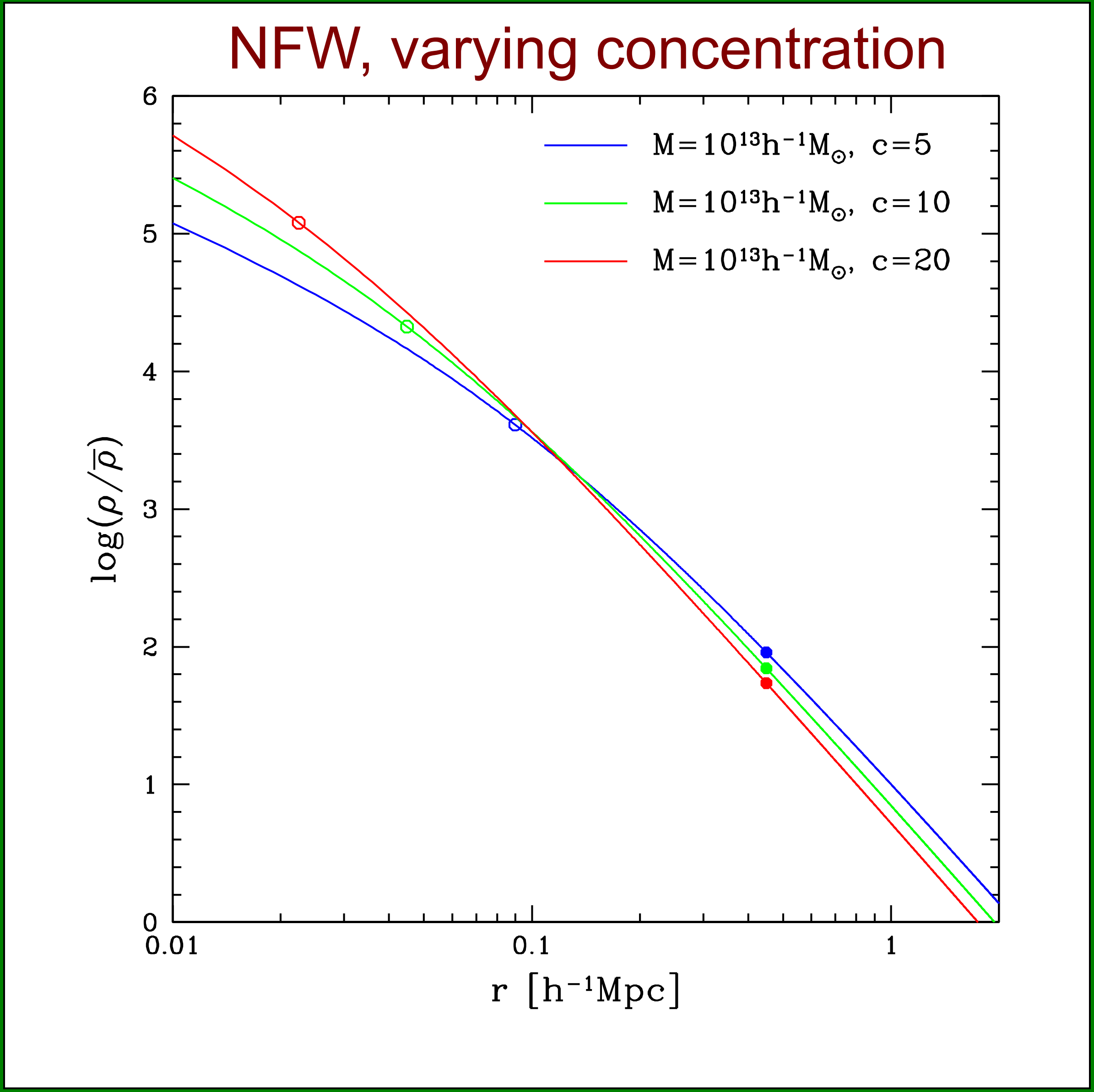
$$c \equiv \frac{R_{vir}}{r_s}$$

$$M_{vir} = \int_0^{R_{vir}} 4\pi r^2 \rho(r) dr$$

Halo properties: Structure (density profile)

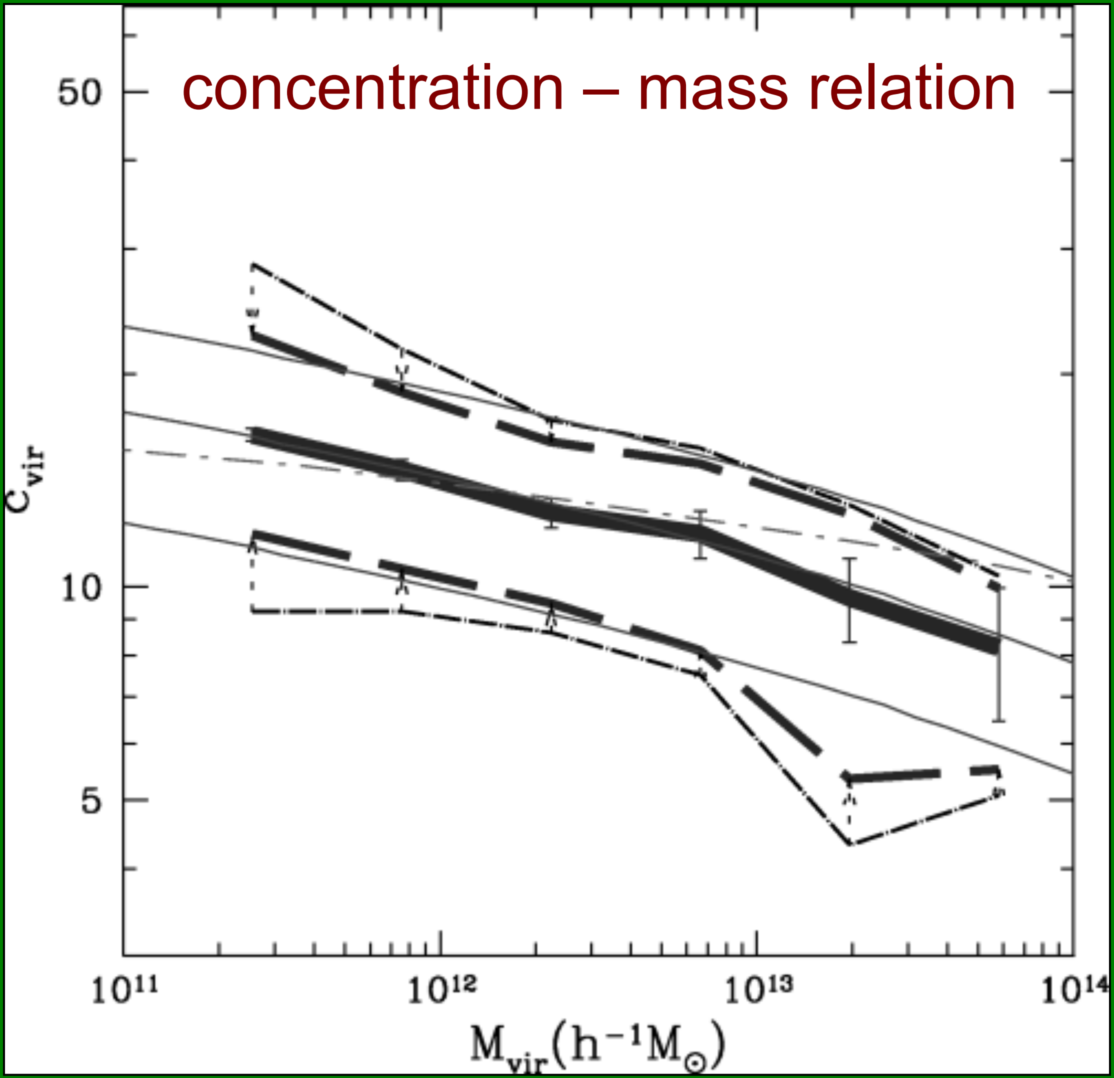


Halo properties: Structure (density profile)



Halo properties: Structure (density profile)

c



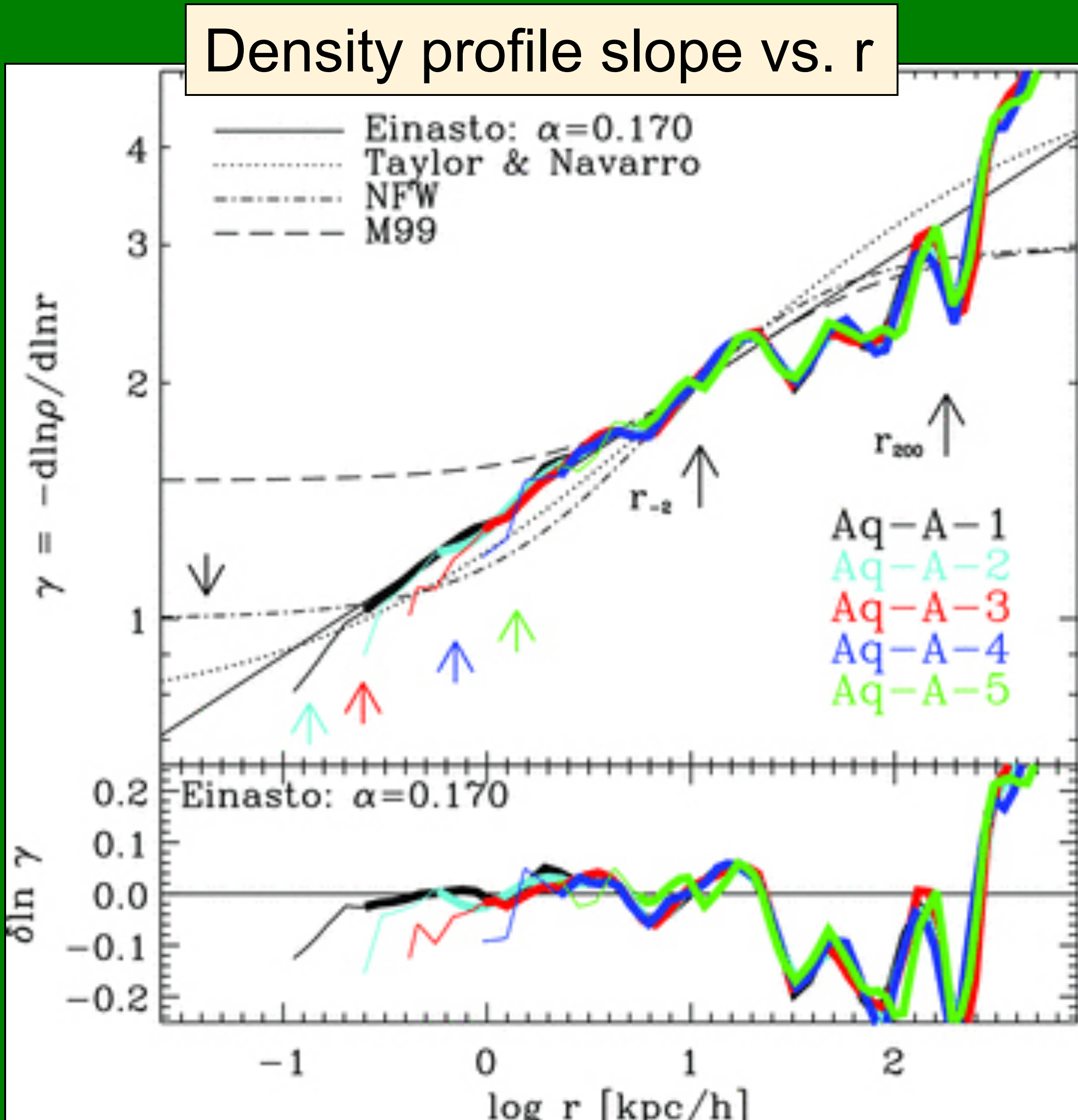
M_{vir}

Bullock et al. (2001)

$$c \equiv R_{vir}/r_s$$

$$c \approx \frac{c_*}{(1+z)}\left(M/M_*\right)^{-0.13}$$

Halo properties: Structure (density profile)



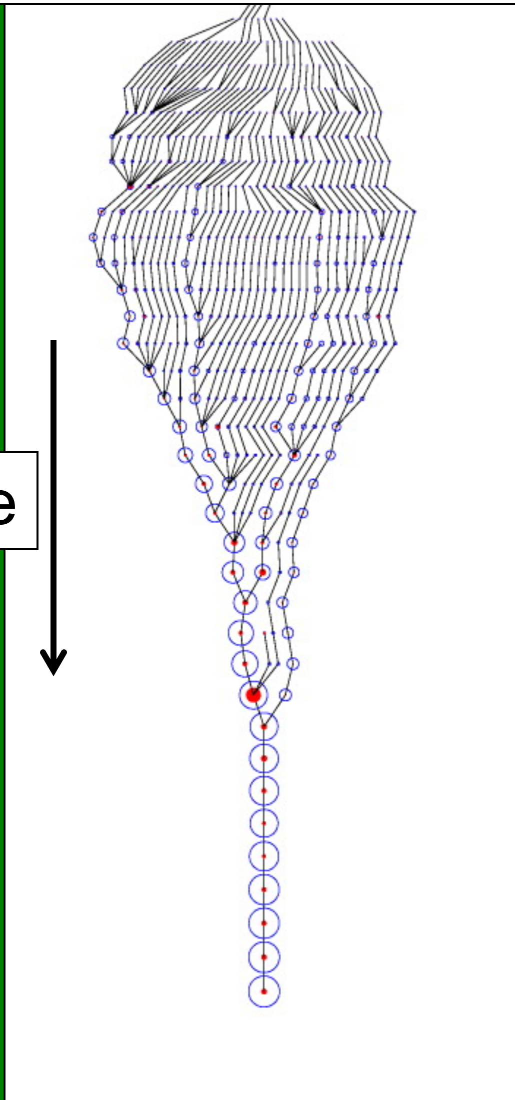
Einasto profile

$$\rho = \rho_{-2} e^{-2/\alpha} e^{\left[\left(r/r_{-2}\right)^{\alpha} - 1\right]}$$

Navarro et al. (2010)

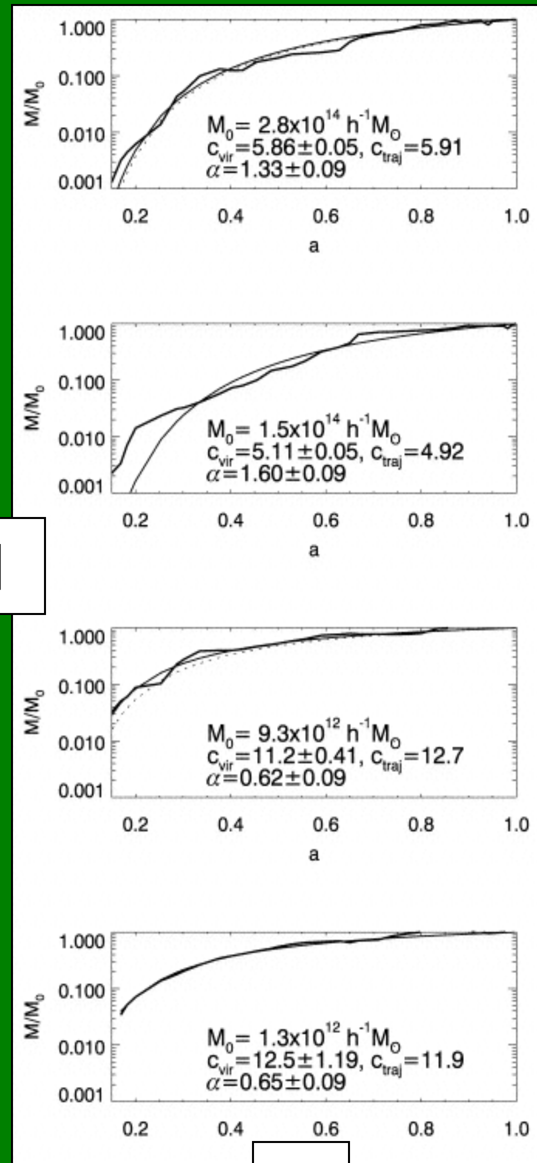
Halo properties: History (merger tree)

Assembly History



time

M

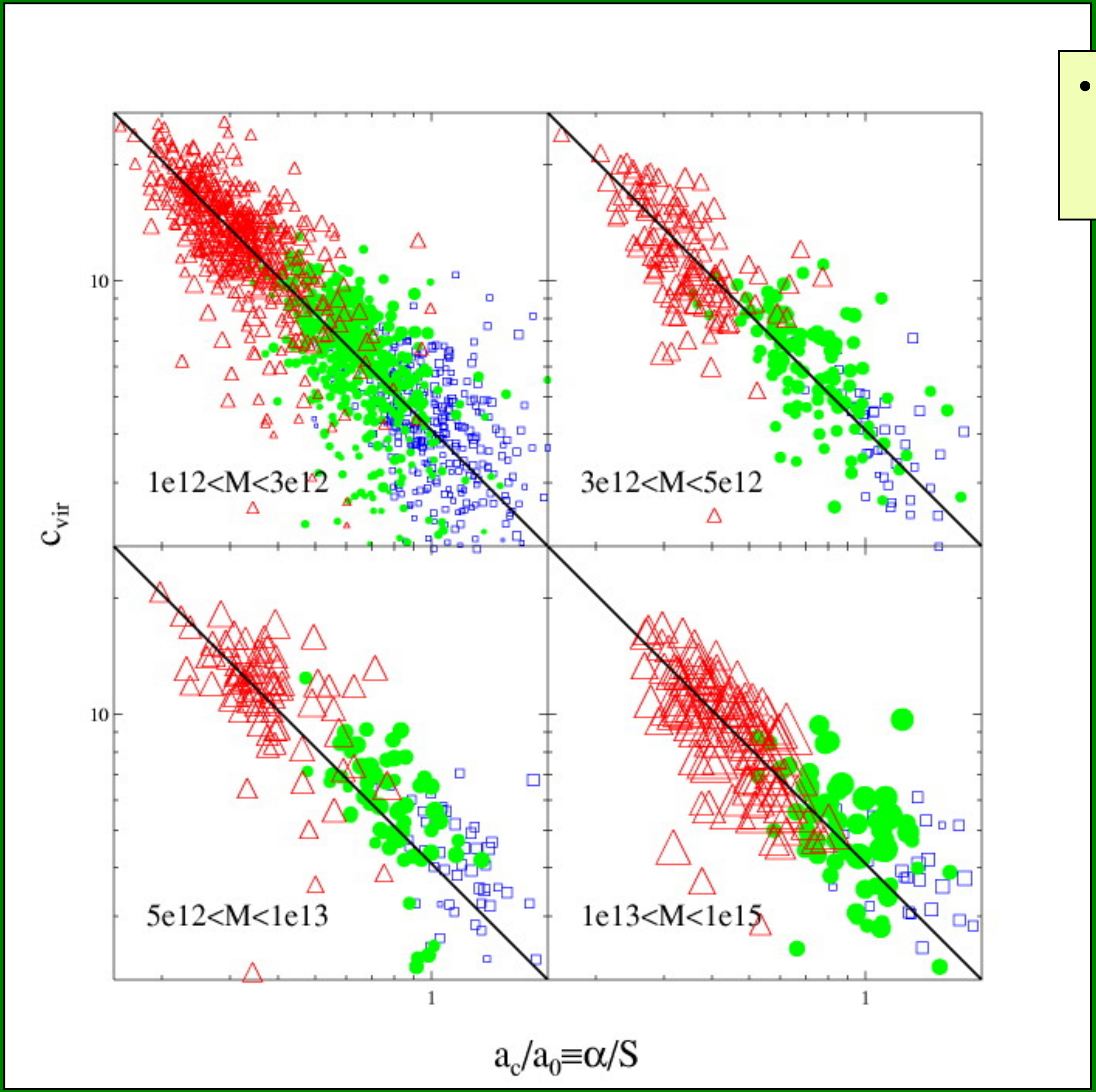


a

- High mass halos have accreted more of their mass recently relative to low mass halos.

Wechsler et al. (2002)

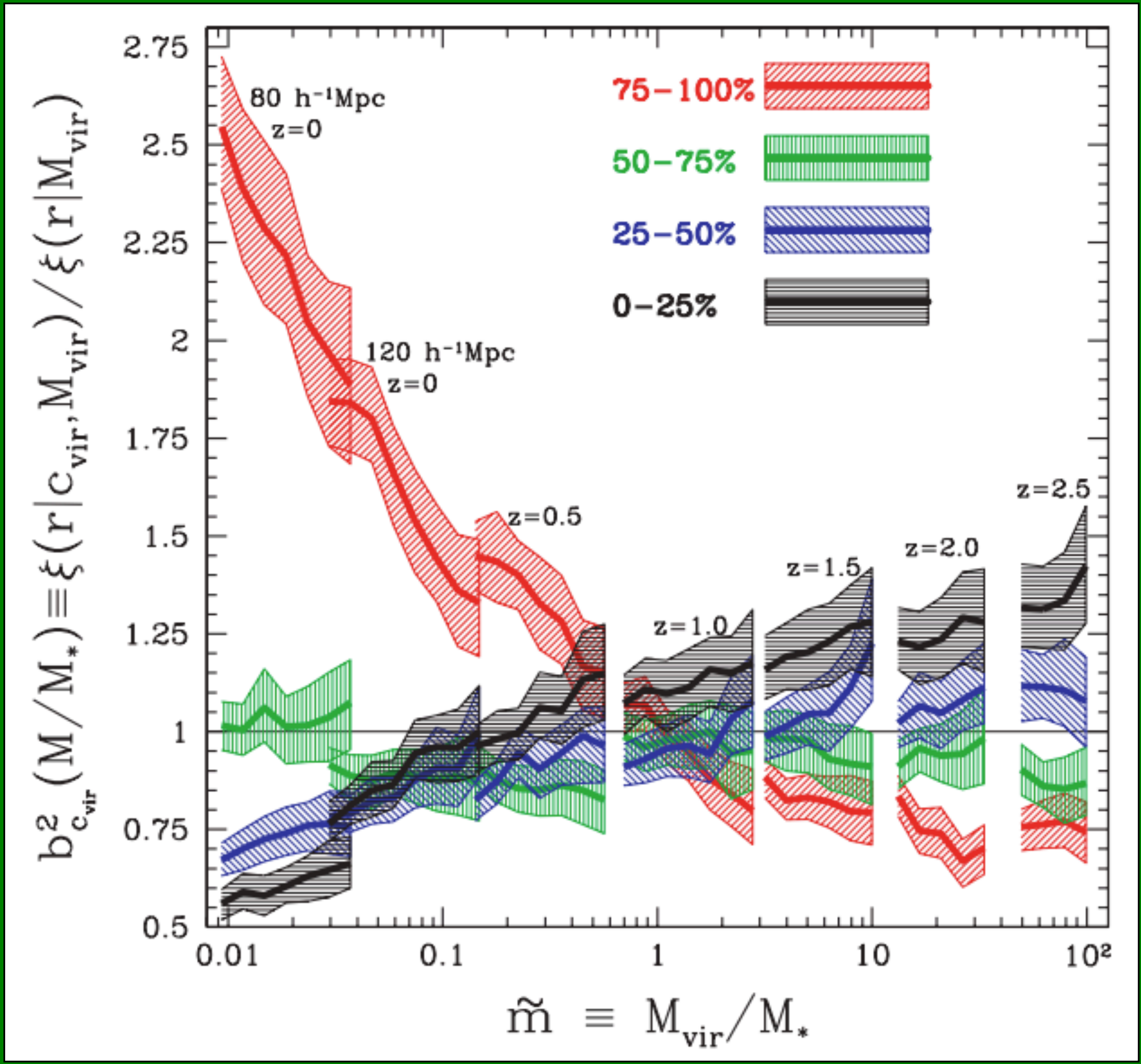
Halo properties: History (merger tree)



Wechsler et al. (2002)

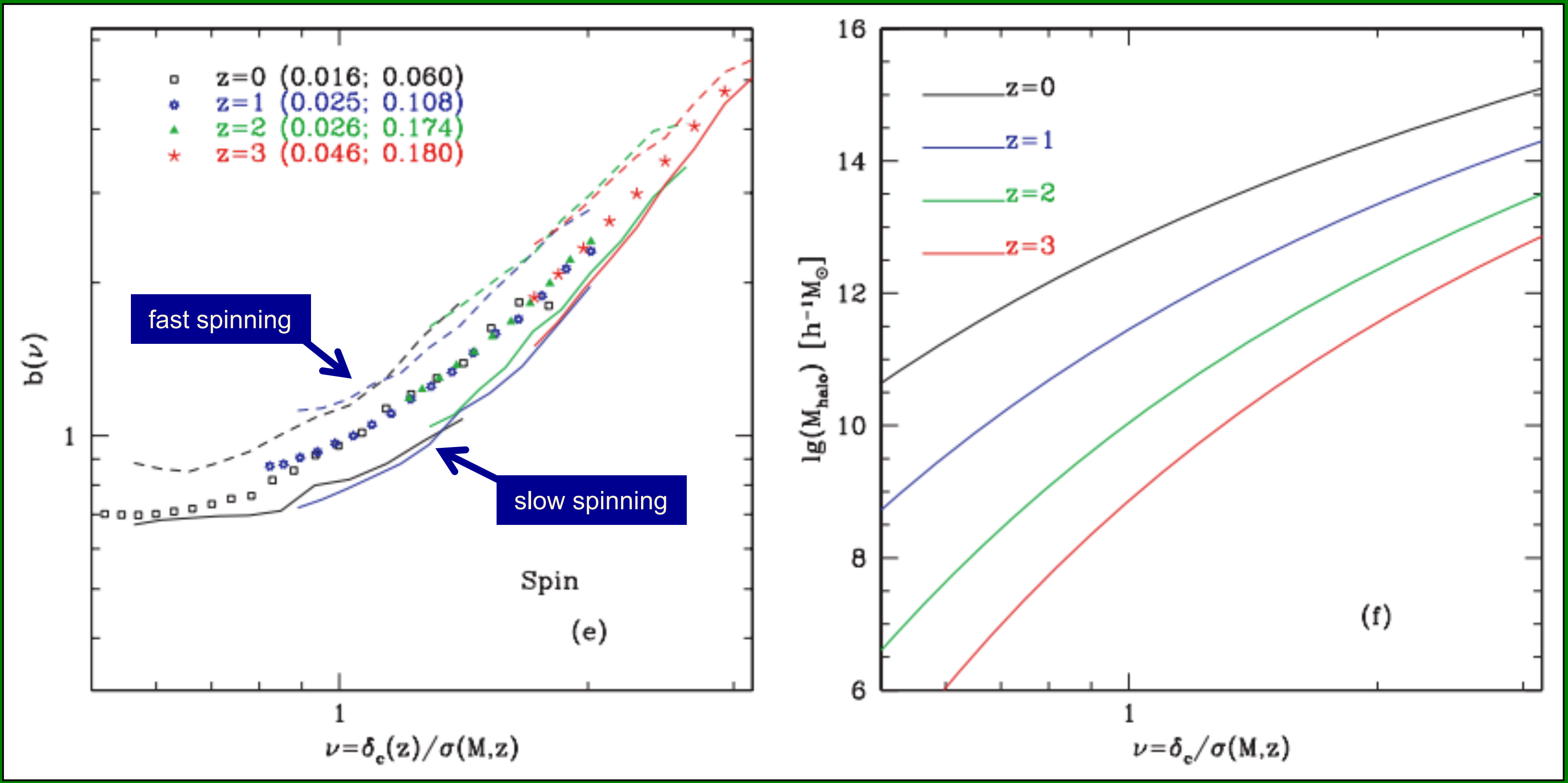
- Halo concentrations are determined by their accretion history.

Halo properties: Assembly Bias



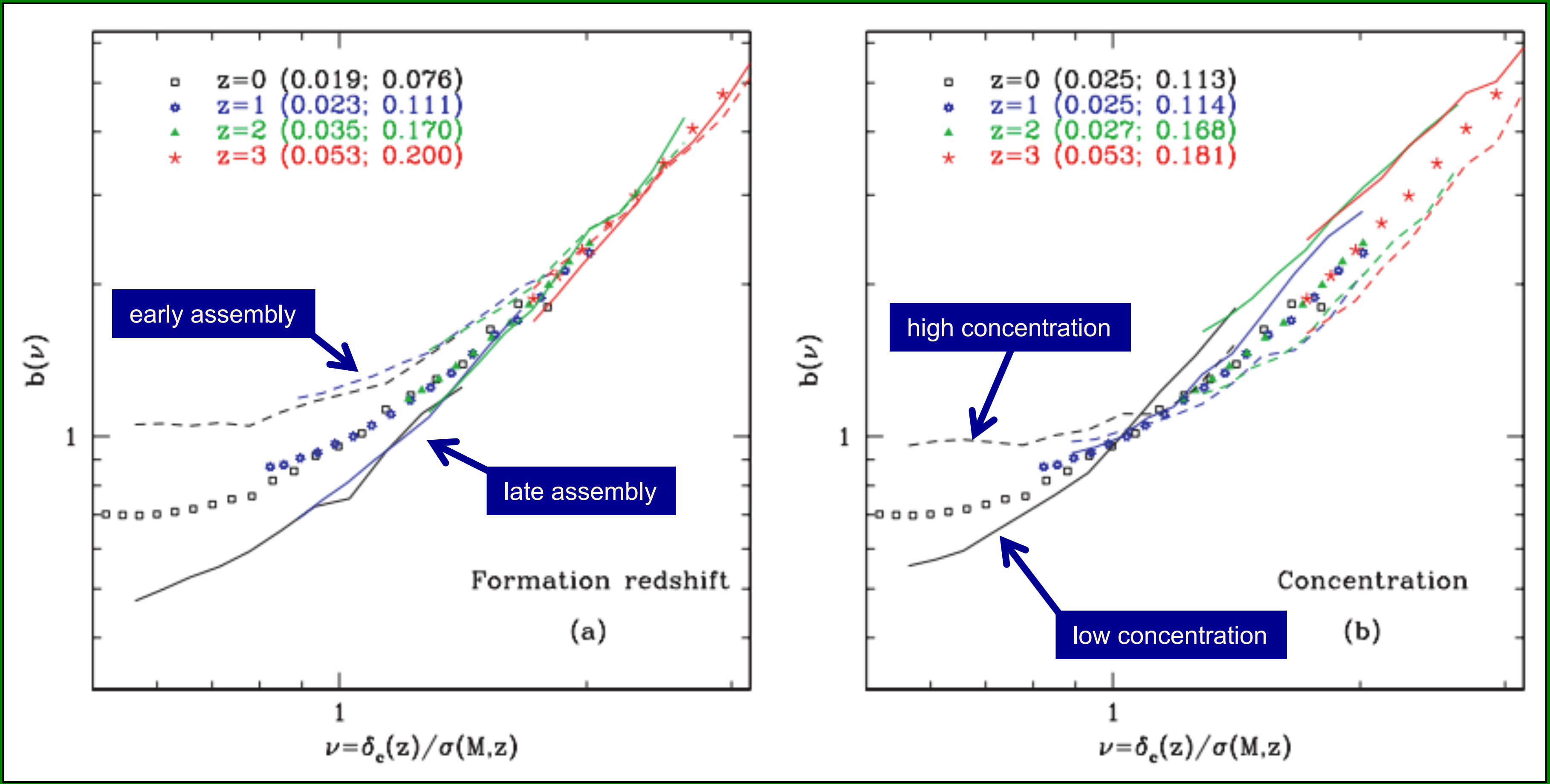
Wechsler et al. (2006)

Halo properties: Assembly Bias



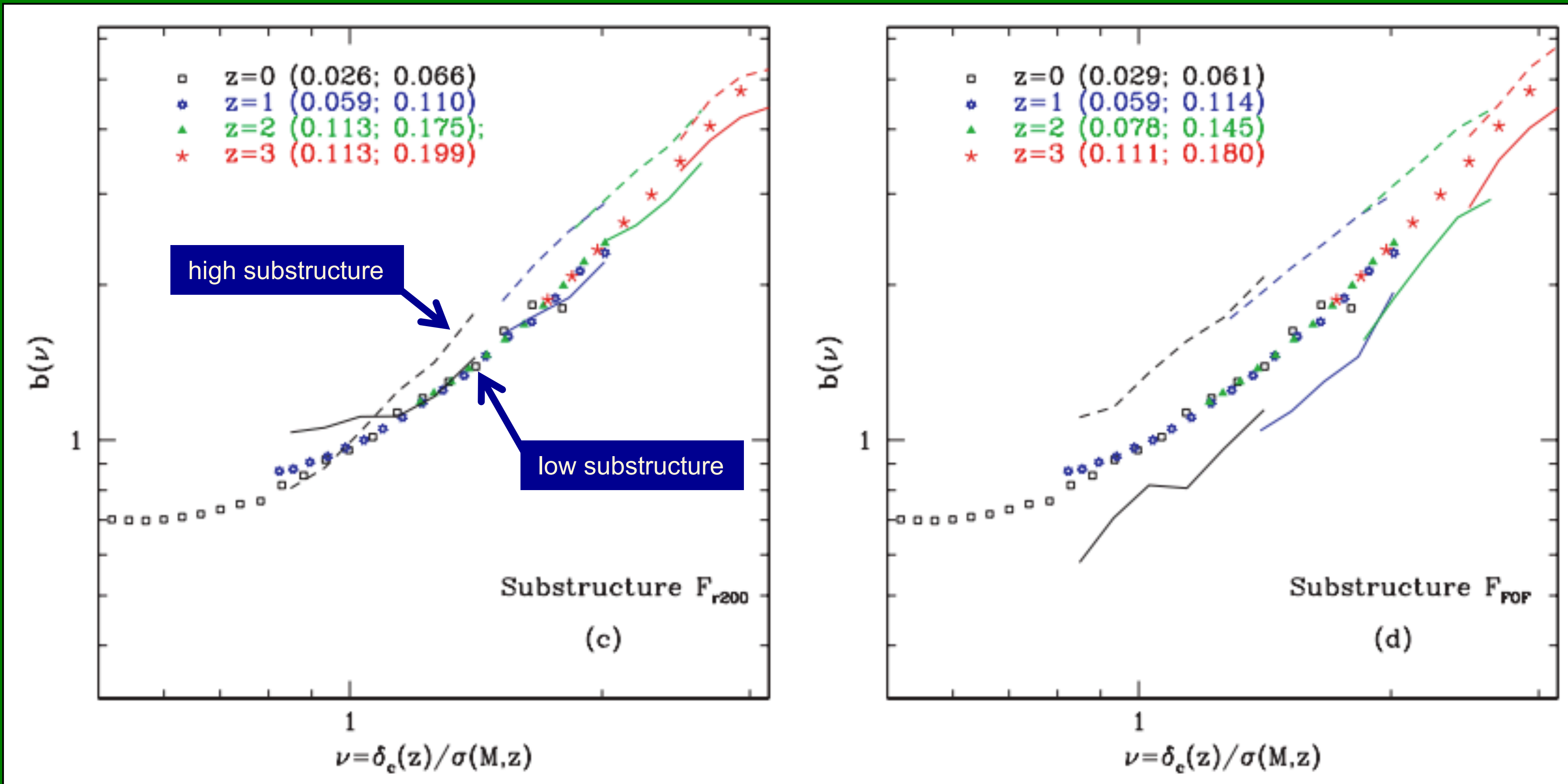
Gao & White (2007)

Halo properties: Assembly Bias



Gao & White (2007)

Halo properties: Assembly Bias



Gao & White (2007)