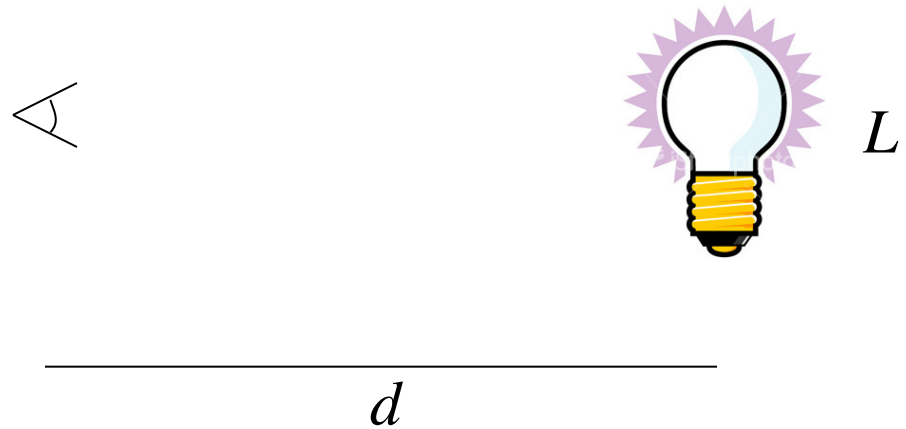


Determining distance

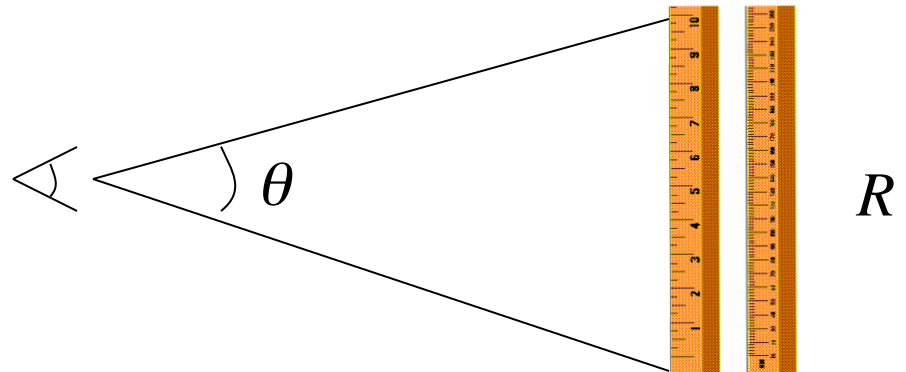
Standard candle

$$d = \left(\frac{L}{4\pi f} \right)^{\frac{1}{2}}$$



Standard ruler

$$d = \frac{R}{\theta}$$

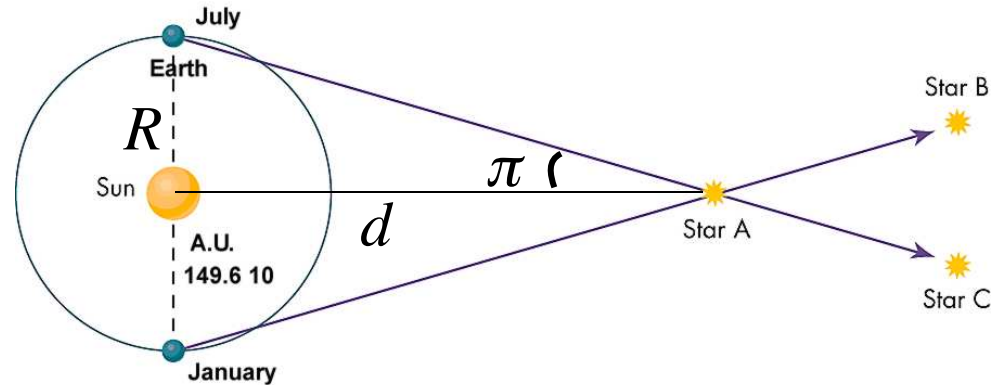


Determining distance: Parallax

RULER

$$\tan \pi = \frac{R}{d} \approx \pi$$

$$R = 1AU = 1.5 \times 10^{13} \text{ cm}$$

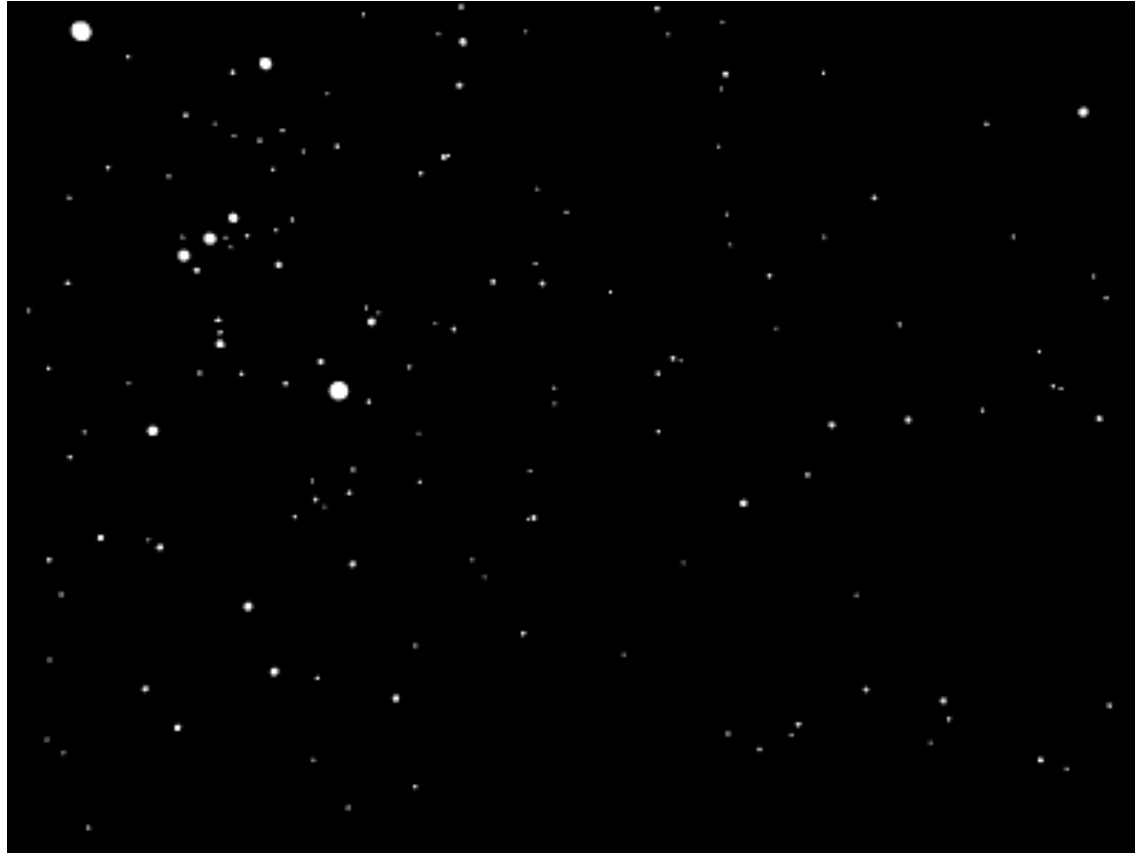


Define new distance unit: parsec (**par**allax-**sec**ond)

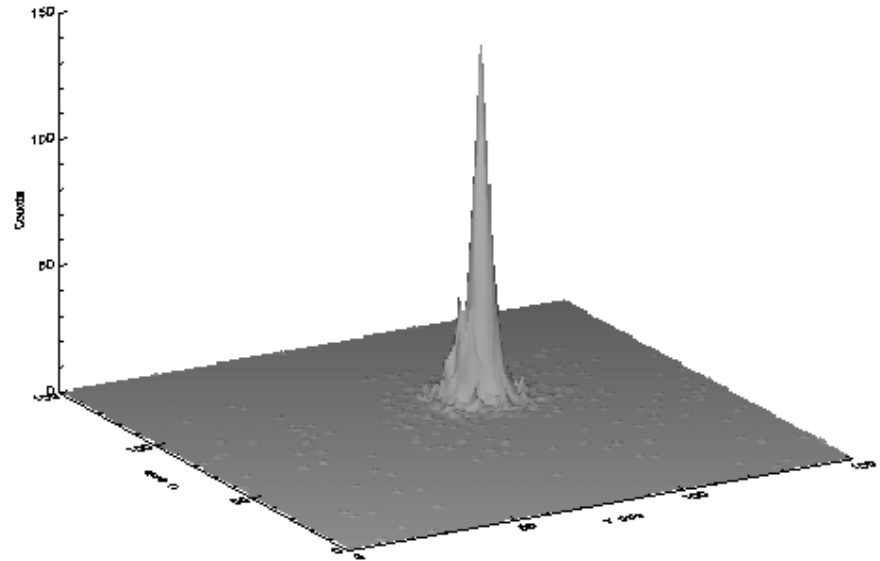
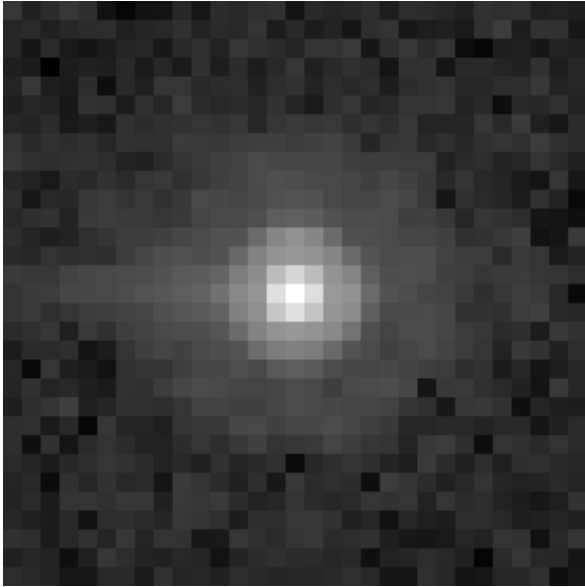
$$1pc = \frac{1AU}{\tan(1'')} = 206,265AU = 3.26ly$$

$$\left(\frac{d}{1pc} \right) = \frac{1}{\pi''}$$

Determining distance: Parallax



Determining distance: Parallax



Point spread function (PSF)

Determining distance: Parallax

Need high angular precision to probe far away stars.

$$d = \frac{1}{\pi}$$

Error propagation:

$$\sigma_d = \sqrt{\left(\frac{\partial d}{\partial \pi}\right)^2} \sigma_\pi = \sqrt{\left(-\frac{1}{\pi^2}\right)^2} \sigma_\pi = \frac{\sigma_\pi}{\pi^2} = \frac{\sigma_\pi}{\pi} d$$

$$\frac{\sigma_d}{d} = \frac{\sigma_\pi}{\pi}$$

At what distance do we get a given fractional distance error?

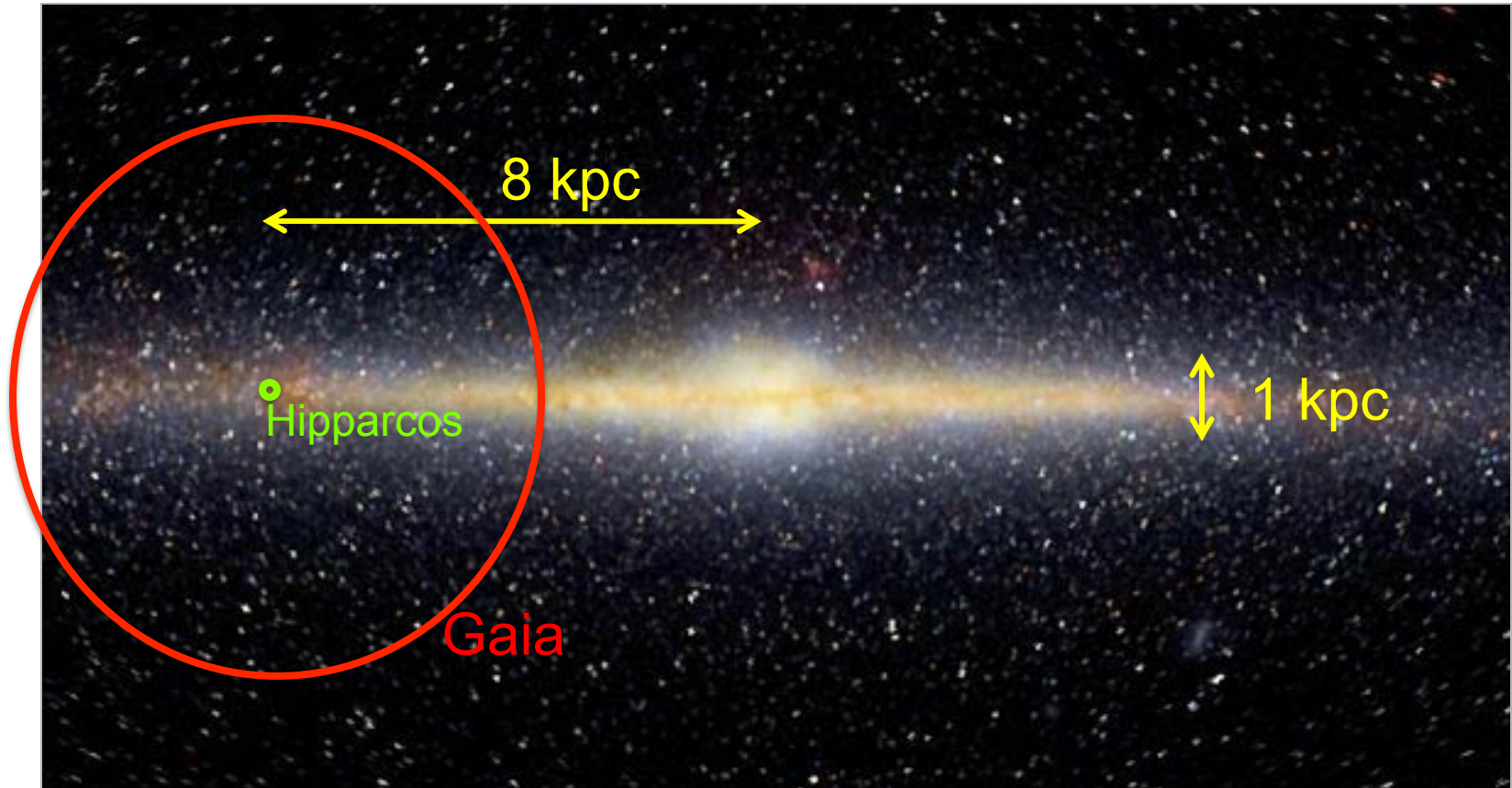
$$d = \left(\frac{\sigma_d}{d}\right) \frac{1}{\sigma_\pi}$$

Determining distance: Parallax

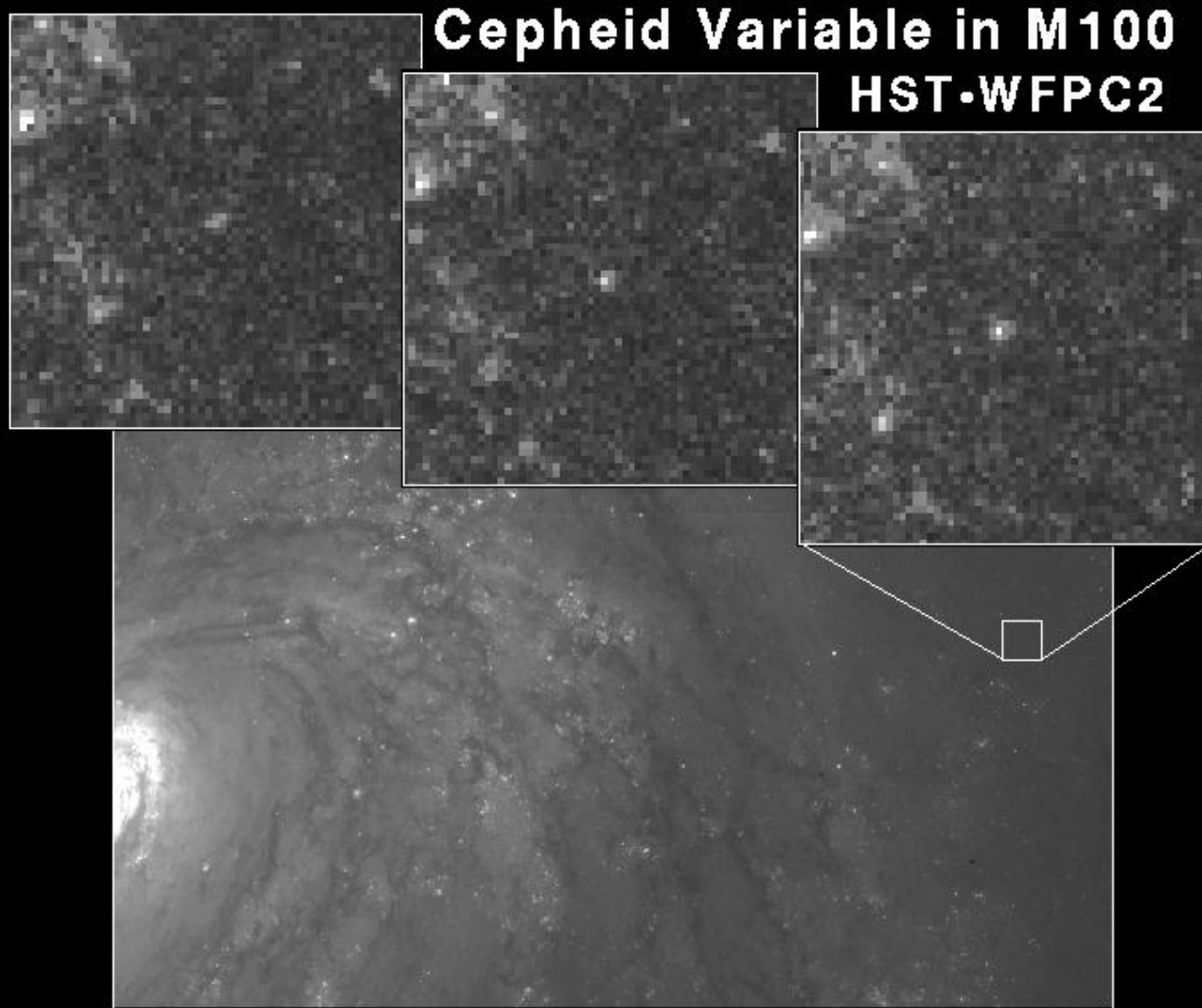
e.g., to get 10% distance errors $d_{\max} = \frac{0.1}{\sigma_{\pi}}$

Mission	Dates	σ_{π}	d_{\max}
Earth telescope		$\sim 0.1 \text{ as}$	1 pc
HST		$\sim 0.01 \text{ as}$	10 pc
Hipparcos	1989-1993	$\sim 1 \text{ mas}$	100 pc
Gaia	2013-2018	$\sim 20 \mu\text{as}$	5 kpc
SIM	cancelled	$\sim 4 \mu\text{as}$	25 kpc

Determining distance: Parallax



Determining distance: variable stars



Determining distance: variable stars

Cepheid variables:

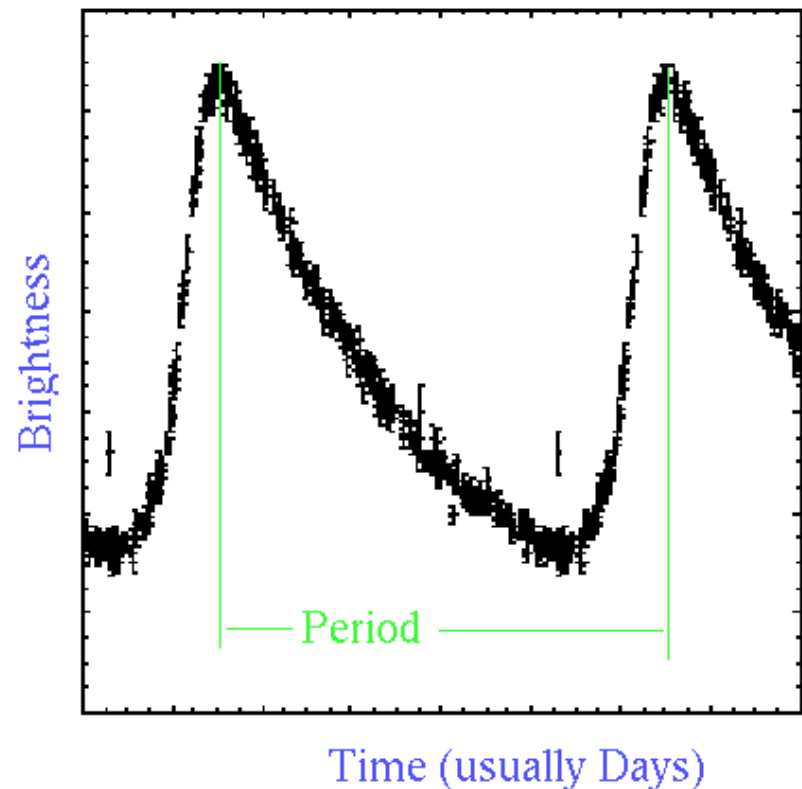
Pop I giants, $M \sim 5\text{-}20 M_{\text{sun}}$

Pulsation due to feedback loop:

An increase in T

- HeIII (doubly ionized He)
- high opacity
- radiation can't escape
- even higher T and P
- atmosphere expands
- low T
- HeII (singly ionized He)
- low opacity
- atmosphere contracts
- rinse and repeat...

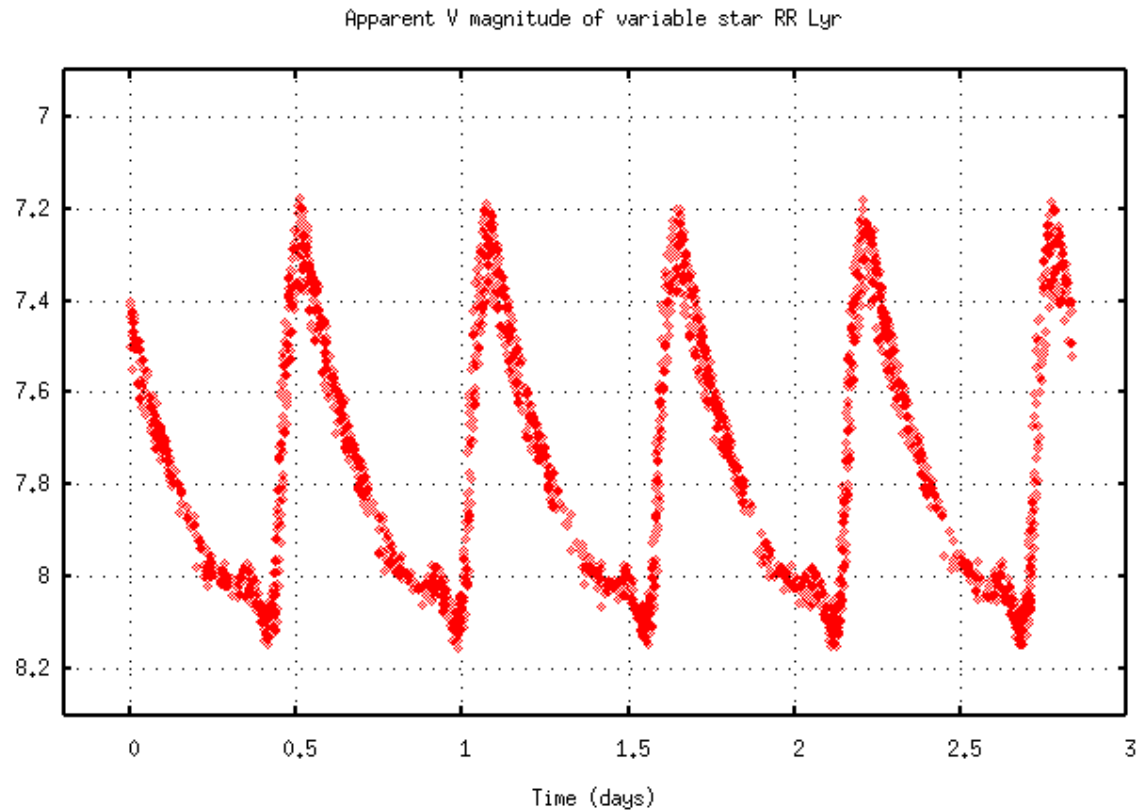
Data from a Well-Measured Cepheid



Determining distance: variable stars

RR-Lyrae variables:

Pop II dwarfs, $M \sim 0.5 M_{\text{sun}}$

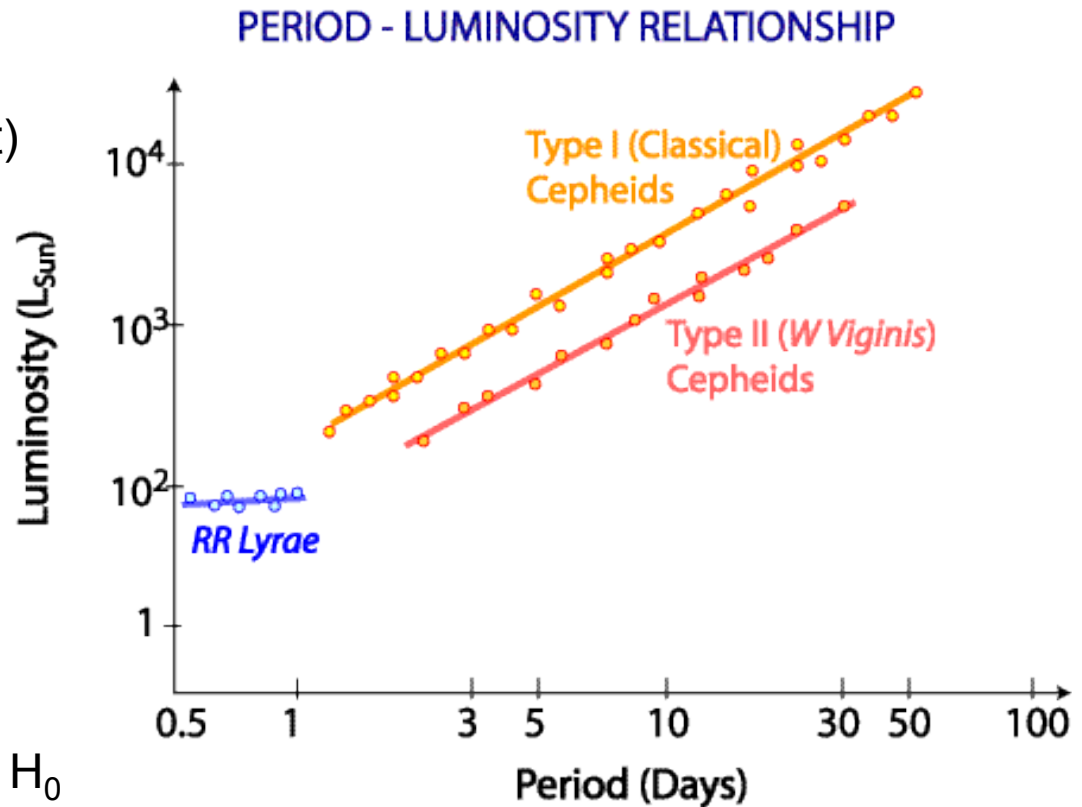


Variable stars have a tight period-luminosity relation

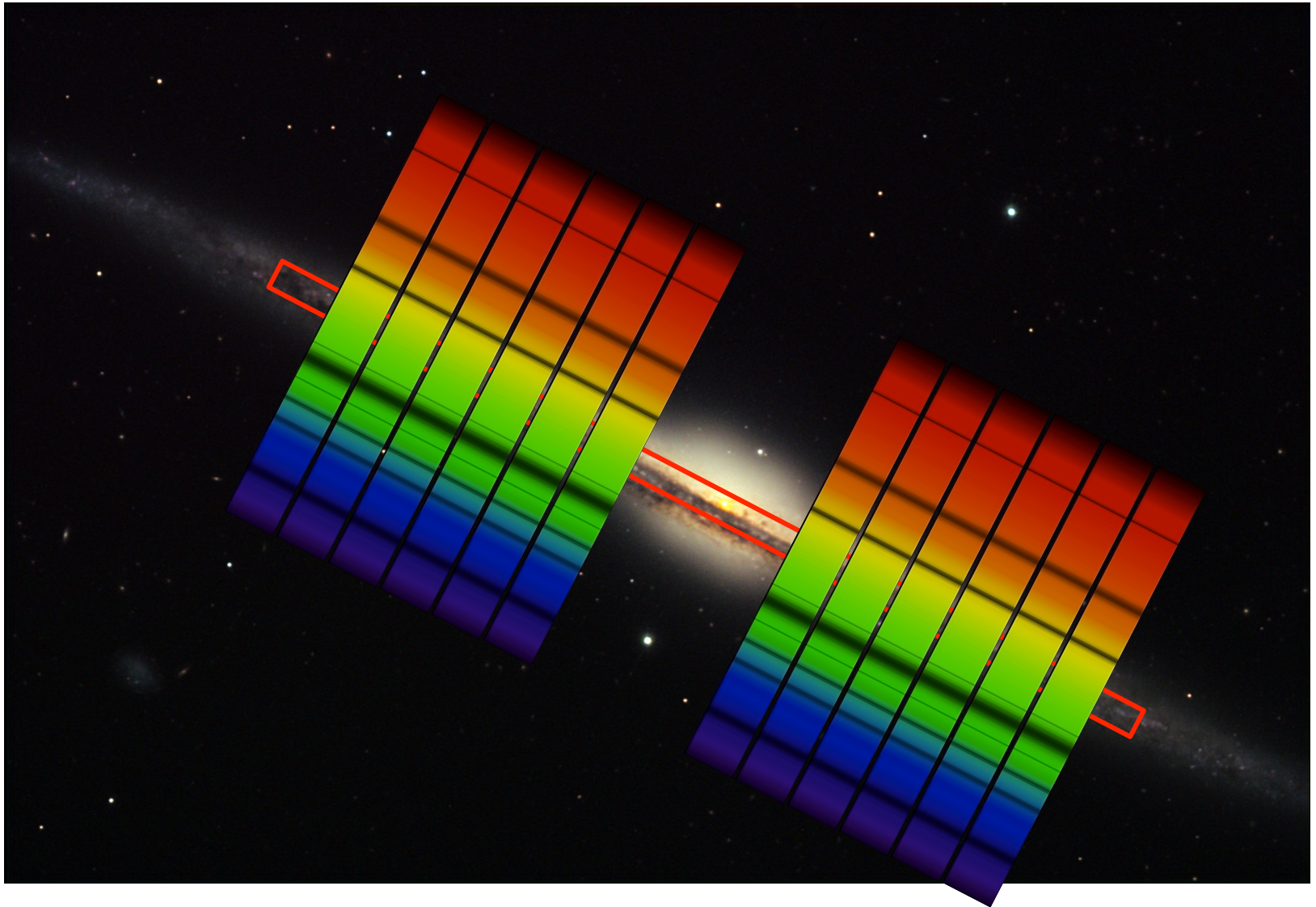
- Measure lightcurves: flux(t)
- Get period P
- From P-L relation, get L
- Use L to get distance

Very powerful method.
Cepheids can be seen very far away. Used to measure H_0

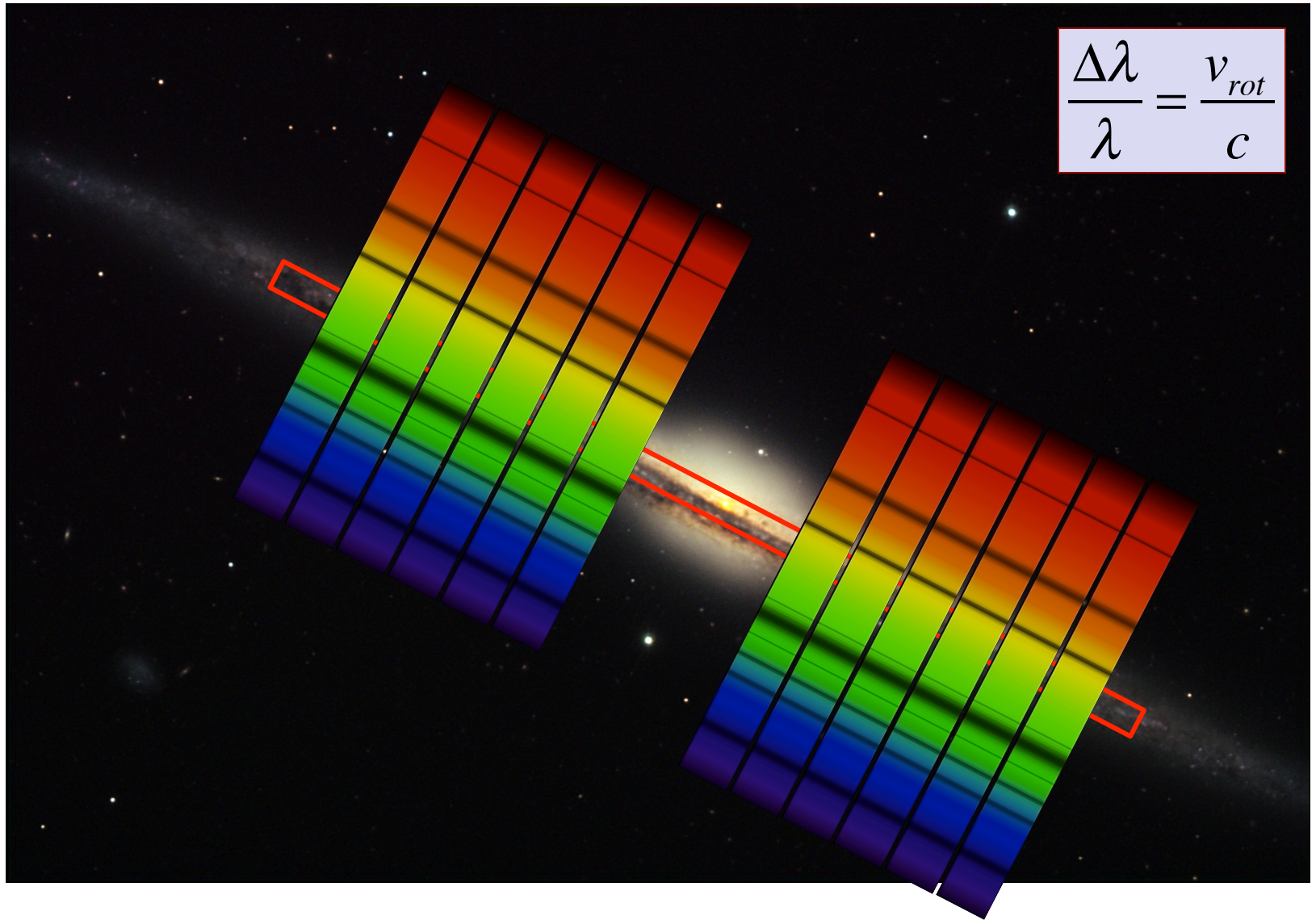
P-L relation is calibrated on local variables with parallax measurements



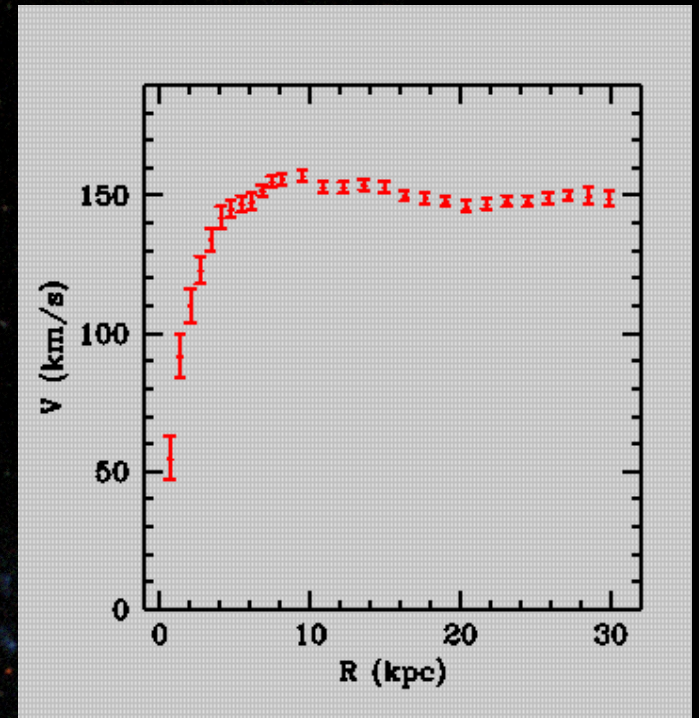
Determining distance: Tully-Fisher



Determining distance: Tully-Fisher



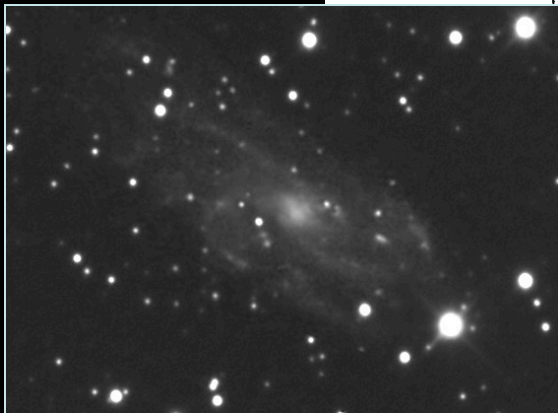
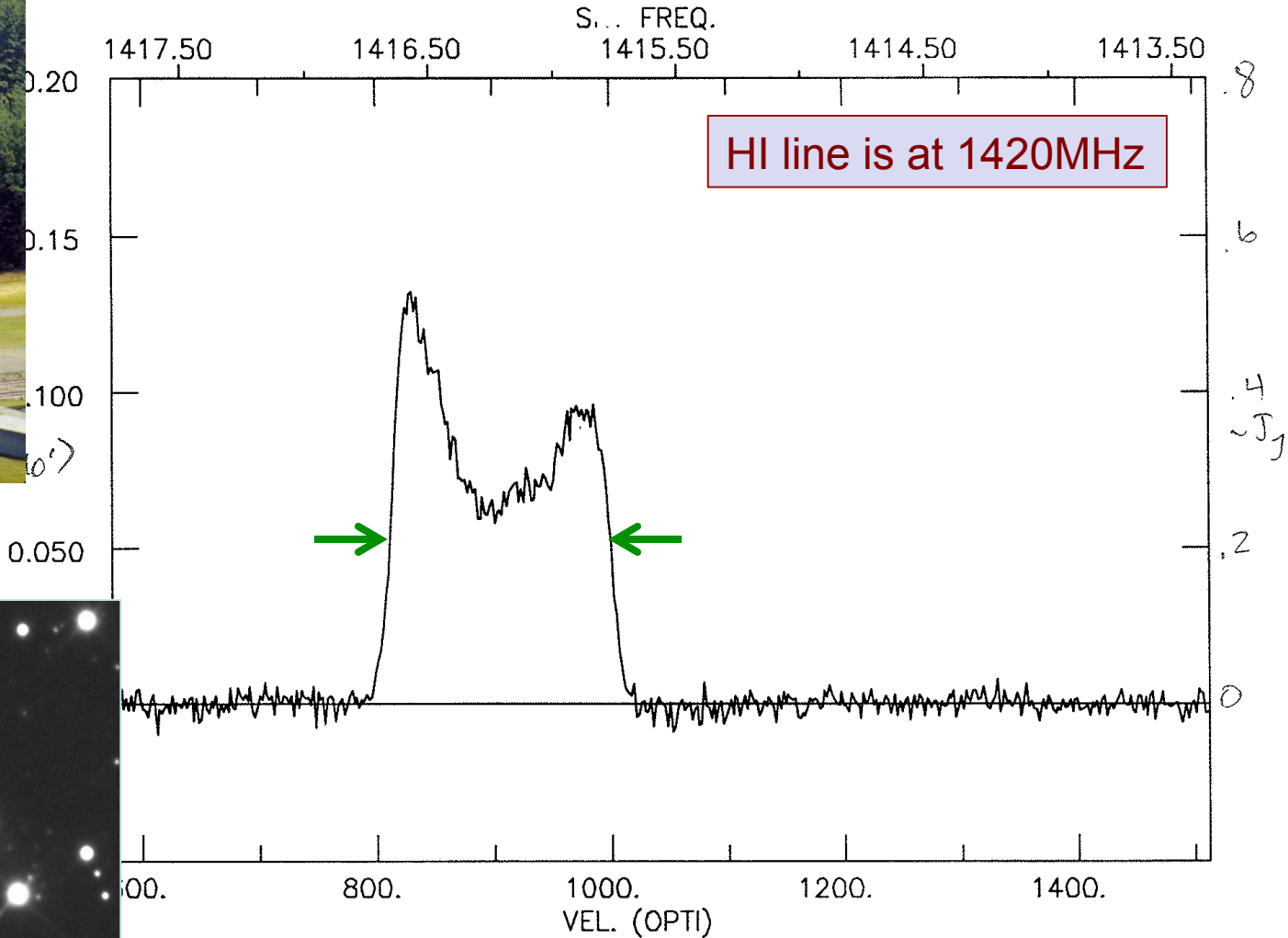
Determining distance: Tully-Fisher



NGC 3198 rotation curve

Determining distance: Tully-Fisher

HI emission line width



Determining distance: Tully-Fisher



HI absorption line width

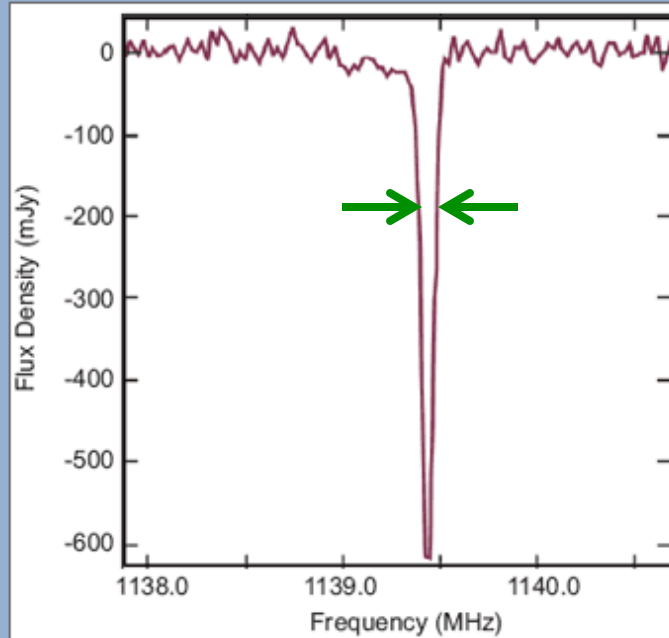


Figure 1. The HI 21 cm absorption line toward the source PKS 1413+135 at $z=0.25$ with the EVLA+VLA.

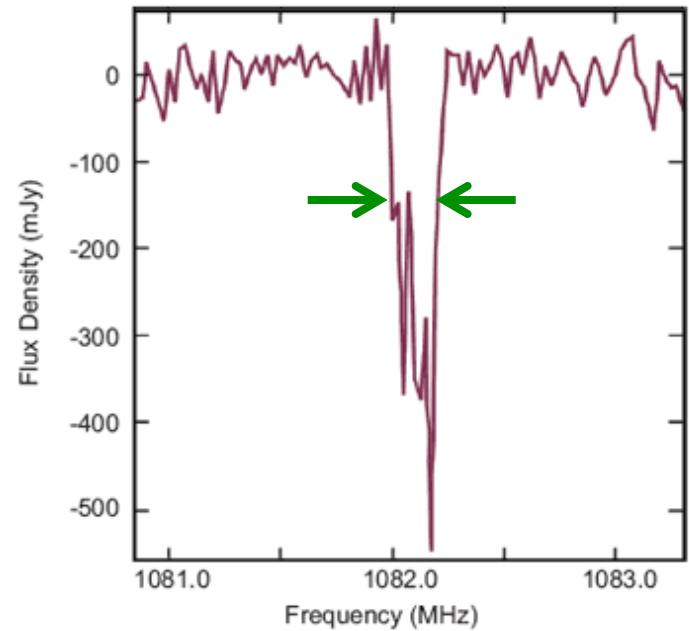


Figure 2. The HI 21 cm absorption line toward the source PKS 1127-145 at $z=0.31$ with the EVLA.

Determining distance: Tully-Fisher

A New Method of Determining Distances to Galaxies

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² National Radio Astronomy Observatory**, P.O. Box 2, Green Bank, W. Va. 24944, USA

Received July 15, 1975, revised April 26, 1976

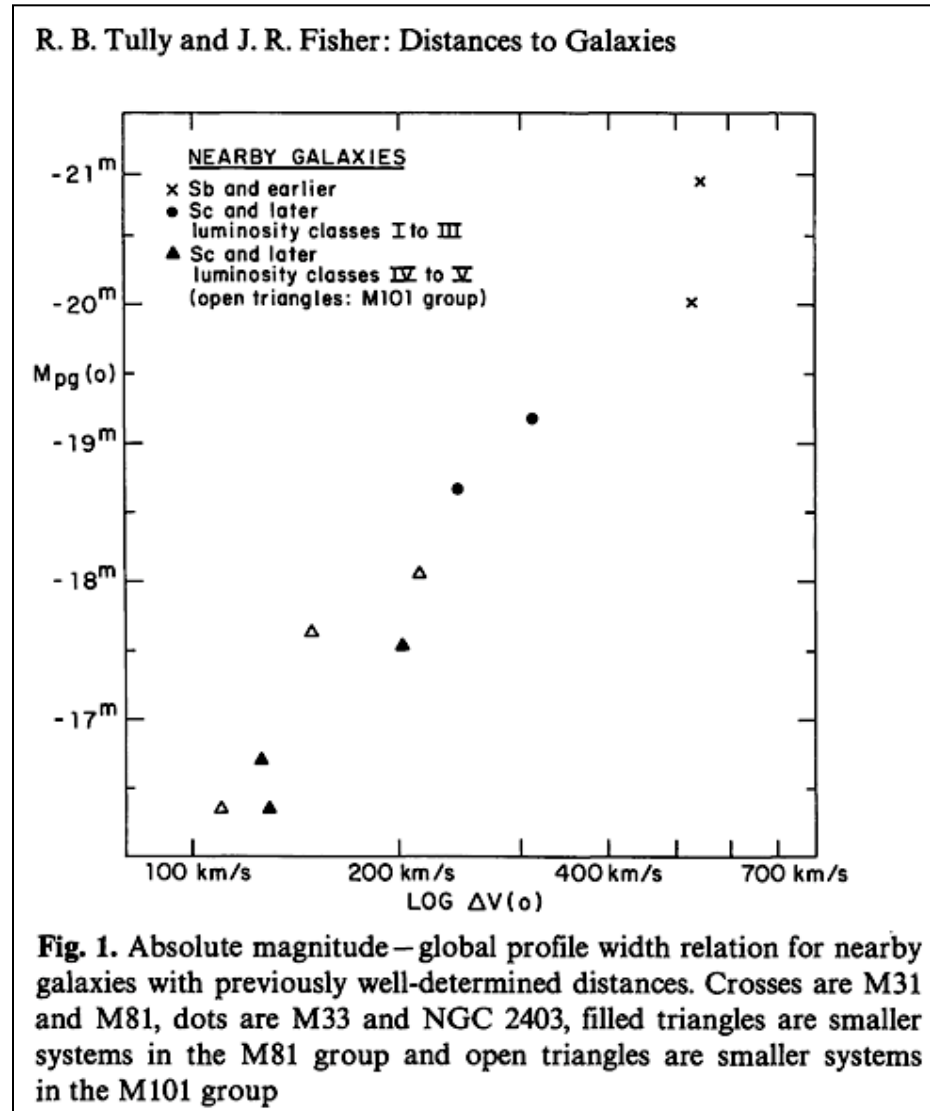
Summary. A good correlation between a distance-independent observable, global galaxian H I profile widths, and absolute magnitudes or diameters of galaxies offers a new extragalactic distance tool, as well as potentially being fundamental to an understanding of galactic structure. The relationships are calibrated with members of the Local Group, the M81 group, and the M101 group and have been used to derive distances to the Virgo cluster ($\mu_0 = 30^m.6 \pm 0^m.2$) and the Ursa Major cluster ($\mu_0 = 30^m.5 \pm 0^m.35$). A preliminary estimate of the Hubble constant is $H_0 = 80$ km/s/Mpc.

Key words: galaxies — distances — neutral hydrogen

total mass and t
correlation is prim
systems that have
than later systems
with luminosity, v
This point is im
structure of galax
for the measurem

The basic diff
and presumably th
notice, is that if
extremely well kn
observational sca
tion of little use.
effect in two ways

$$L = C v_{rot}^{\alpha}$$



Determining distance: Tully-Fisher

$$v_{\text{rot}}^2 = \frac{Gm(< r)}{r}$$

$$m(< r) = \int_0^r \rho(r) 4\pi r^2 dr$$



Flat rotation curve \rightarrow density profile is a singular isothermal sphere (SIS)

$$\rho(r) = \frac{C}{r^2}$$

$$m(< r) = \int_0^r \frac{C}{r^2} 4\pi r^2 dr = 4\pi Cr$$

$$M = 4\pi CR$$

$$v_{\text{rot}}^2 = \frac{G4\pi Cr}{r} = 4\pi GC$$

$$v_{\text{rot}}^2 = \frac{GM}{R}$$

Determining distance: Tully-Fisher

Dark matter halo definition:

$$M = \frac{4}{3} \pi R^3 \rho(< R)$$

$$= \frac{4}{3} \pi R^3 \Delta \bar{\rho}_0$$

$$\Delta \approx 200$$

$$R = \left(\frac{3M}{4\pi\Delta\bar{\rho}_0} \right)^{1/3}$$

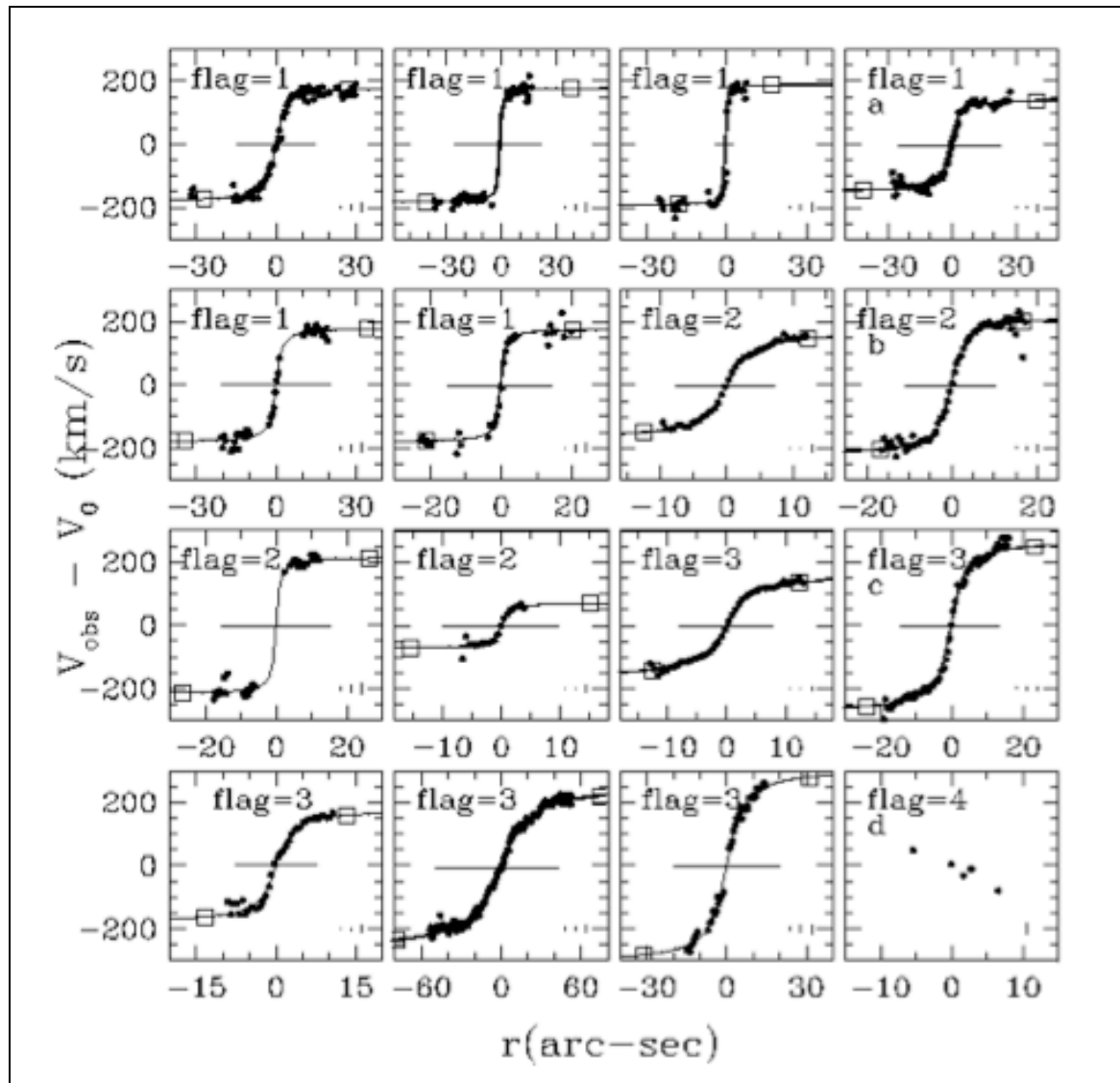


$$v_{\text{rot}}^2 = \frac{GM}{R} = G \left(\frac{4\pi\Delta\bar{\rho}_0}{3} \right)^{1/3} M^{2/3}$$

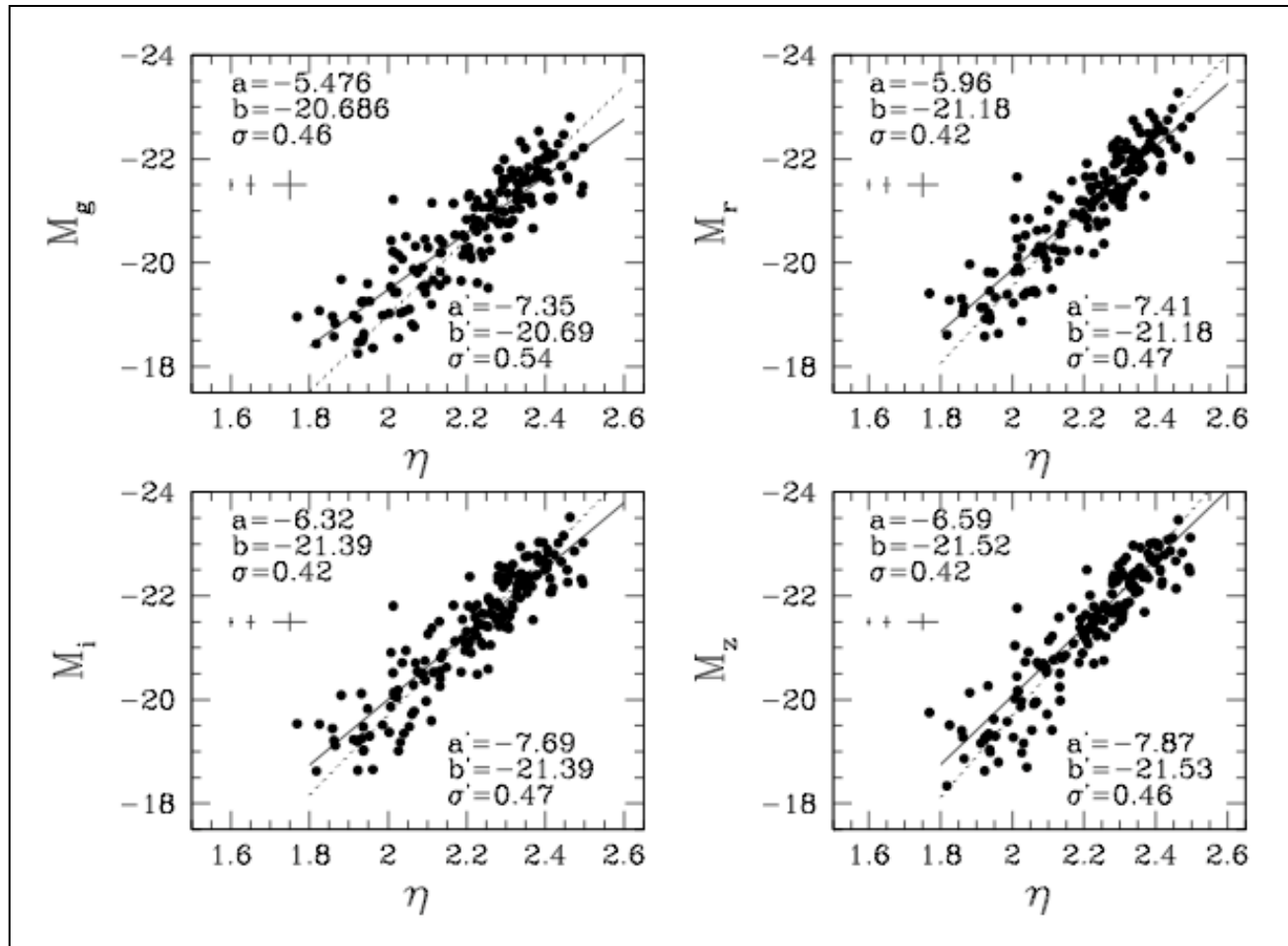
$$\rightarrow M = \left(\frac{3}{4\pi G^3 \Delta \bar{\rho}_0} \right)^{1/2} v_{\text{rot}}^3$$

$$L \propto M^\alpha \rightarrow L \propto v_{\text{rot}}^{3\alpha}$$

Determining distance: Tully-Fisher



Determining distance: Tully-Fisher



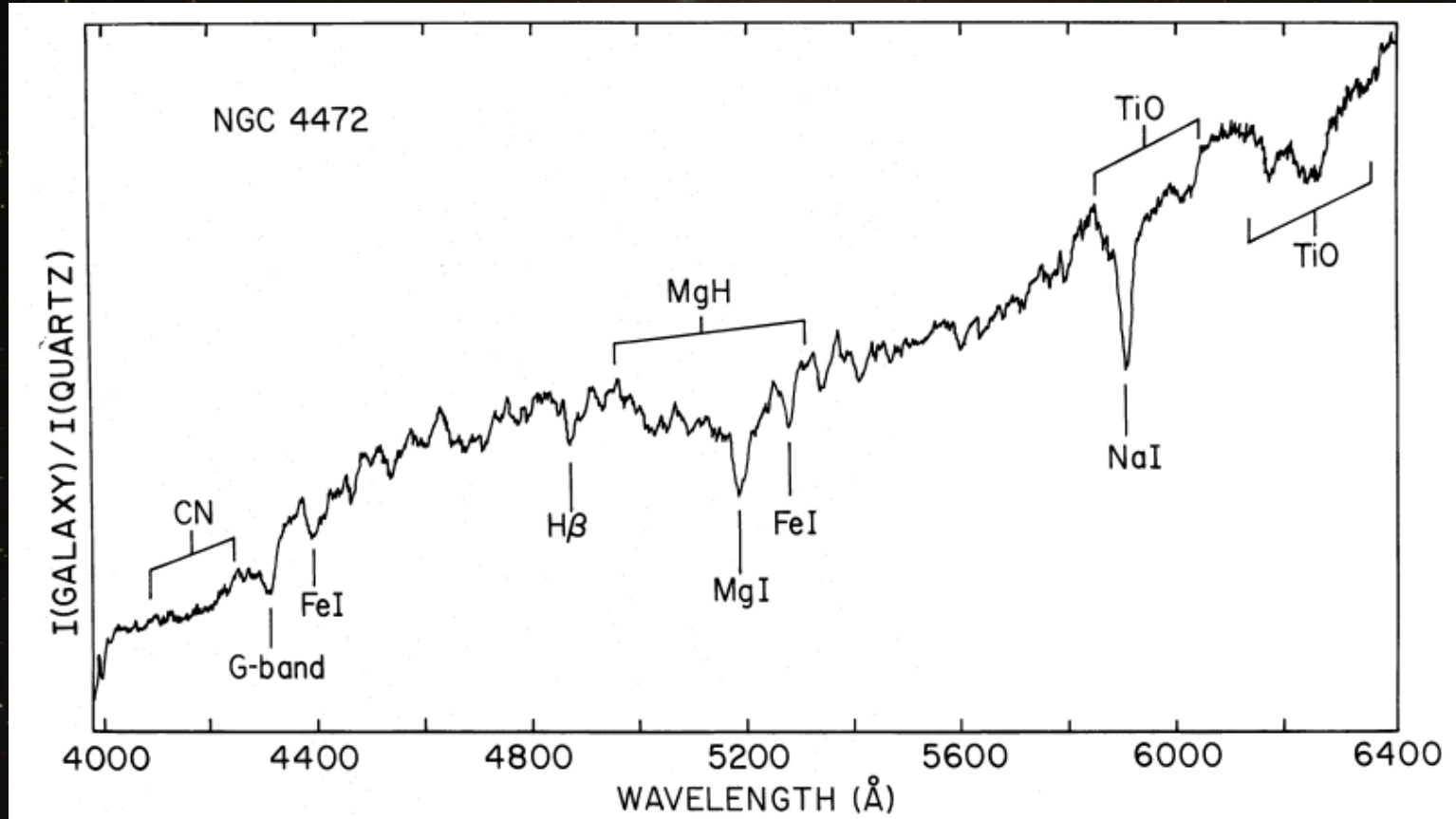
Pizagno et al. (2007)

$$L = Cv_{rot}^\alpha \rightarrow M_i = -2.5 \log(Cv_{rot}^\alpha) + c = -2.5\alpha \log v_{rot} + (c - 2.5 \log C)$$

Determining distance: Faber-Jackson



Determining distance: Faber-Jackson



Determining distance: Faber-Jackson

VELOCITY DISPERSIONS AND MASS-TO-LIGHT RATIOS FOR ELLIPTICAL GALAXIES*

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Received 1975 June 30; revised 1975 August 28

ABSTRACT

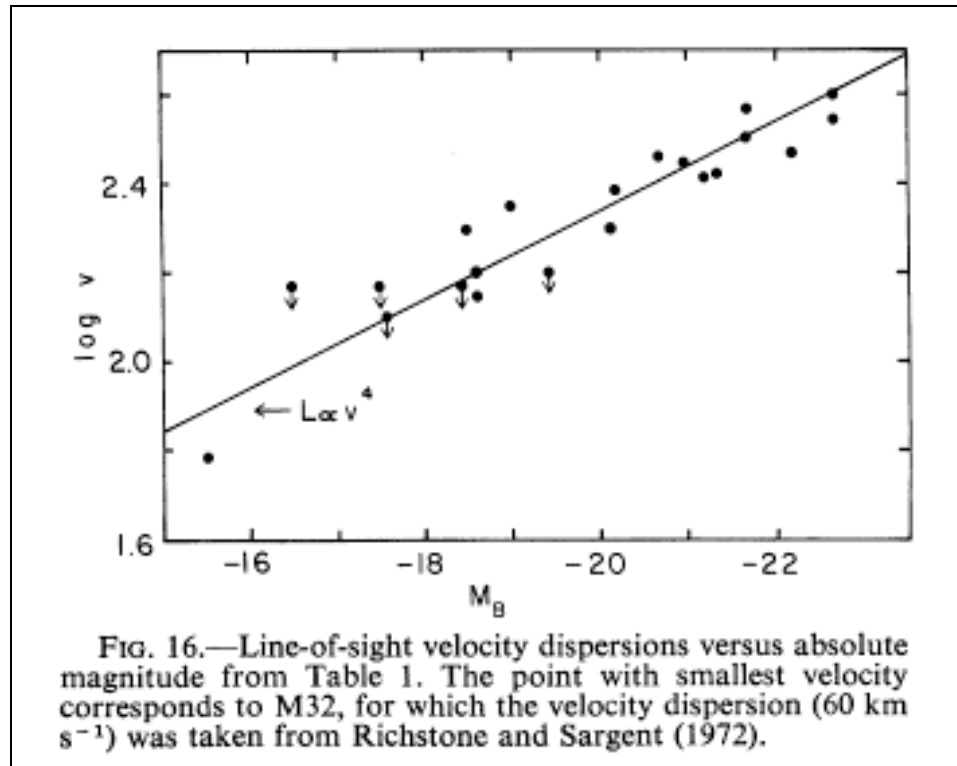
Velocity dispersions for 25 galaxies have been measured using conventional and Fourier techniques. The resultant velocity system is probably accurate to 10–15 percent. Internal rms errors are on the order of 10 percent. Using unpublished data of King, we have computed core values of M/L_B . For luminous ellipticals with $M_B < -20$, M/L_B averages $7(H/50 \text{ km s}^{-1} \text{ Mpc}^{-1})$, considerably smaller than previous estimates. This value agrees well with M/L_B for early-type spirals, indicating that there is no large discontinuity in M/L_B between ellipticals and early-type spirals. This result is consistent with the observed small color differences between ellipticals and Sa's.

Velocity dispersions increase with luminosity for normal elliptical galaxies of moderate ellipticity. The data also suggest that M/L_B generally increases with luminosity. This conclusion is consistent with predictions based on King's data on core radii and central surface brightness (to be discussed fully in a separate paper). This increase in M/L_B might be due at least in part to the known increase in metal abundance with luminosity for normal elliptical galaxies.

The close correlation between luminosity and dynamical properties for normal ellipticals is further evidence that the ellipticals are very nearly a one-parameter family with total mass as the most important independent variable.

Subject headings: galaxies: internal motions — galaxies: stellar content

$$L = C\sigma_v^\alpha$$



Determining distance: Fundamental Plane

SPECTROSCOPY AND PHOTOMETRY OF ELLIPTICAL GALAXIES. I. A NEW DISTANCE ESTIMATOR¹

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Lick Observatory and Board of Studies in Astronomy and Astrophysics, University of California, Santa Cruz

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Royal Greenwich Observatory

AND

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Department of Physics and Astronomy, Dartmouth College

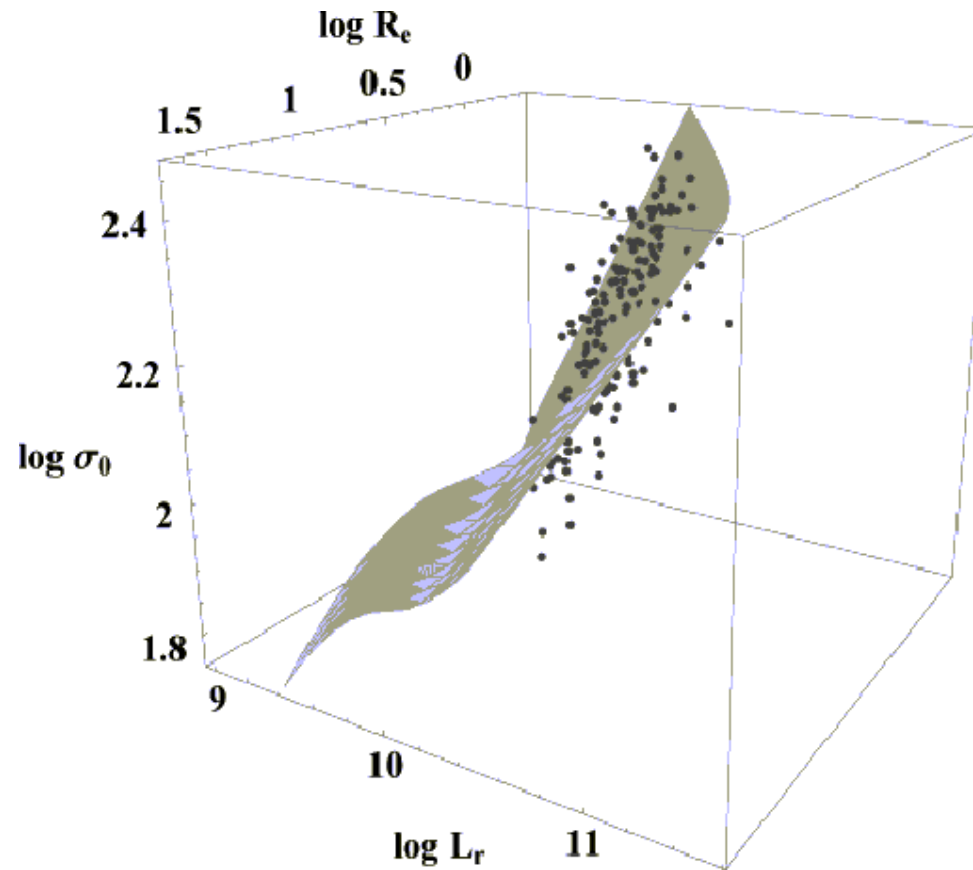
Received 1986 May 7; accepted 1986 July 24

ABSTRACT

Kinematic and photometric data have been obtained for 97 elliptical galaxies in six rich clusters. These data show that ellipticals describe a plane in three dimensions which, when viewed edge-on, projects a smaller scatter than the Faber-Jackson relationship between luminosity and velocity dispersion σ . This plane is approximately given by $L \propto \sigma^{8/3} \Sigma_e^{-3/5}$, where Σ_e is the surface brightness within the effective radius A_e , or equivalently $A_e \propto \sigma^{1.325} \Sigma_e^{-0.825}$.

We present a new photometric parameter D_n , the diameter which encloses an integrated surface brightness Σ , that correlates as well with σ as any linear combination of L (or A_e) and Σ . Thus, D_n effectively replaces two parameters with one. We show that the D_n - σ relation can be used to find relative distances of ellipticals with rms errors of $\lesssim 25\%$ for a single galaxy and $\lesssim 10\%$ for rich clusters. This accuracy is comparable to that of the infrared Tully-Fisher method used to find distances to spiral galaxies.

Determining distance: Fundamental Plane



Determining distance: Fundamental Plane

RULER

$$L = C\sigma_v^\alpha I^\beta$$

$$R = S\sigma_v^\gamma \times I^\delta$$

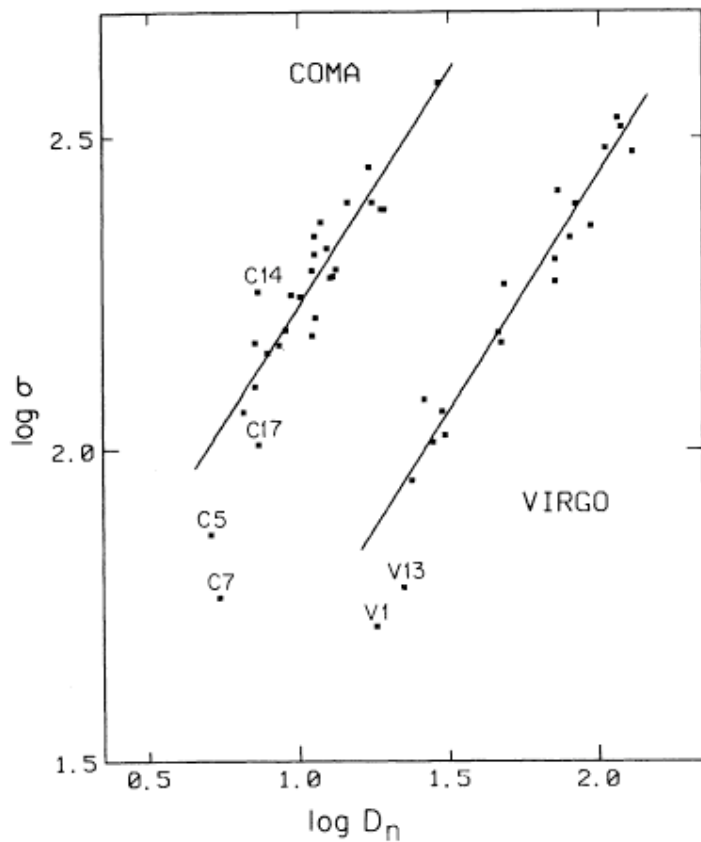


FIG. 1b

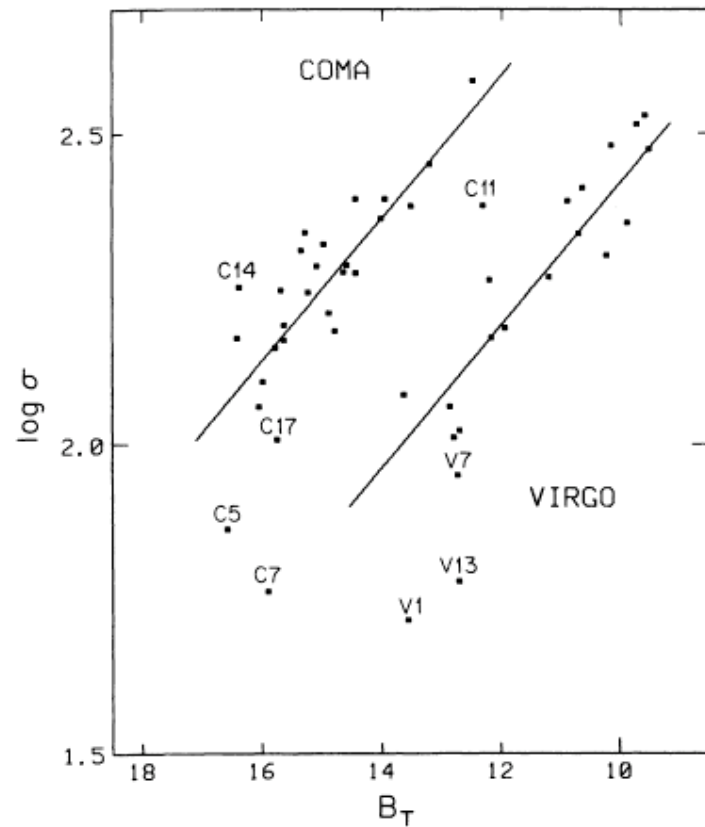
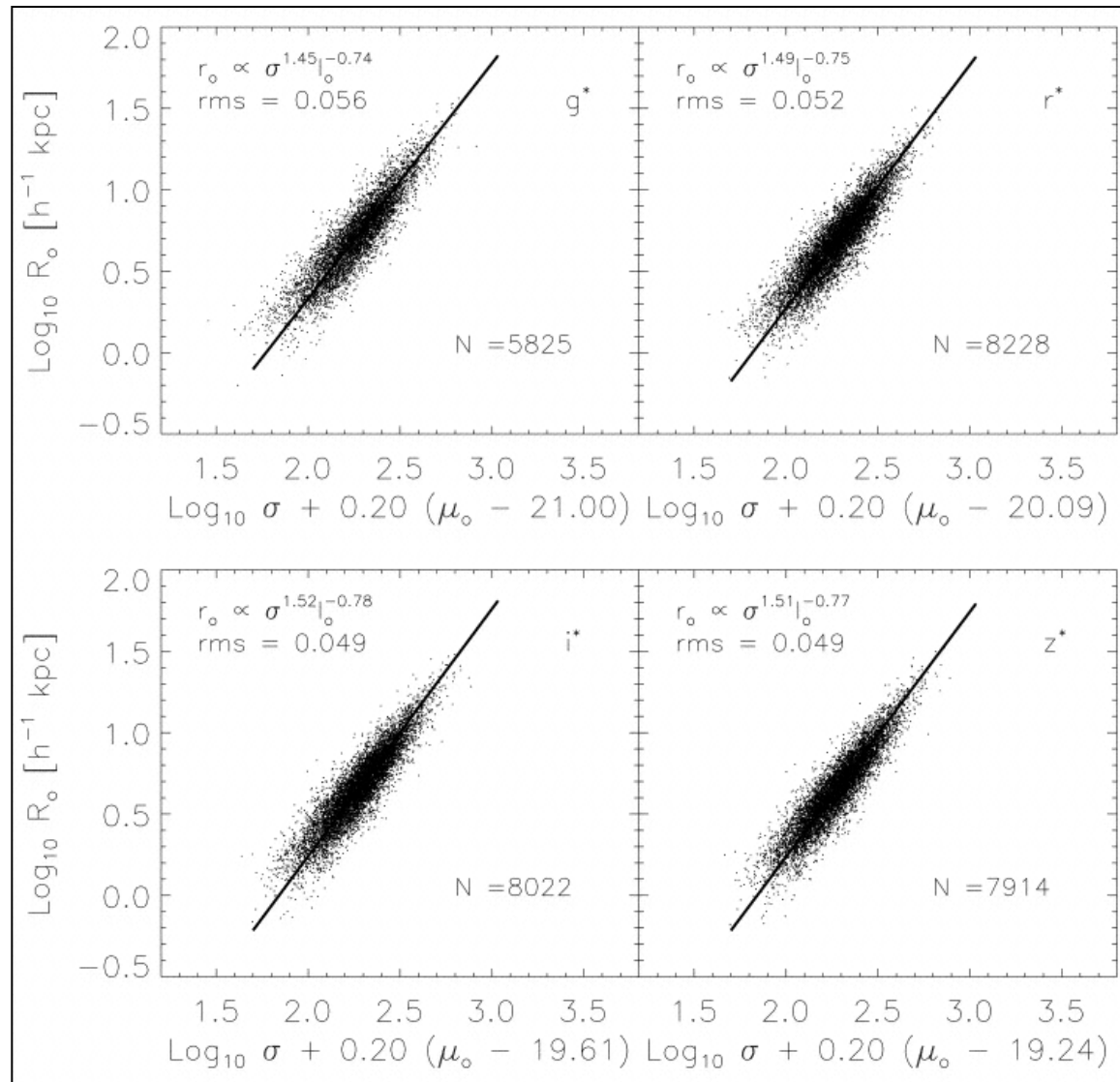


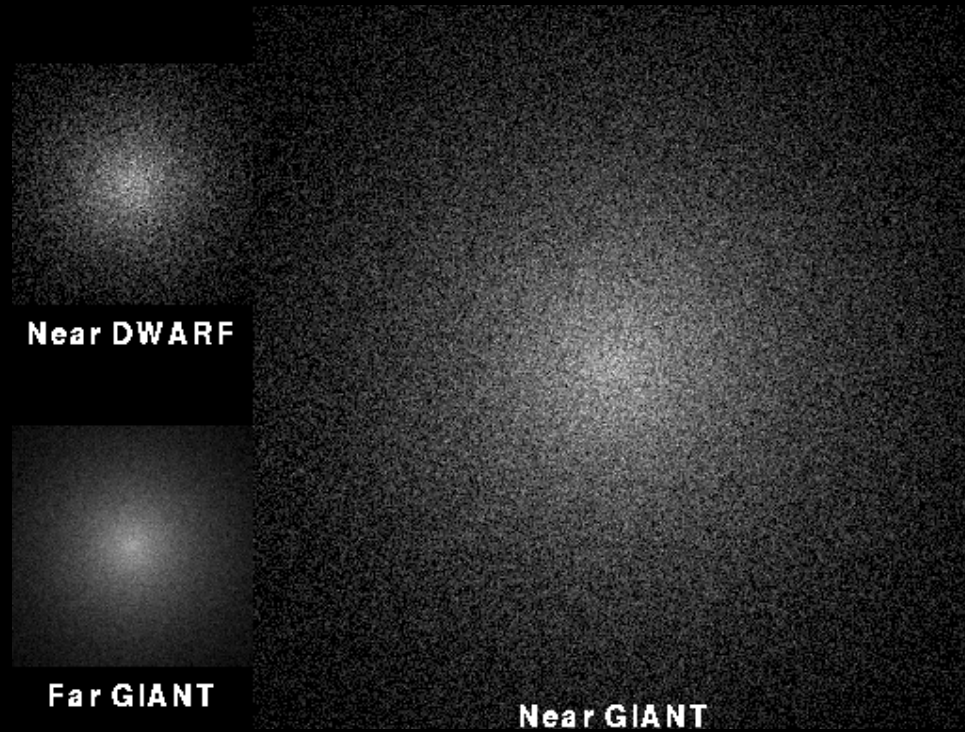
FIG. 1a

FIG. 1.—(a) B_T , the total blue magnitude, vs. $\log \sigma$, the central velocity dispersion, for ellipticals in the Coma and Virgo clusters. These are the variables of the Faber-Jackson relationship. The lines $\log \sigma = -0.114B_T + C$, where $C = 3.561$ for Virgo and $C = 3.960$ for Coma, are best median fits, as described in the text. The rms scatters in B_T from these lines are 0.57 mag for Virgo and 0.69 mag for Coma. (b) $\log D_n$, the diameter within which the integrated blue surface brightness is $20.75 B$ mag arcsec $^{-2}$, vs. $\log \sigma$ for the same galaxies. The horizontal scales correspond to a factor of 10 in distance in both figures. The lines $\log \sigma = 0.750 \log D_n + C$, where $C = 0.934$ for Virgo and $C = 1.475$ for Coma, are best median fits. The rms scatter in $\log D_n$ is 0.059 for Virgo and 0.072 for Coma, a factor of 2 smaller scatter than with the Faber-Jackson relationship.

Determining distance: Fundamental Plane



Determining distance: Surface Brightness Fluctuations



Determining distance: Surface Brightness Fluctuations

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A NEW TECHNIQUE FOR MEASURING EXTRAGALACTIC DISTANCES^{a)}

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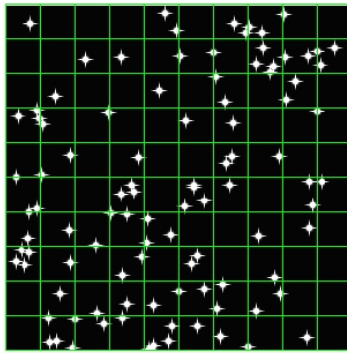
Received 26 April 1988

ABSTRACT

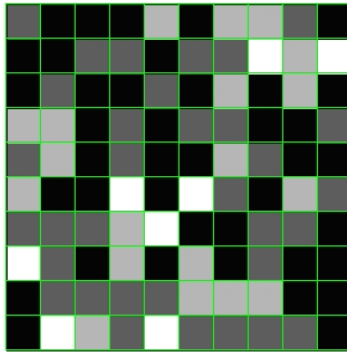
We describe a relatively direct technique of determining extragalactic distances. The method relies on measuring the luminosity fluctuations that arise from the counting statistics of the stars contributing the flux in each pixel of a high-signal-to-noise CCD image of a galaxy. The amplitude of these fluctuations is inversely proportional to the distance of the galaxy. This approach bypasses most of the successive stages of calibration required in the traditional extragalactic distance ladder; the only serious drawback to this method is that it requires an accurate knowledge of the bright end ($M_V < 3$) of the luminosity function. Potentially, this method can produce accurate distances of elliptical galaxies and spiral bulges at distances out to about 20 Mpc. In this paper, we explain how to calculate the value of the fluctuations, taking into account various sources of contamination and the effects of finite spatial resolution, and we demonstrate, via simulations and CCD images of M32 and N3379, the feasibility and limitations of this technique.

Determining distance: Surface Brightness Fluctuations

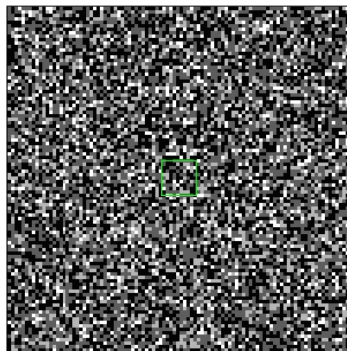
Nearby Galaxy



Galaxy star field



What the CCD sees



More CCD pixels

\bar{f} Star flux $\bar{f}/9$

n Star density $9n$

Surface Brightness

$n\bar{f}$

$n\bar{f}$

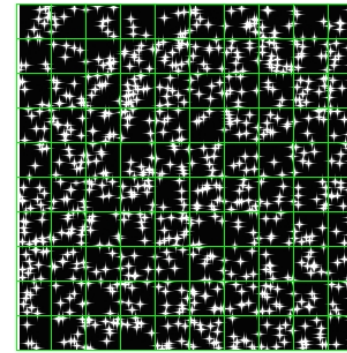
Rms fluctuation
(inversely prop. to distance)

$\sqrt{n} \bar{f}$

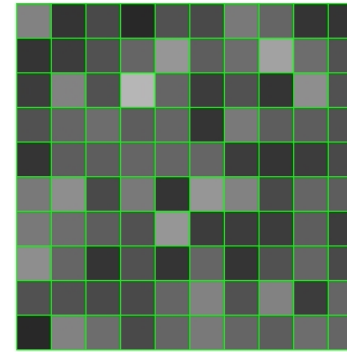
$\sqrt{9n} \bar{f}/9$

$= \frac{1}{3} \sqrt{n} \bar{f}$

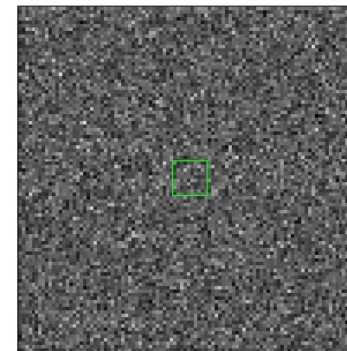
Same Galaxy
Three times the distance



Galaxy star field

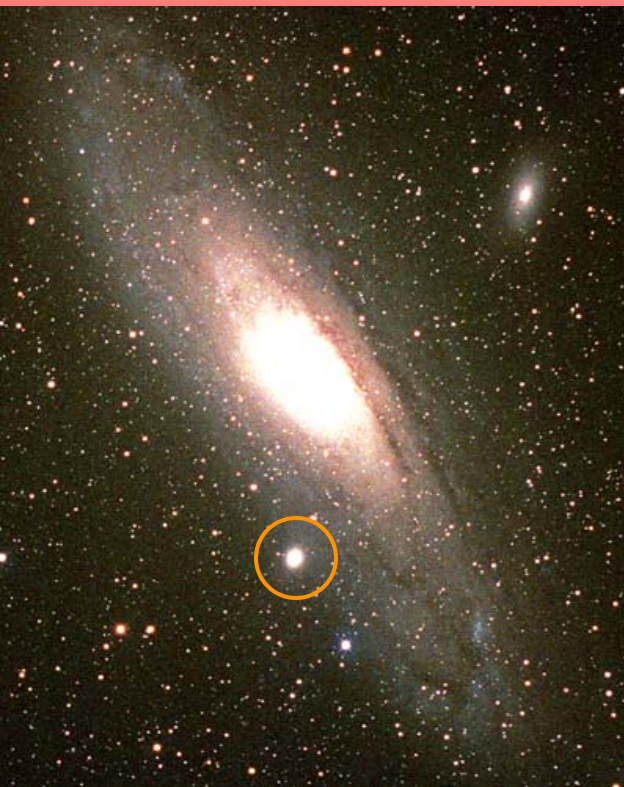


What the CCD sees



More CCD pixels

Determining distance: Surface Brightness Fluctuations

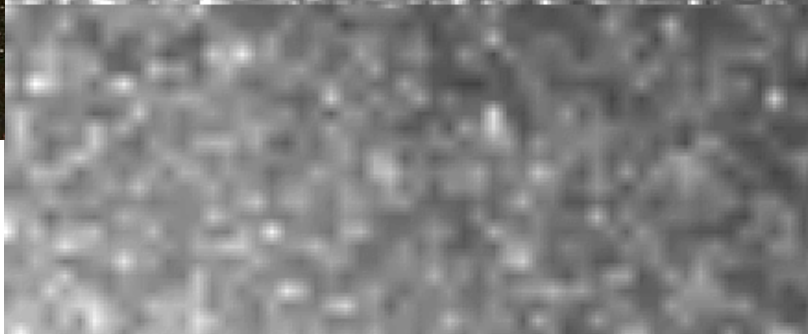


M32



$d=0.76$ Mpc

x2

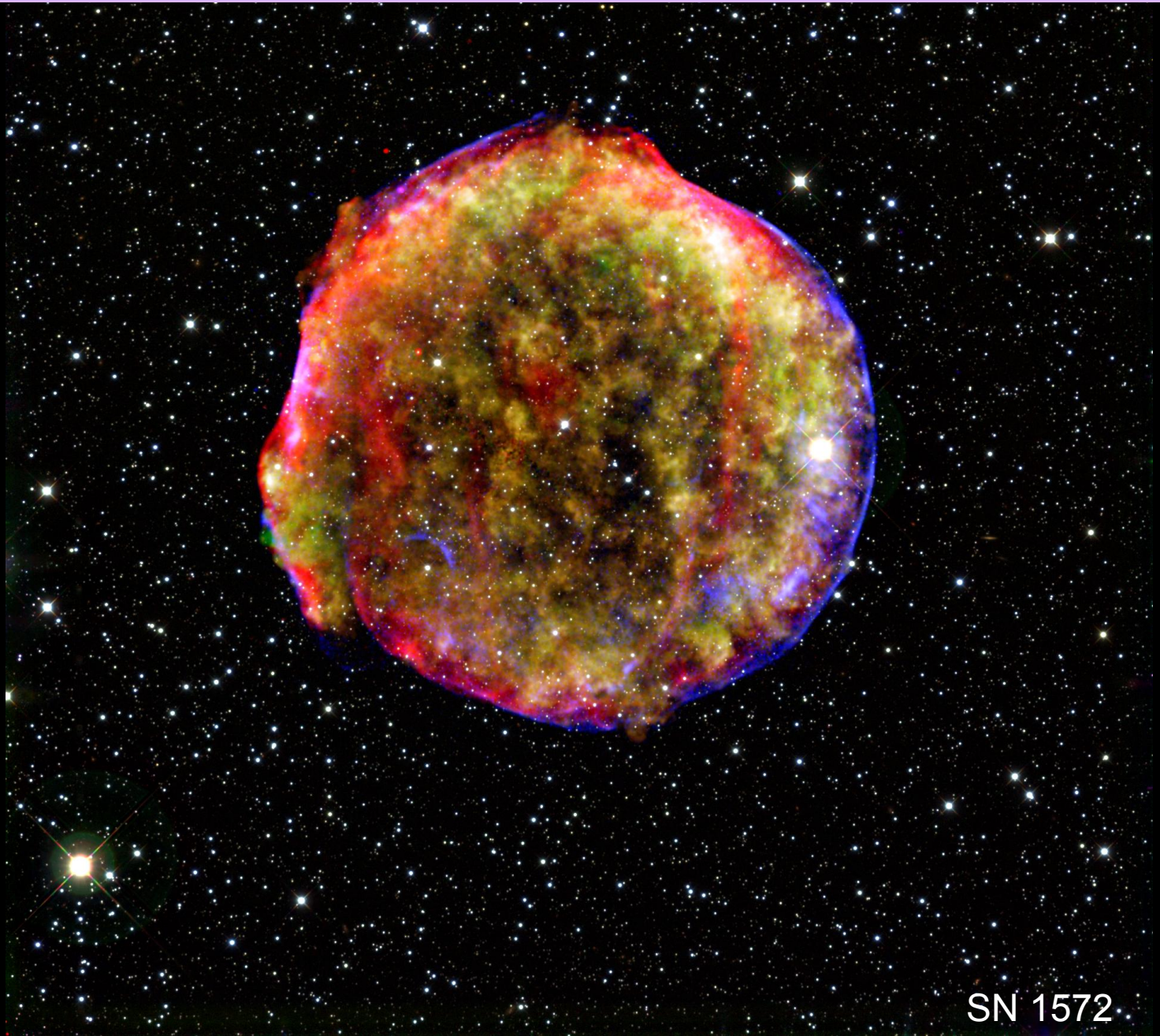


x4



x8

Determining distance: Supernovae type Ia



SN 1572

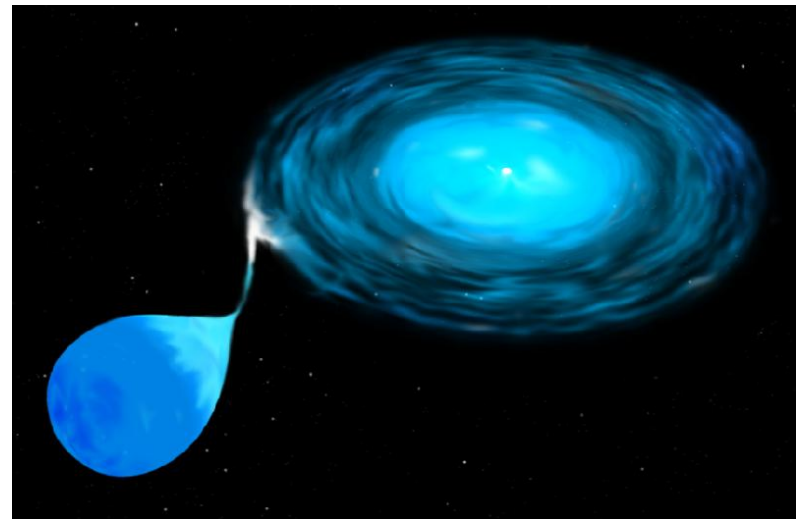
Determining distance: Supernovae type Ia

White dwarfs are made of a C/O core that is supported by electron degeneracy pressure.

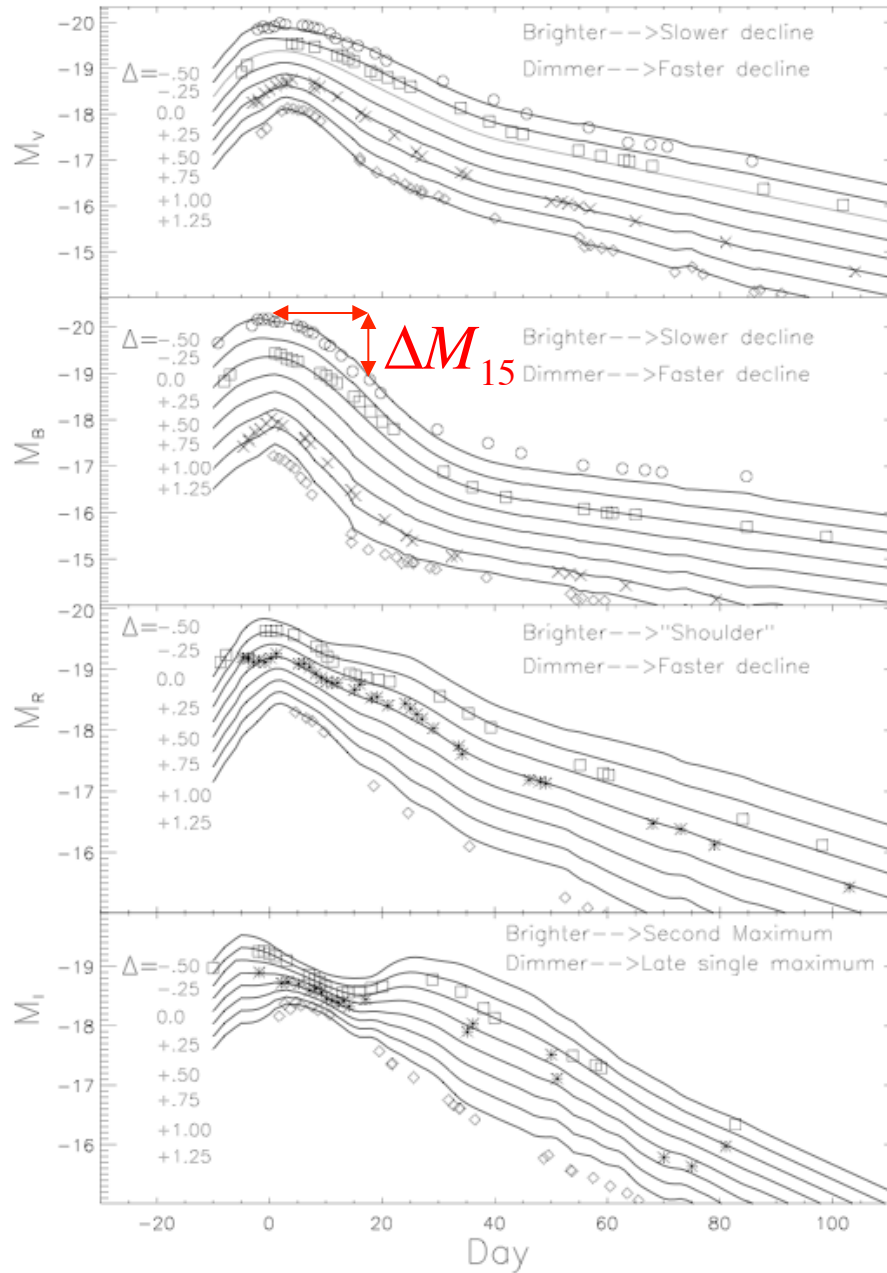
WD masses cannot exceed $1.4 M_{\text{sun}}$ (Chandrasekhar limit) because then gravity wins.

WD that accretes enough mass to surpass this limit, collapses, heats up, and fuses all its C/O in a fast runaway reaction.

The energy released unbinds the star. **SN Ia**

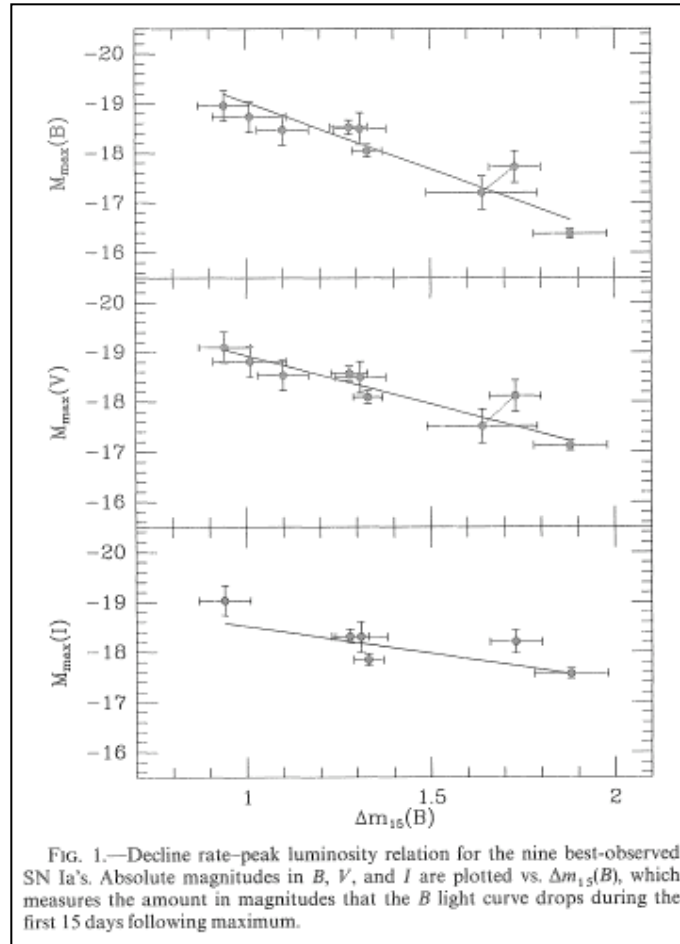


Determining distance: Supernovae type Ia



Determining distance: Supernovae type Ia

CANDLE



Phillips (1993)

The Distance Ladder

Method	Scatter	Reach	Systematics
Parallax	$\sim d$	<1 kpc	
Cepheids	5-10%	30 Mpc	Metallicity
SBF	5-10%	50 Mpc	Stellar LF
Tully-Fisher	10-20%	>100 Mpc	Mass-to-light
FP/ D_n -sigma	10-20%	>100 Mpc	Kinematics
SN Ia	5-10%	>1000 Mpc	Dust

The Distance Ladder

SN Ia

Tully-Fisher/ D_n -sigma



Cepheids/RR Lyrae

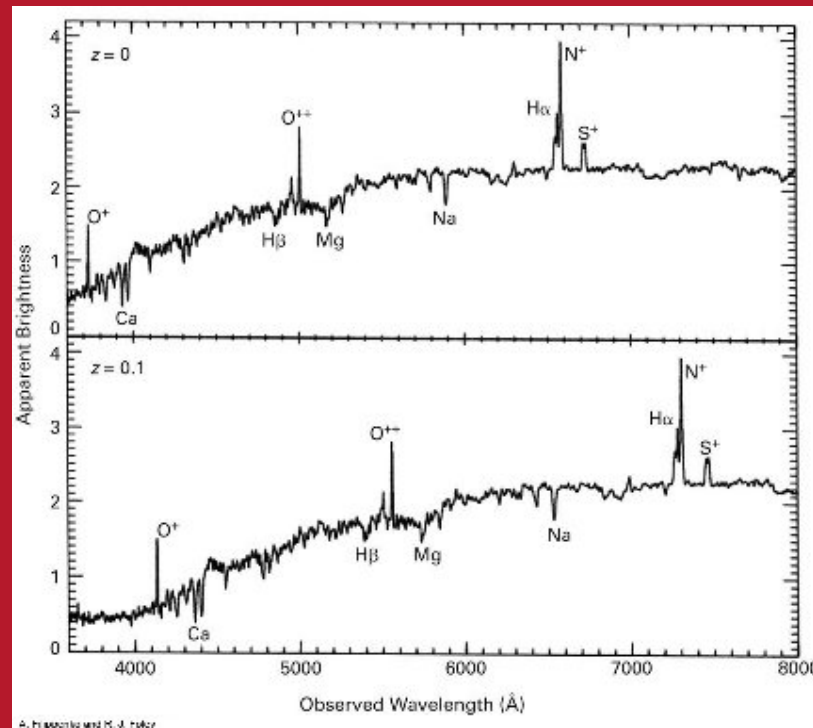


parallax

Redshift

We can measure galaxies' radial velocity using the Doppler effect.

Doppler effect: when an object is moving away from (or toward) us, the frequency of light that we see from it is shifted.



galaxy spectrum \rightarrow Doppler shift (redshift) \rightarrow radial velocity

Redshift

$$z = \frac{\lambda_{\text{obs}} - \lambda_{\text{emit}}}{\lambda_{\text{emit}}}$$

Relativistic Doppler Effect

$$1 + z = \sqrt{\frac{1 + v_r/c}{1 - v_r/c}}$$

$$z \approx \frac{v_r}{c}$$

Hubble Law

corrected for solar motion. The result, 745 km./sec. for a distance of 1.4×10^6 parsecs, falls between the two previous solutions and indicates a value for K of 530 as against the proposed value, 500 km./sec.

Secondly, the scatter of the individual nebulae can be examined by assuming the relation between distances and velocities as previously determined. Distances can then be calculated from the velocities corrected for solar motion, and absolute magnitudes can be derived from the apparent magnitudes. The results are given in table 2 and may be compared with the distribution of absolute magnitudes among the nebulae in table 1, whose distances are derived from other criteria. N. G. C. 404

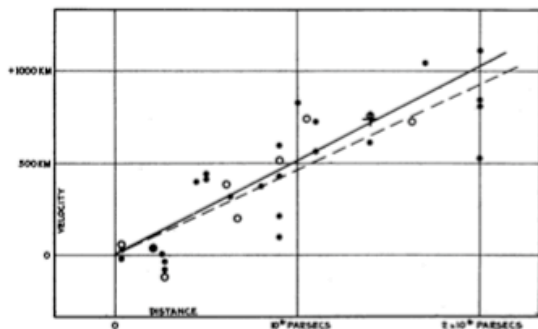


FIGURE 1
Velocity-Distance Relation among Extra-Galactic Nebulae.

Radial velocities, corrected for solar motion, are plotted against distances estimated from involved stars and mean luminosities of nebulae in a cluster. The black discs and full line represent the solution for solar motion using the nebulae individually; the circles and broken line represent the solution combining the nebulae into groups; the cross represents the mean velocity corresponding to the mean distance of 22 nebulae whose distances could not be estimated individually.

can be excluded, since the observed velocity is so small that the peculiar motion must be large in comparison with the distance effect. The object is not necessarily an exception, however, since a distance can be assigned for which the peculiar motion and the absolute magnitude are both within the range previously determined. The two mean magnitudes, -15.3 and -15.5 , the ranges, 4.9 and 5.0 mag., and the frequency distributions are closely similar for these two entirely independent sets of data; and even the slight difference in mean magnitudes can be attributed to the selected, very bright, nebulae in the Virgo Cluster. This entirely unforced agreement supports the validity of the velocity-distance relation in a very

evident matter. Finally, it is worth recording that the frequency distribution of absolute magnitudes in the two tables combined is comparable with those found in the various clusters of nebulae.

The results establish a roughly linear relation between velocities and distances among nebulae for which velocities have been previously published, and the relation appears to dominate the distribution of velocities. In order to investigate the matter on a much larger scale, Mr. Humason at Mount Wilson has initiated a program of determining velocities of the most distant nebulae that can be observed with confidence. These, naturally, are the brightest nebulae in clusters of nebulae. The first definite result,⁴ $v = +3779$ km./sec. for N. G. C. 7619, is thoroughly consistent with the present conclusions. Corrected for the solar motion, this velocity is $+3910$, which, with $K = 500$, corresponds to a distance of 7.8×10^6 parsecs. Since the apparent magnitude is 11.8, the absolute magnitude at such a distance is -17.65 , which is of the right order for the brightest nebulae in a cluster. A preliminary distance, derived independently from the cluster of which this nebula appears to be a member, is of the order of 7×10^6 parsecs.

New data to be expected in the near future may modify the significance of the present investigation or, if confirmatory, will lead to a solution having many times the weight. For this reason it is thought premature to discuss in detail the obvious consequences of the present results. For example, if the solar motion with respect to the clusters represents the rotation of the galactic system, this motion could be subtracted from the results for the nebulae and the remainder would represent the motion of the galactic system with respect to the extra-galactic nebulae.

The outstanding feature, however, is the possibility that the velocity-distance relation may represent the de Sitter effect, and hence that numerical data may be introduced into discussions of the general curvature of space. In the de Sitter cosmology, displacements of the spectra arise from two sources, an apparent slowing down of atomic vibrations and a general tendency of material particles to scatter. The latter involves an acceleration and hence introduces the element of time. The relative importance of these two effects should determine the form of the relation between distances and observed velocities; and in this connection it may be emphasized that the linear relation found in the present discussion is a first approximation representing a restricted range in distance.

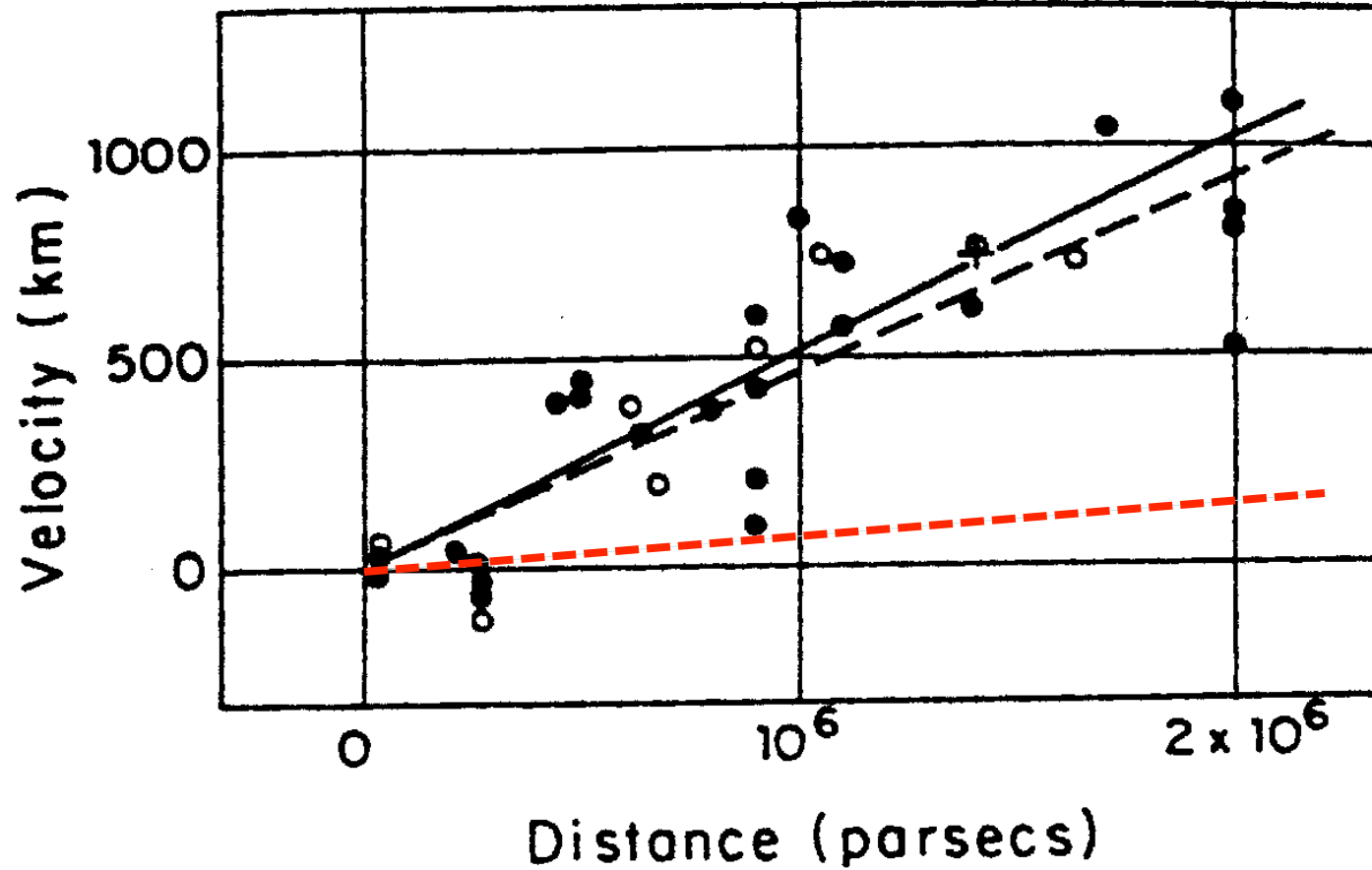
¹ *Mt. Wilson Contr.*, No. 324; *Astroph. J.*, Chicago, Ill., **64**, 1926 (321).

² *Harvard Coll. Obs. Circ.*, 294, 1926.

³ *Mon. Not. R. Astr. Soc.*, **85**, 1925 (865-894).

⁴ These PROCEEDINGS, **15**, 1929 (167).

Hubble Law

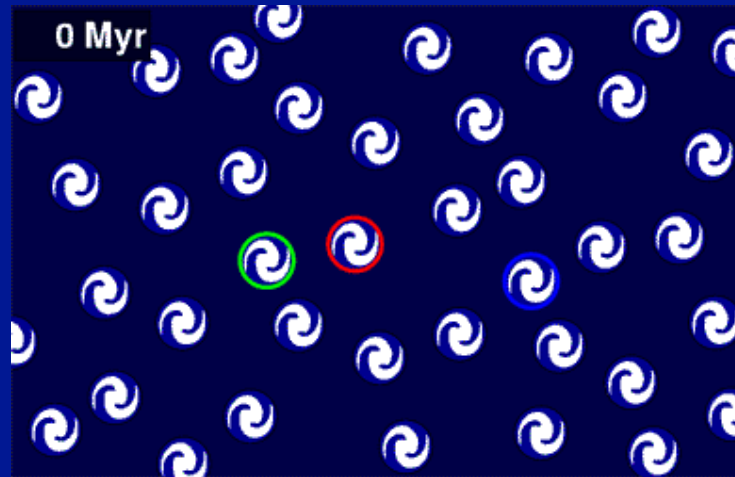
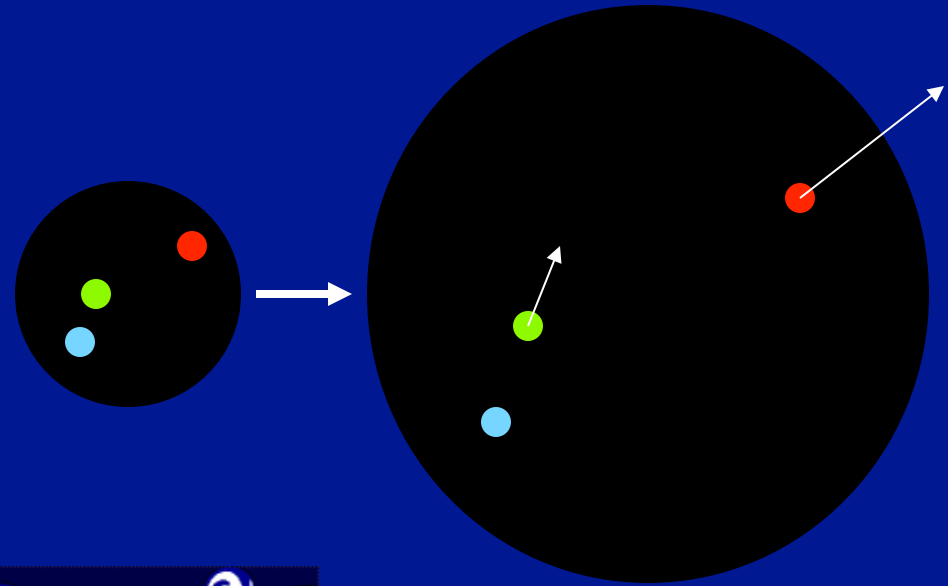


$$v_r = H_0 \times d$$

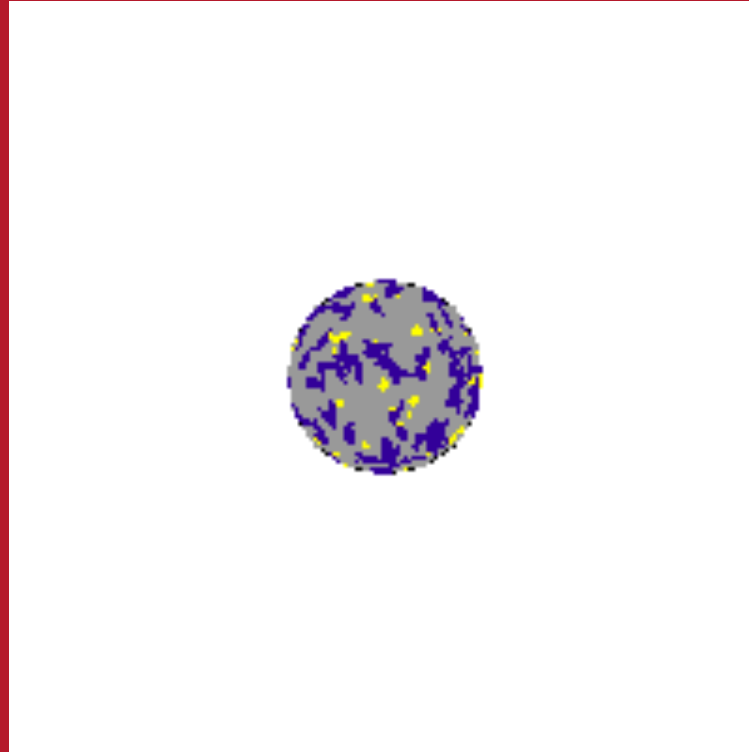
$$\left(H_0 \approx 500 \frac{\text{km}}{\text{s}} \text{Mpc}^{-1} \right)$$

Hubble Law

The **expansion** of the universe is such that galaxies' recessional speeds are proportional to their distance.



Cosmological Redshift



Cosmological Redshift

$$z = \frac{\lambda_{\text{obs}} - \lambda_{\text{emit}}}{\lambda_{\text{emit}}} \rightarrow 1 + z = \frac{\lambda_{\text{obs}}}{\lambda_{\text{emit}}}$$

Cosmic scale factor:

$$a(t) = \frac{R(t)}{R(t_0)}$$

Wavelengths stretch with scale factor:

$$1 + z = \frac{a(t_0)}{a(t)} = \frac{1}{a(t)}$$

Hubble Law

$$v_r = H_0 \times d$$

$$H_0 \approx 70 \frac{\text{km}}{\text{s}} \text{Mpc}^{-1}$$

$$H_0 \equiv 100h \frac{\text{km}}{\text{s}} \text{Mpc}^{-1}$$

$$h \approx 0.7$$

Hubble Law

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FINAL RESULTS FROM THE *HUBBLE SPACE TELESCOPE* KEY PROJECT TO MEASURE THE HUBBLE CONSTANT¹

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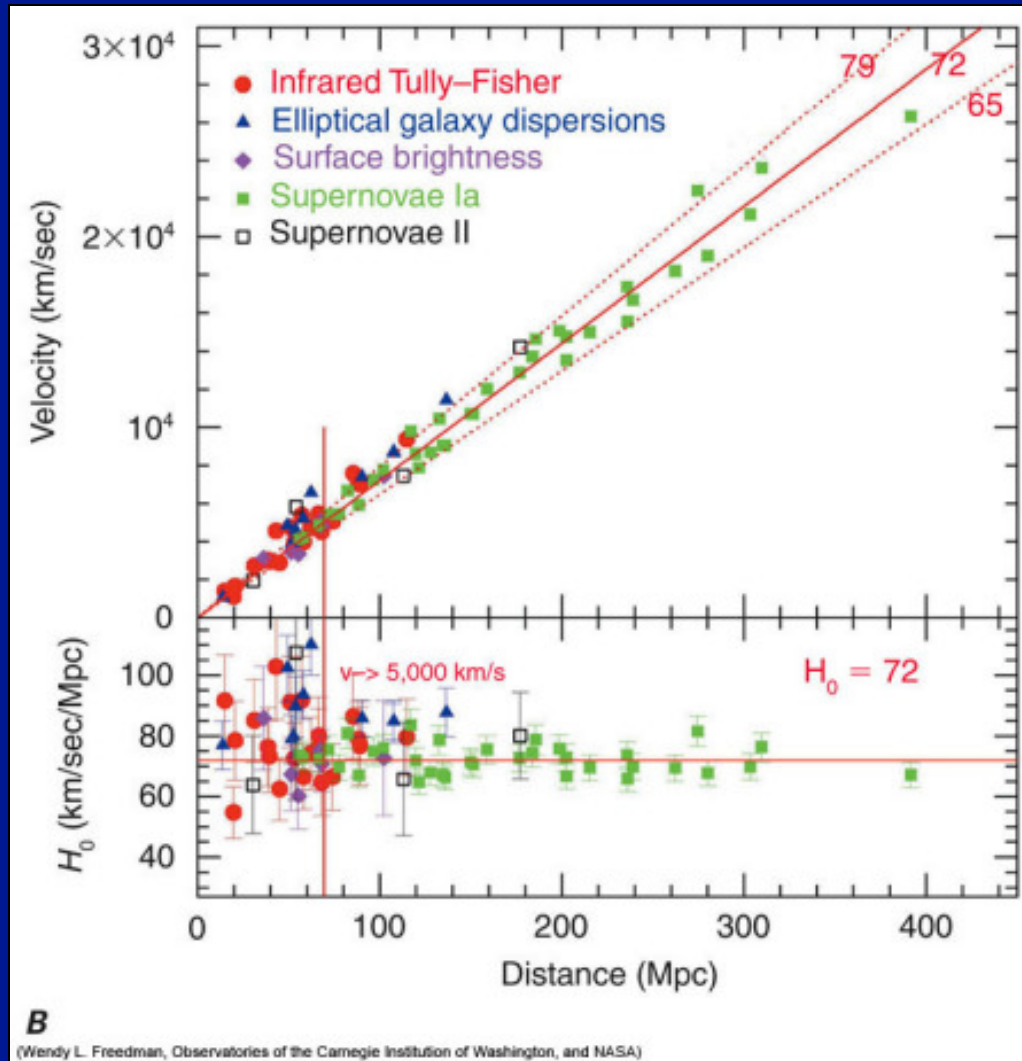
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ABSTRACT

We present here the final results of the *Hubble Space Telescope* (*HST*) Key Project to measure the Hubble constant. We summarize our method, the results, and the uncertainties, tabulate our revised distances, and give the implications of these results for cosmology. Our results are based on a Cepheid calibration of several secondary distance methods applied over the range of about 60–400 Mpc. The analysis presented here benefits from a number of recent improvements and refinements, including (1) a larger LMC Cepheid sample to define the fiducial period-luminosity (PL) relations, (2) a more recent *HST* Wide Field and Planetary Camera 2 (WFPC2) photometric calibration, (3) a correction for Cepheid metallicity, and (4) a correction for incompleteness bias in the observed Cepheid PL samples. We adopt a distance modulus to the LMC (relative to which the more distant galaxies are measured) of $\mu_0(\text{LMC}) = 18.50 \pm 0.10$ mag, or 50 kpc. New, revised distances are given for the 18 spiral galaxies for which Cepheids have been discovered as part of the Key Project, as well as for 13 additional galaxies with published Cepheid data. The new calibration results in a Cepheid distance to NGC 4258 in better agreement with the maser distance to this galaxy. Based on these revised Cepheid distances, we find values (in $\text{km s}^{-1} \text{Mpc}^{-1}$) of $H_0 = 71 \pm 2$ (random) ± 6 (systematic) (Type Ia supernovae), $H_0 = 71 \pm 3 \pm 7$ (Tully-Fisher relation), $H_0 = 70 \pm 5 \pm 6$ (surface brightness fluctuations), $H_0 = 72 \pm 9 \pm 7$ (Type II supernovae), and $H_0 = 82 \pm 6 \pm 9$ (fundamental plane). We combine these results for the different methods with three different weighting schemes, and find good agreement and consistency with $H_0 = 72 \pm 8 \text{ km s}^{-1} \text{Mpc}^{-1}$. Finally, we compare these results with other, global methods for measuring H_0 .

Subject headings: Cepheids — cosmology: observations — distance scale —
galaxies: distances and redshifts

Hubble Law



Hubble Law

