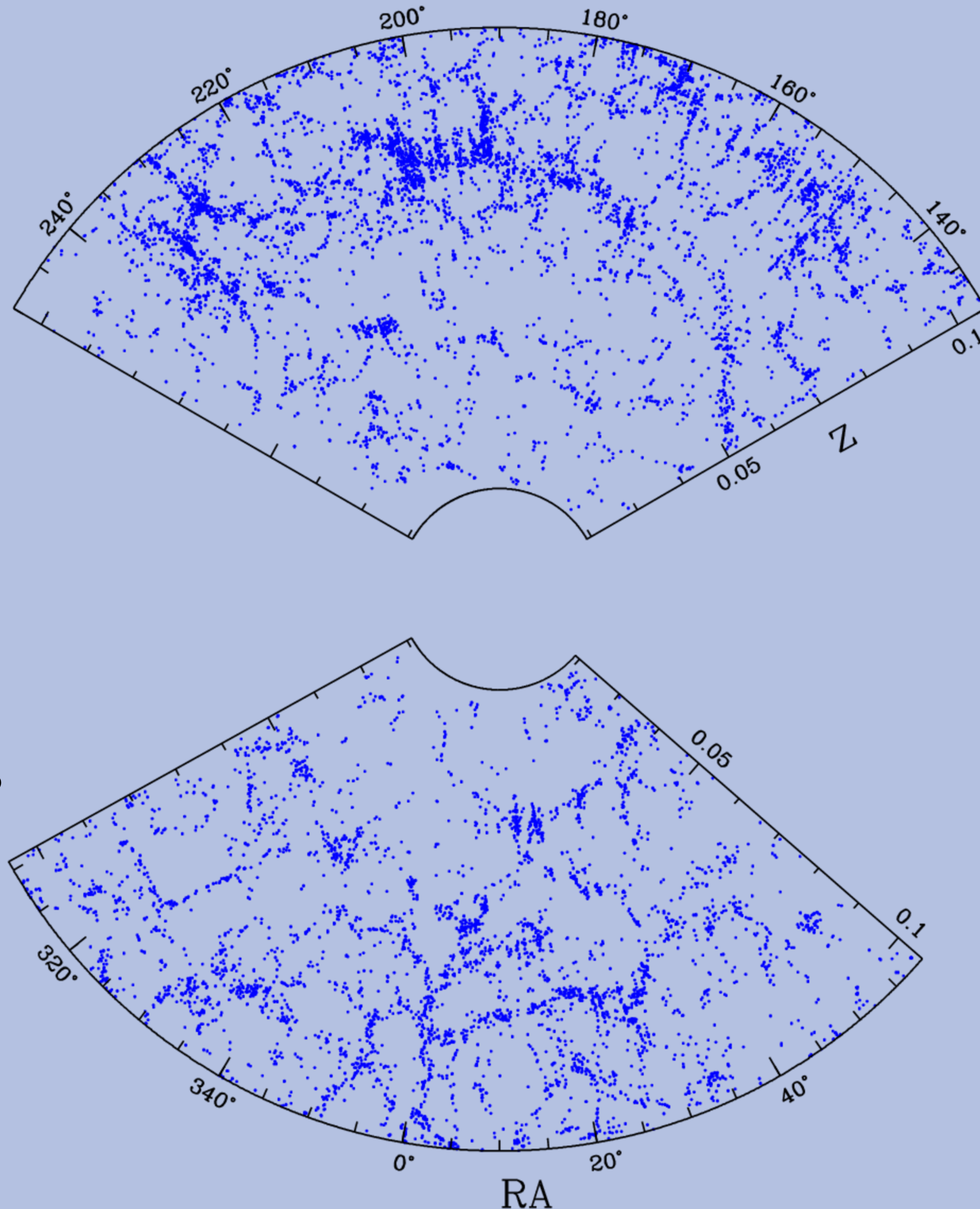
# Statistics of the Galaxy Distribution

measure the environment
around individual galaxies

or

measure average statistics
for a sample of galaxies

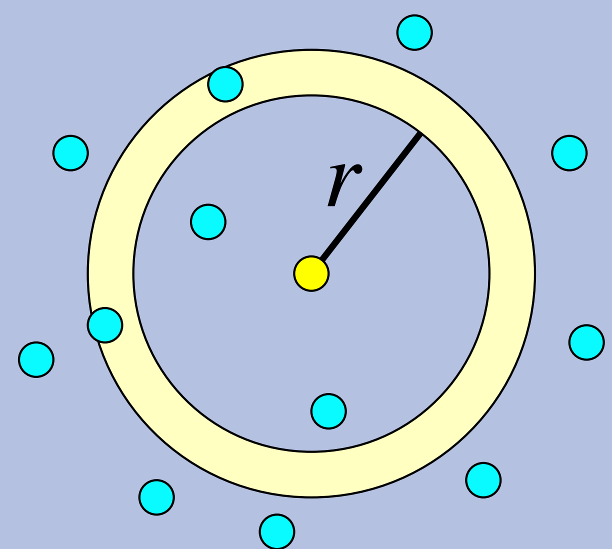


# The 2-point Correlation Function

The *excess* probability that two galaxies are separated by a distance ***r*** relative to that for a random distribution.

For a random point distribution of number density ***n***, the number of points at a distance between ***r*** and ***r+dr*** from any one point is:

$$n \cdot dV = n \cdot 4\pi r^2 dr$$




The total number density of pairs at this separation is then:

$$\frac{n}{2} \times n \cdot 4\pi r^2 dr = n^2 2\pi r^2 dr$$

# The 2-point Correlation Function

For a point distribution that is not random, the number density of pairs
At this separation is:

$$n^2 2\pi r^2 dr\left[1+\xi(r)\right]$$

The correlation function is then equal to:

$$\xi(r)=\frac{n_{\text{pairs, data}}(r)}{n_{\text{pairs, rand}}(r)}-1$$

$$\xi(r)=\frac{DD(r)}{RR(r)}-1$$

# The 2-point Correlation Function

## Complications

- When the sample volume is complex, calculate number of random pairs using an actual generated random data set that occupies the same volume as the data.

- In practice, we often use other estimators. For example, the most commonly used estimator for galaxy samples is the Landy-Szalay estimator:

$$\xi(r) = \frac{DD - 2DR + RR}{RR}$$

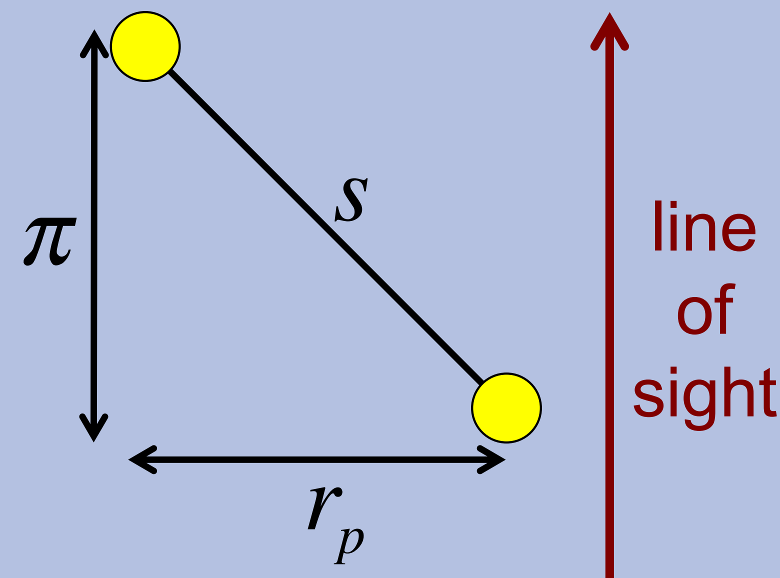
- Must normalize the DD, DR, and RR terms when the number of data and random points are not the same.

# The 2-point Correlation Function

## Complications

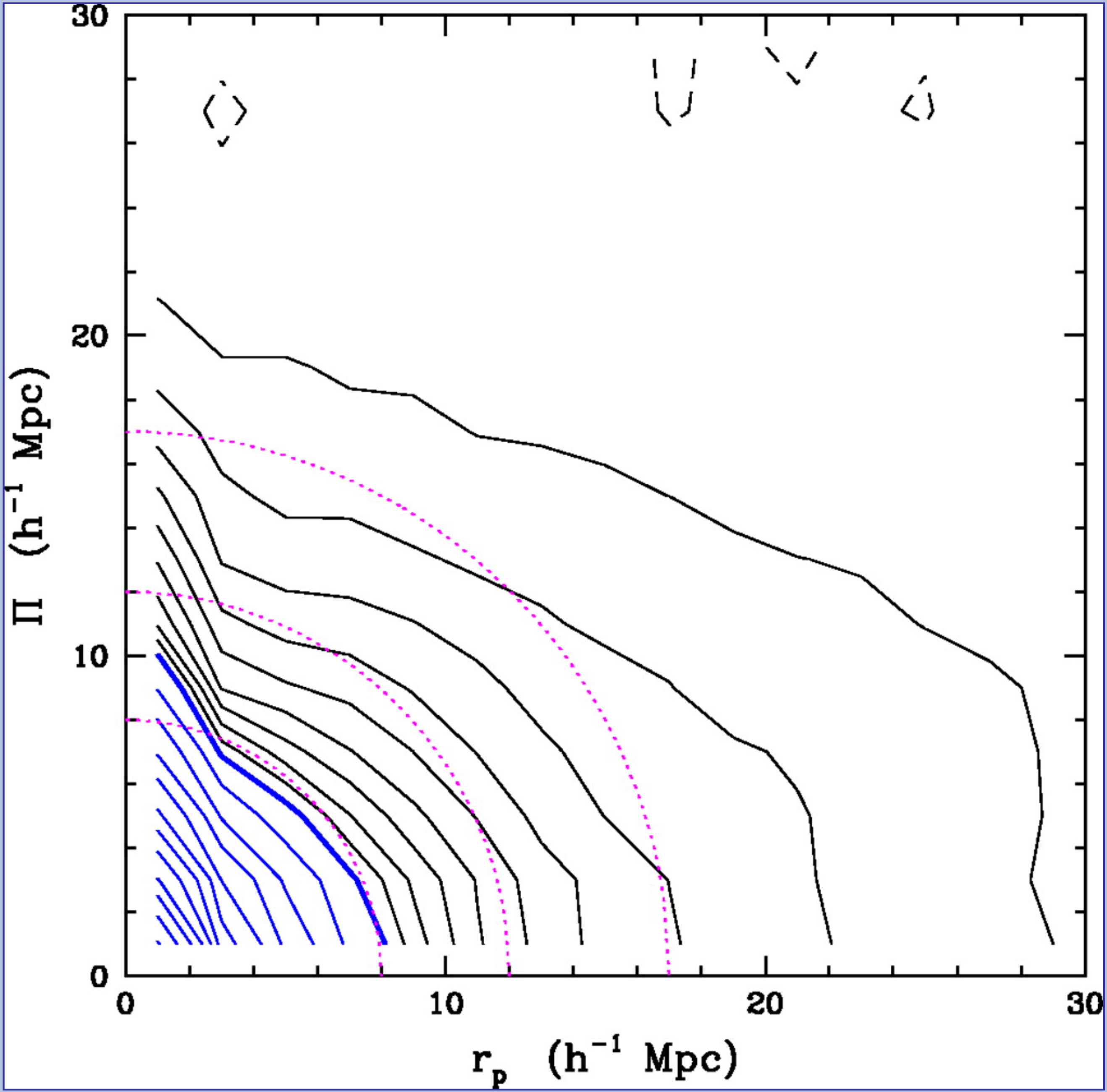
- To deal with redshift distortions, we usually measure a *projected* correlation function.

$$\xi\left(r_p,\pi\right)$$



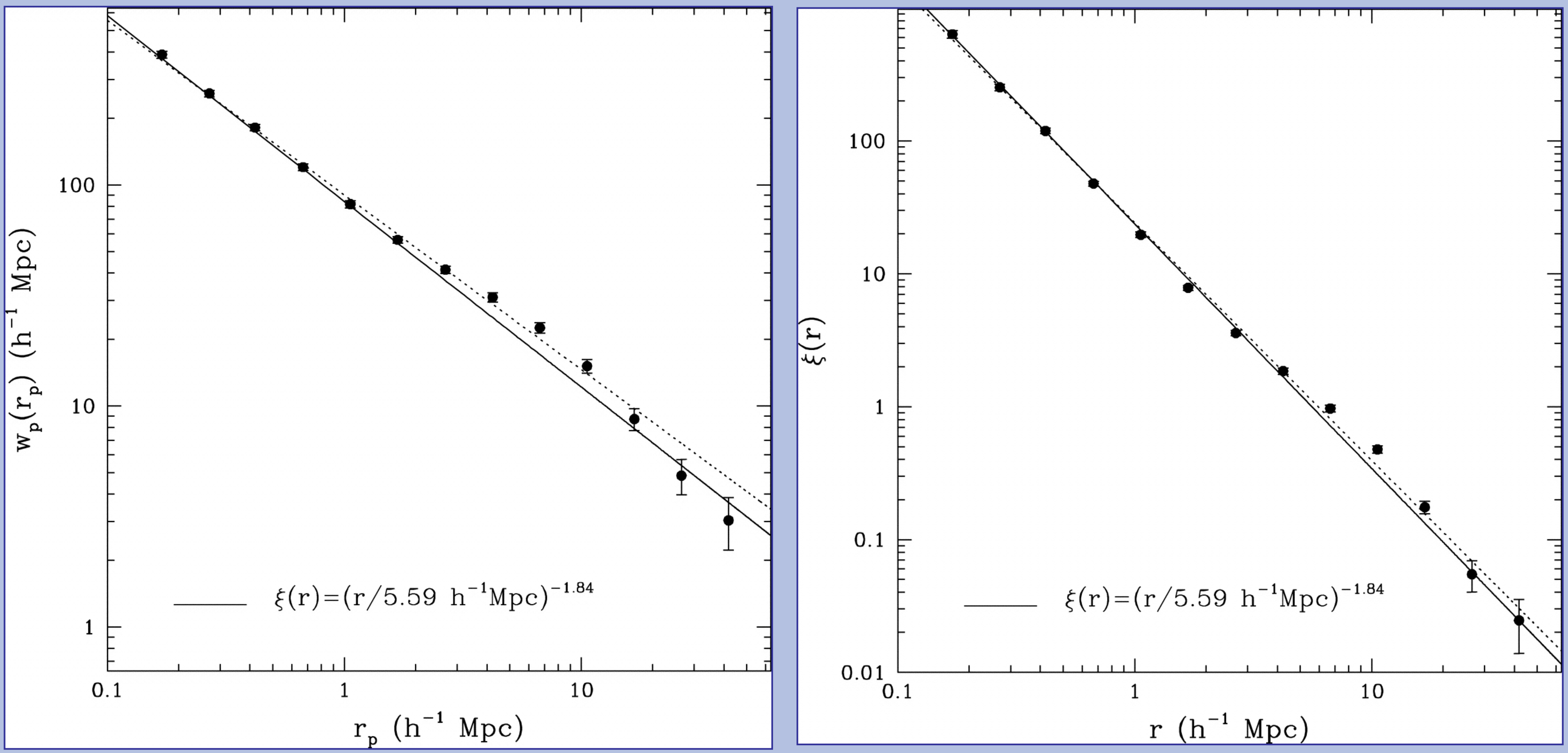

$$w_p\left(r_p\right)=2\int_0^{\pi_{\max}}\xi\left(r_p,\pi\right)d\pi$$

# The 2-point Correlation Function



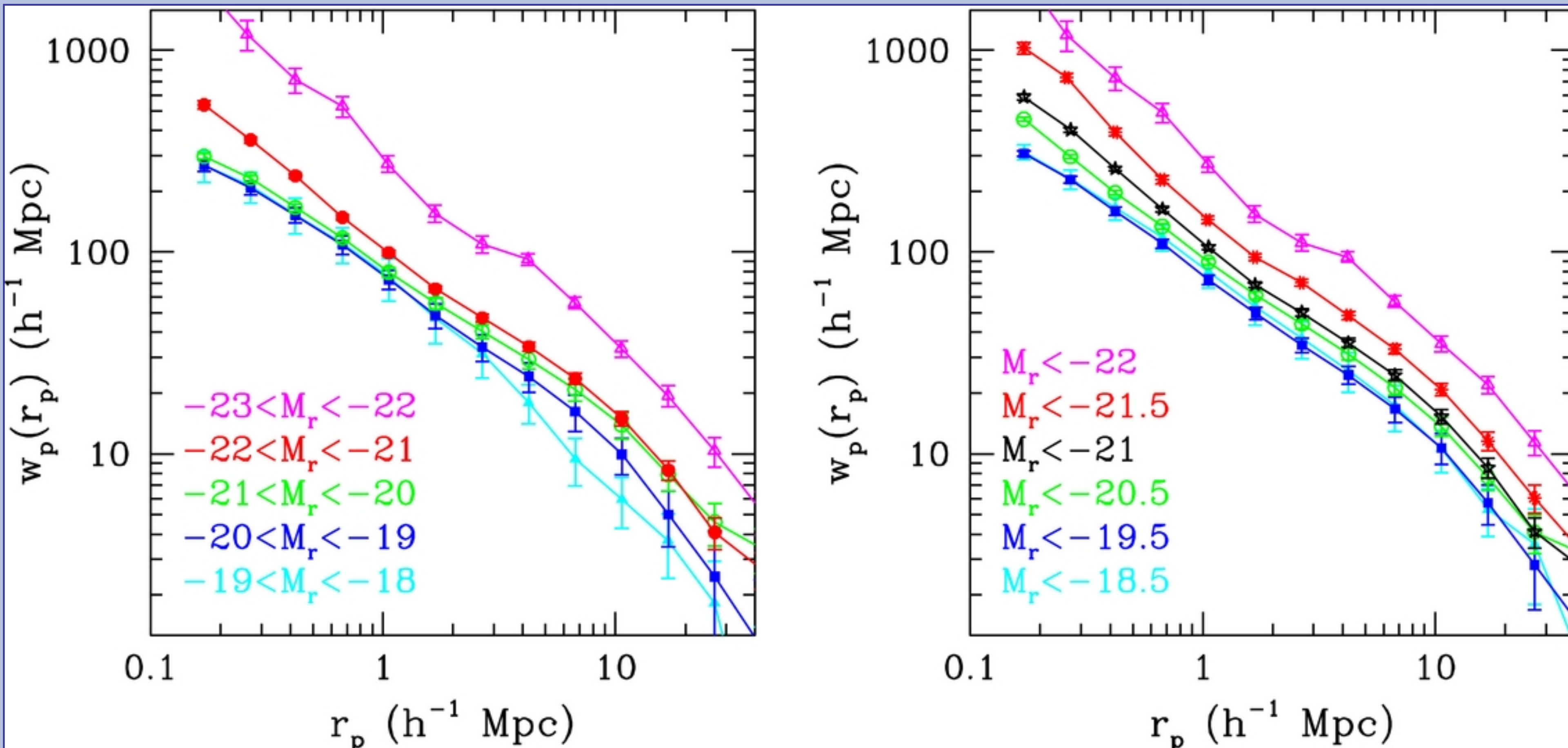

Zehavi et al. (2005)

# The 2-point Correlation Function



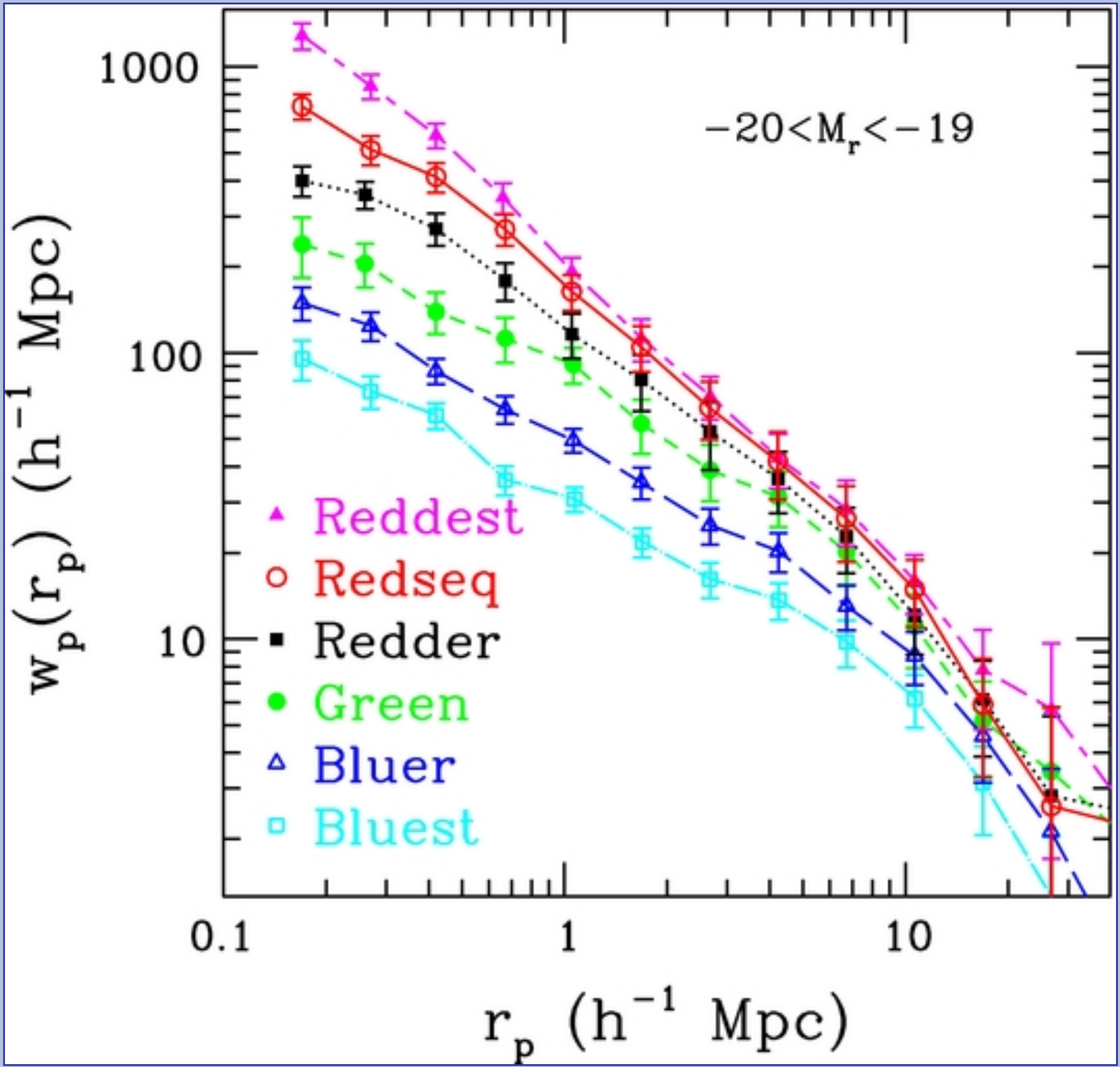

Zehavi et al. (2005)

# The 2-point Correlation Function




Zehavi et al. (2011)

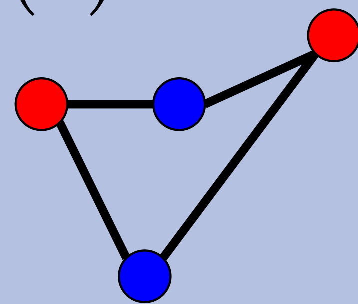
# The 2-point Correlation Function

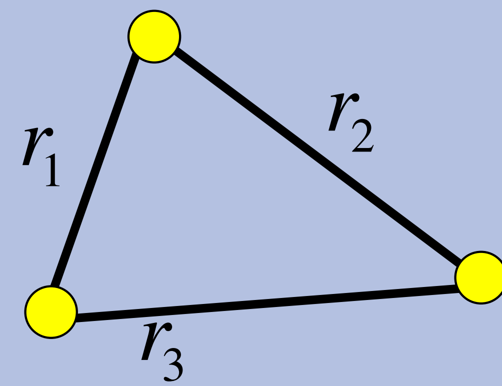


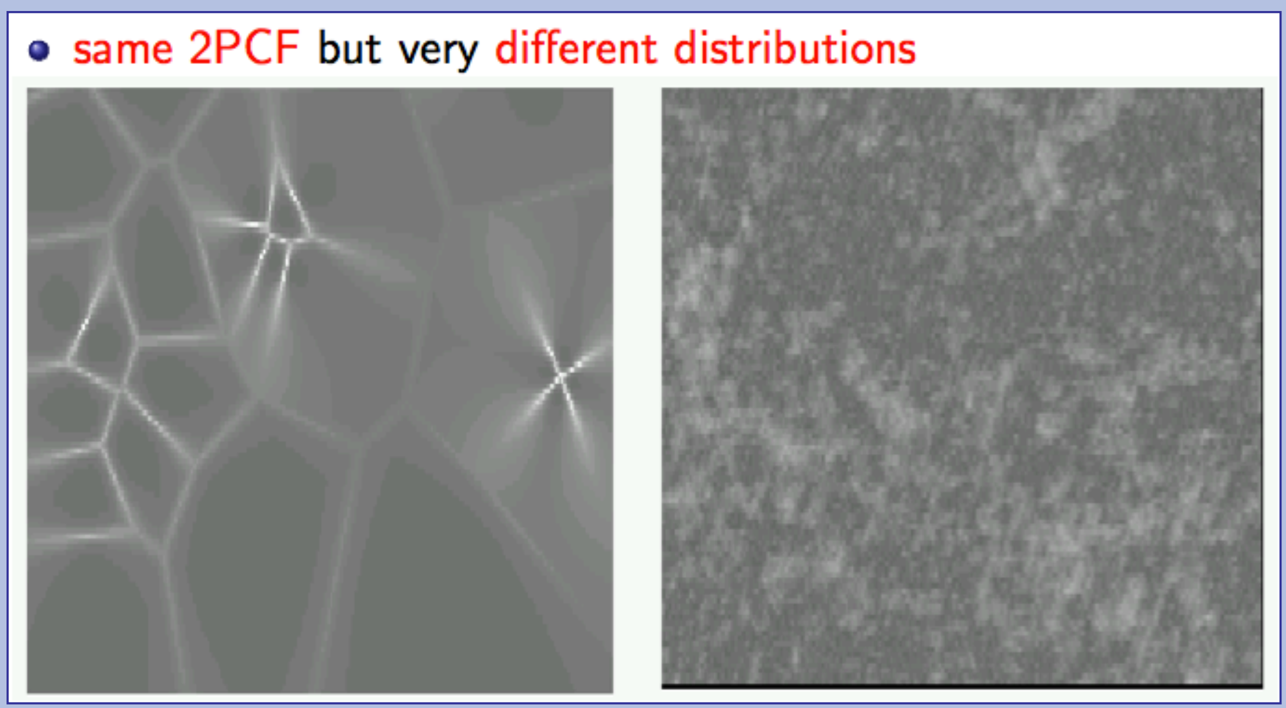

Zehavi et al. (2011)

# Other Correlation Functions

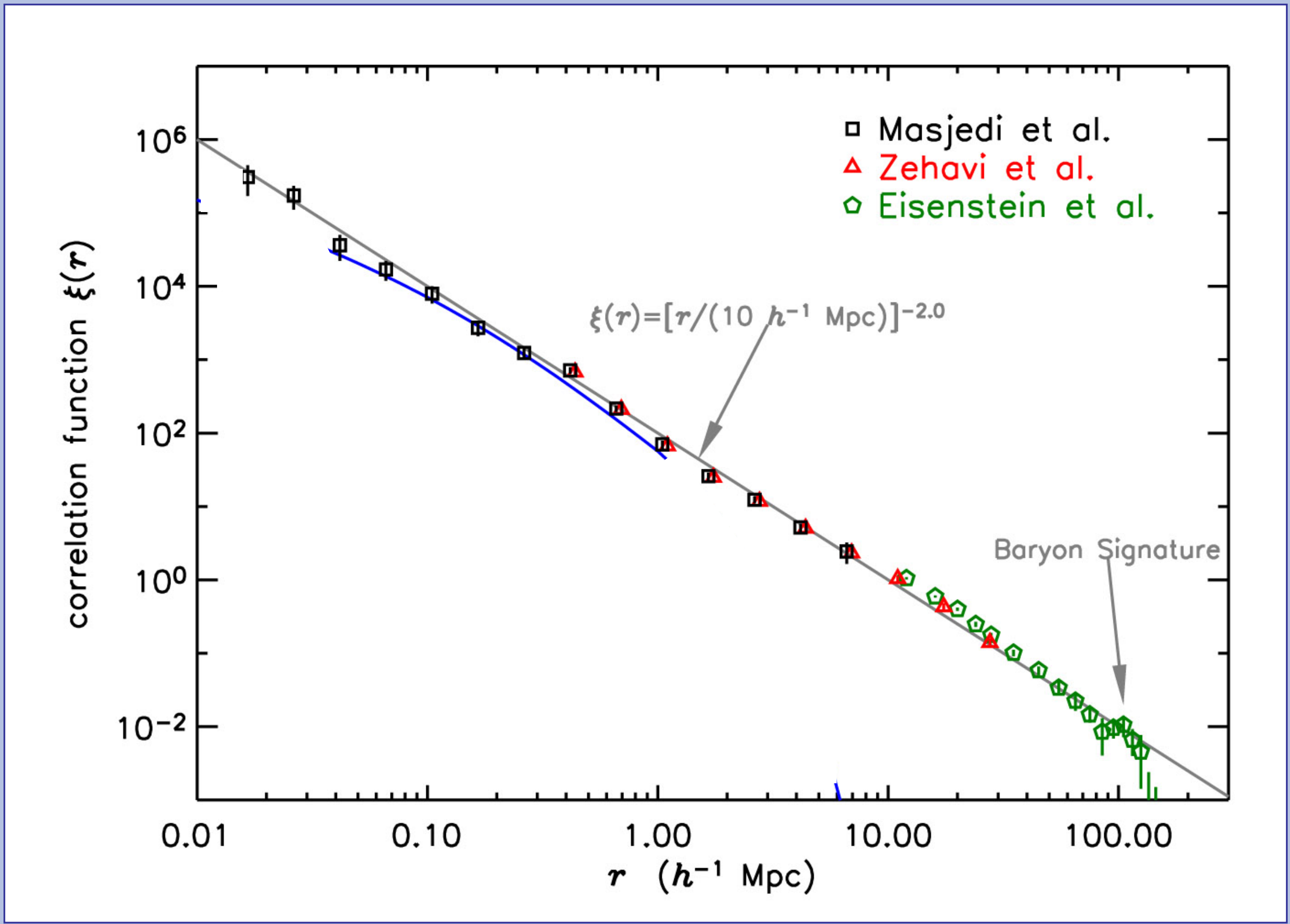
- Angular correlation function $\omega(\theta)$
- Cross-correlation function
- Higher order: three-point, etc





# The 2-point Correlation Function




Masjedi et al. (2006)

# The 2-point Correlation Function

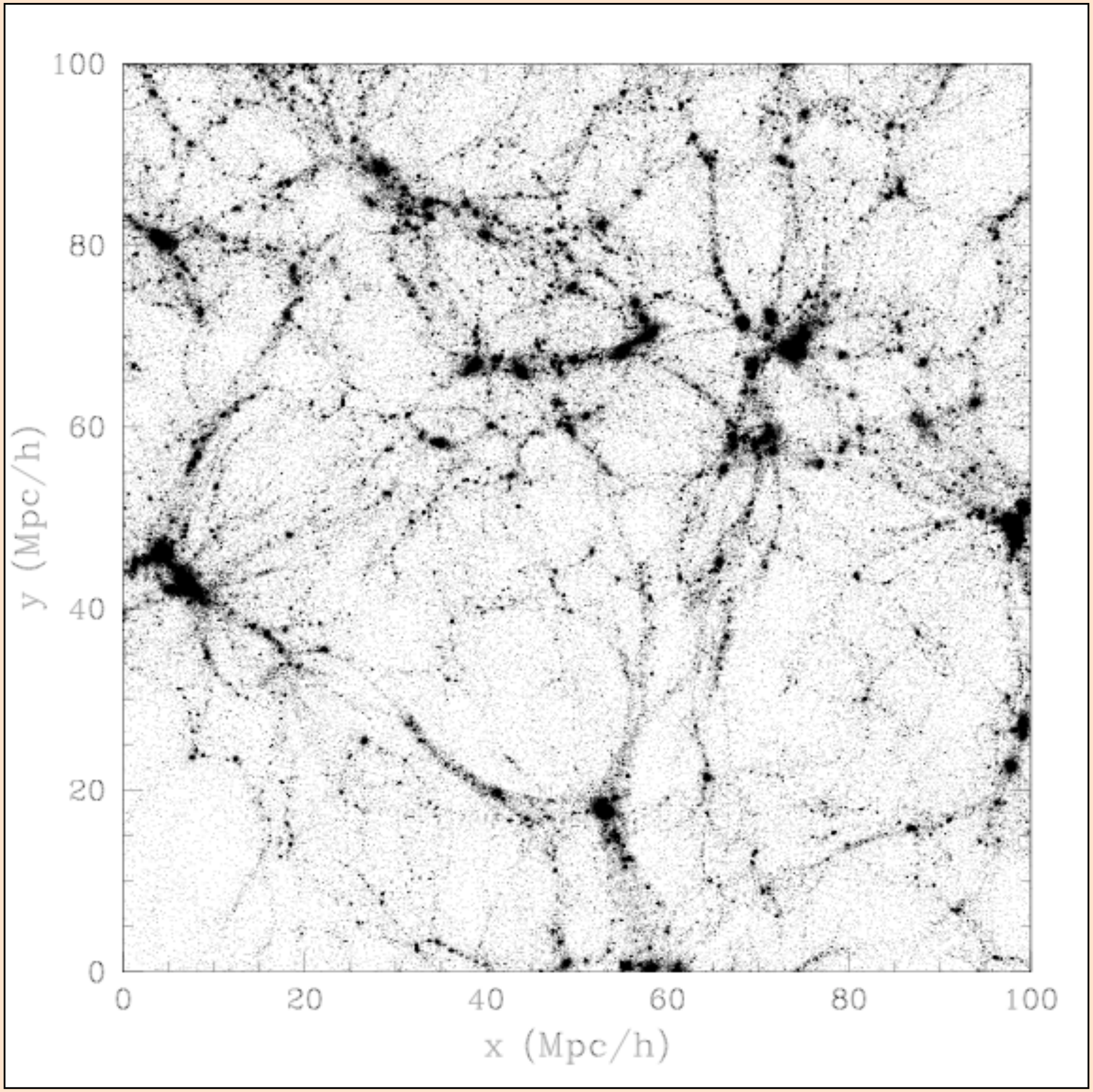
Galaxy "bias" refers to the amount of galaxy clustering relative to the clustering of the underlying dark matter

$$b = \sqrt{\frac{\xi_{\mathrm{galaxy}}}{\xi_{\mathrm{mass}}}}$$

This is a function of scale ***r***, but on large scales it becomes constant.

For example, the bias of Milky Way – like galaxies is ***b*** ~ 1
the bias of Luminous Red Galaxies is ***b*** ~ 2

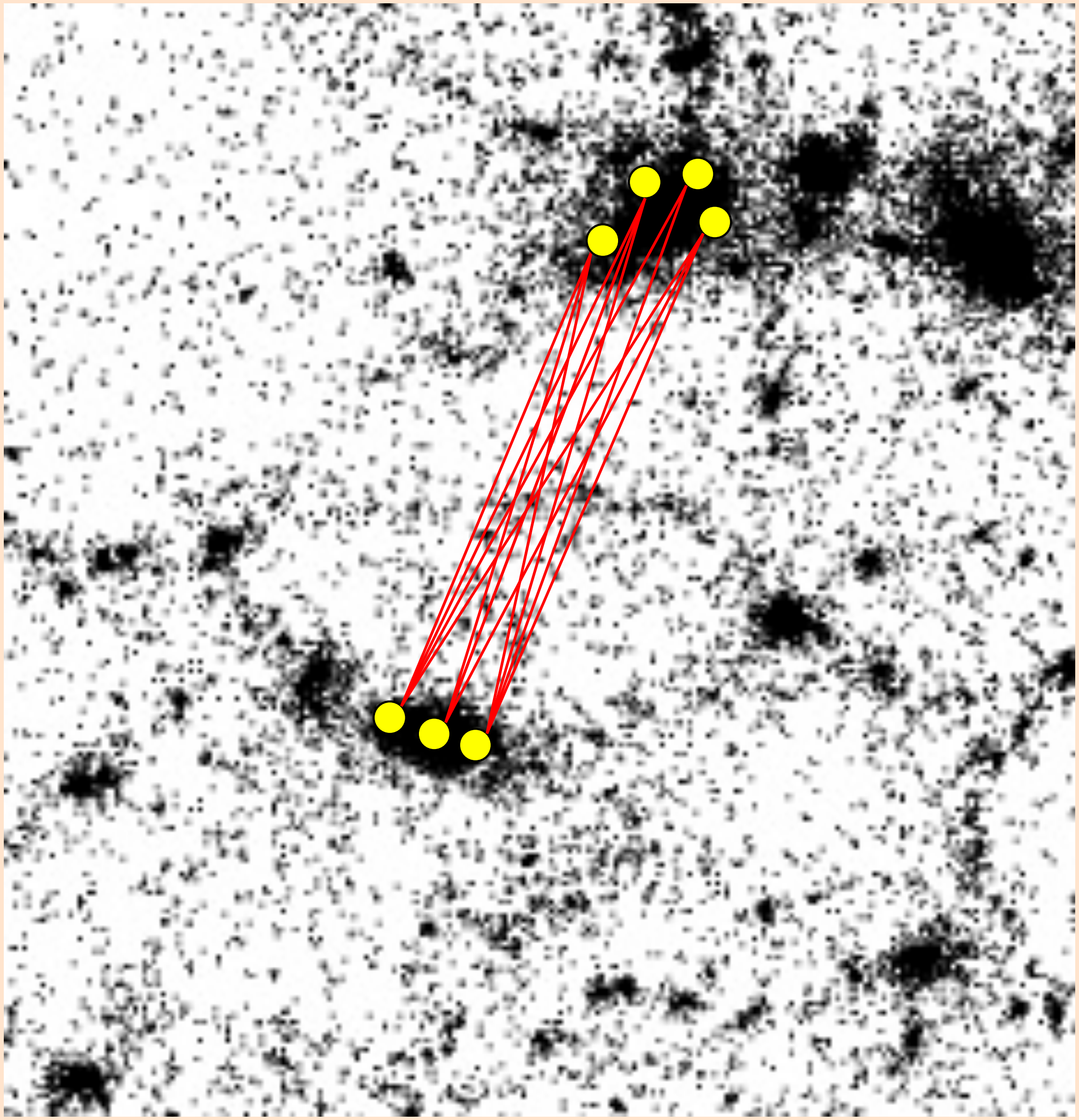
# Power Spectrum




Density field

$$\delta(\vec{x}) = \frac{\rho(\vec{x}) - \bar{\rho}}{\bar{\rho}}$$

# Power Spectrum



$$\boxed{\xi(r) = \frac{DD}{RR} - 1}$$

$$DD = \rho(\vec{x})\rho(\vec{x} + r)$$

$$RR = \bar{\rho}\bar{\rho}$$

# Power Spectrum

$$\xi(r) = \frac{DD}{RR} - 1 \qquad DD = \rho(\vec{x})\rho(\vec{x}+r) \qquad RR = \bar{\rho}\bar{\rho}$$

$$\xi(r) = \langle \frac{\rho(\vec{x})}{\bar{\rho}} \frac{\rho(\vec{x}+r)}{\bar{\rho}} \rangle - 1 \qquad \delta(\vec{x}) = \frac{\rho(\vec{x})}{\bar{\rho}} - 1$$

$$= \langle (\delta(\vec{x}) + 1)(\delta(\vec{x}+r) + 1) \rangle - 1$$

$$= \cancel{\langle \delta(\vec{x}) \rangle} + \cancel{\langle \delta(\vec{x}+r) \rangle} + \langle \delta(\vec{x})\delta(\vec{x}+r) \rangle$$

$$= \langle \delta(\vec{x})\delta(\vec{x}+r) \rangle$$

# Power Spectrum

Density field

$$\delta(\vec{x}) = \frac{\rho(\vec{x}) - \bar{\rho}}{\bar{\rho}}$$

Correlation function

$$\xi(r) = \langle \delta(\vec{x}) \delta(\vec{x} + r) \rangle$$

Fourier density modes

$$\delta_{\vec{k}} = \int \delta(\vec{x}) e^{i\vec{k} \bullet \vec{x}} d^3\vec{x}$$

Power spectrum

$$P(k) = \left\langle \left| \delta_{\vec{k}} \right|^2 \right\rangle$$

$$\boxed{P(k) = \int \xi(r) e^{i\vec{k} \bullet \vec{r}} d^3\vec{x}}$$

# Power Spectrum

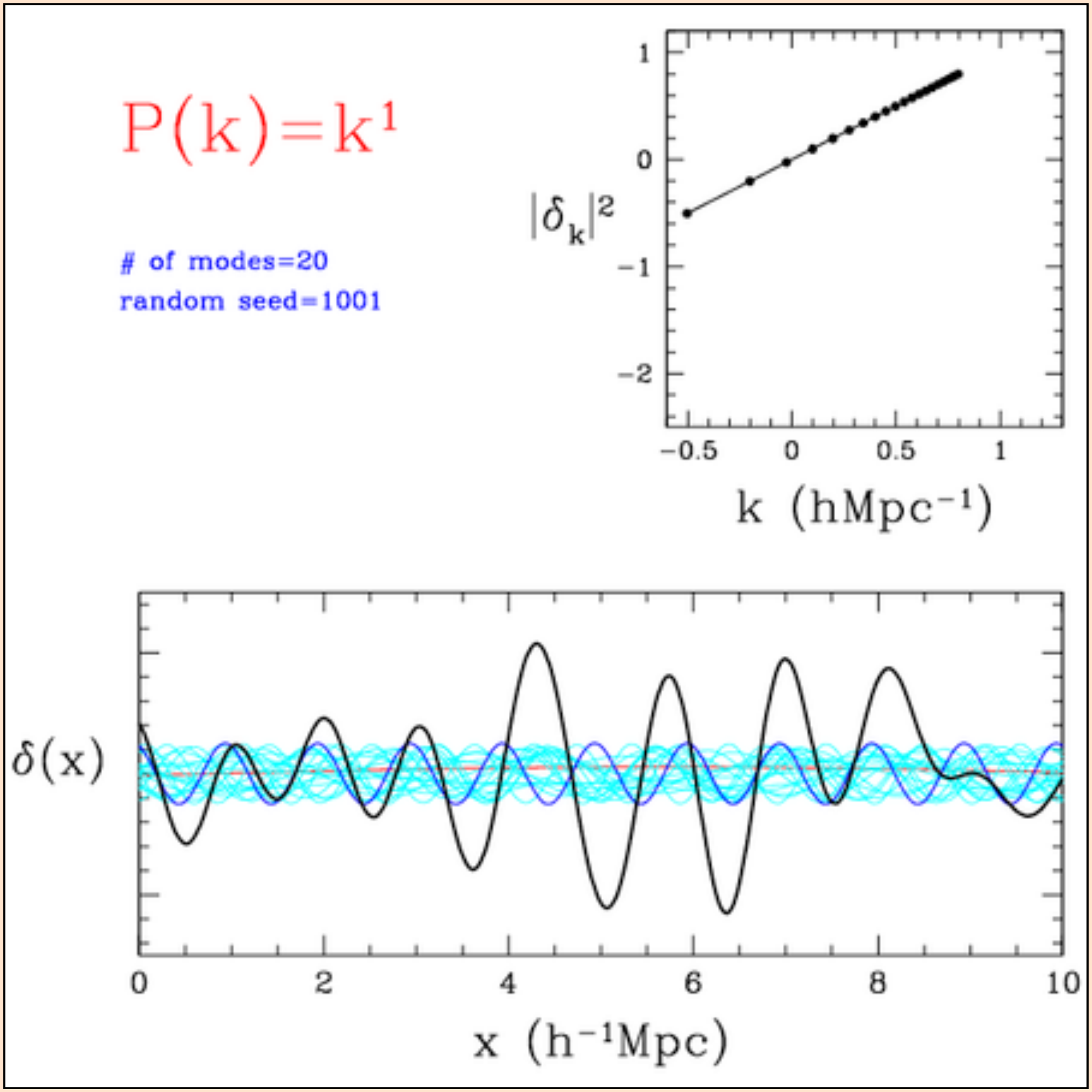
Any density field can be decomposed into an infinite set of modes (i.e., sine waves) $\delta_{\vec{k}}$

Each mode has a

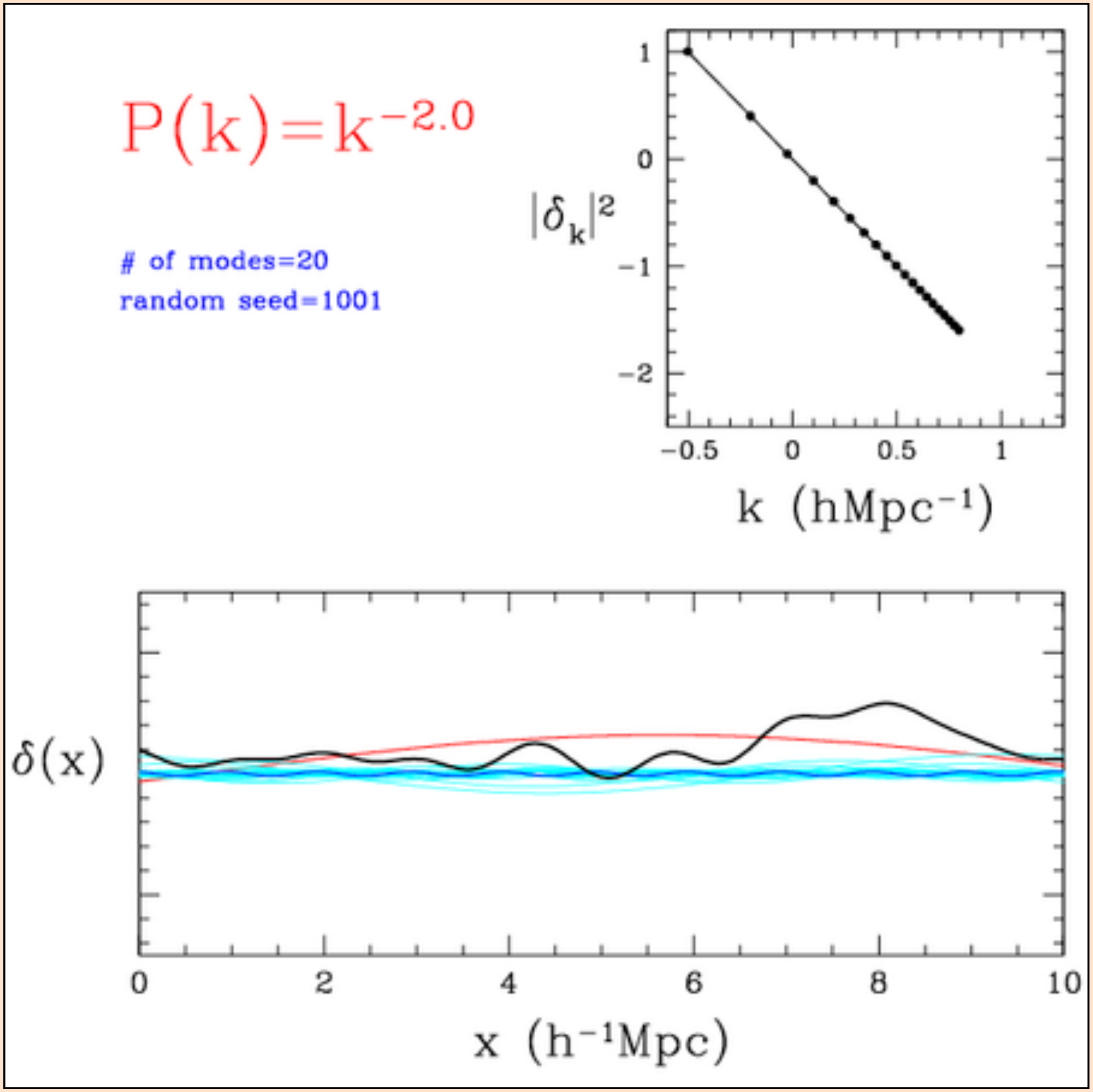
- <u>wavelength</u> $\lambda$ or wavenumber $k = \dfrac{2\pi}{\lambda}$
- <u>amplitude</u> $\left|\delta_{\vec{k}}\right|$
- <u>phase</u> $e^{-i\theta}$

The power spectrum is the amplitude as a function of k

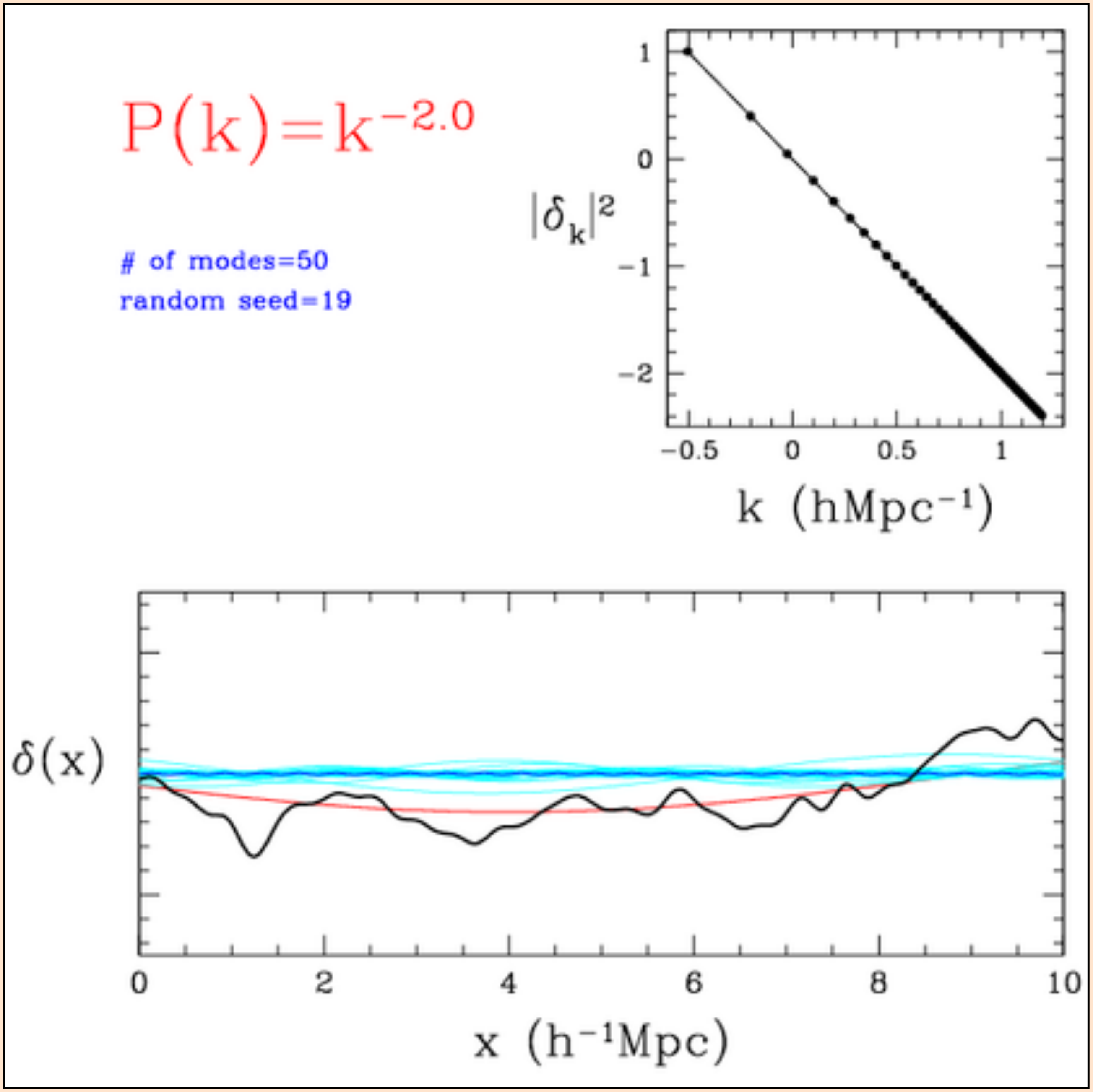
# Power Spectrum



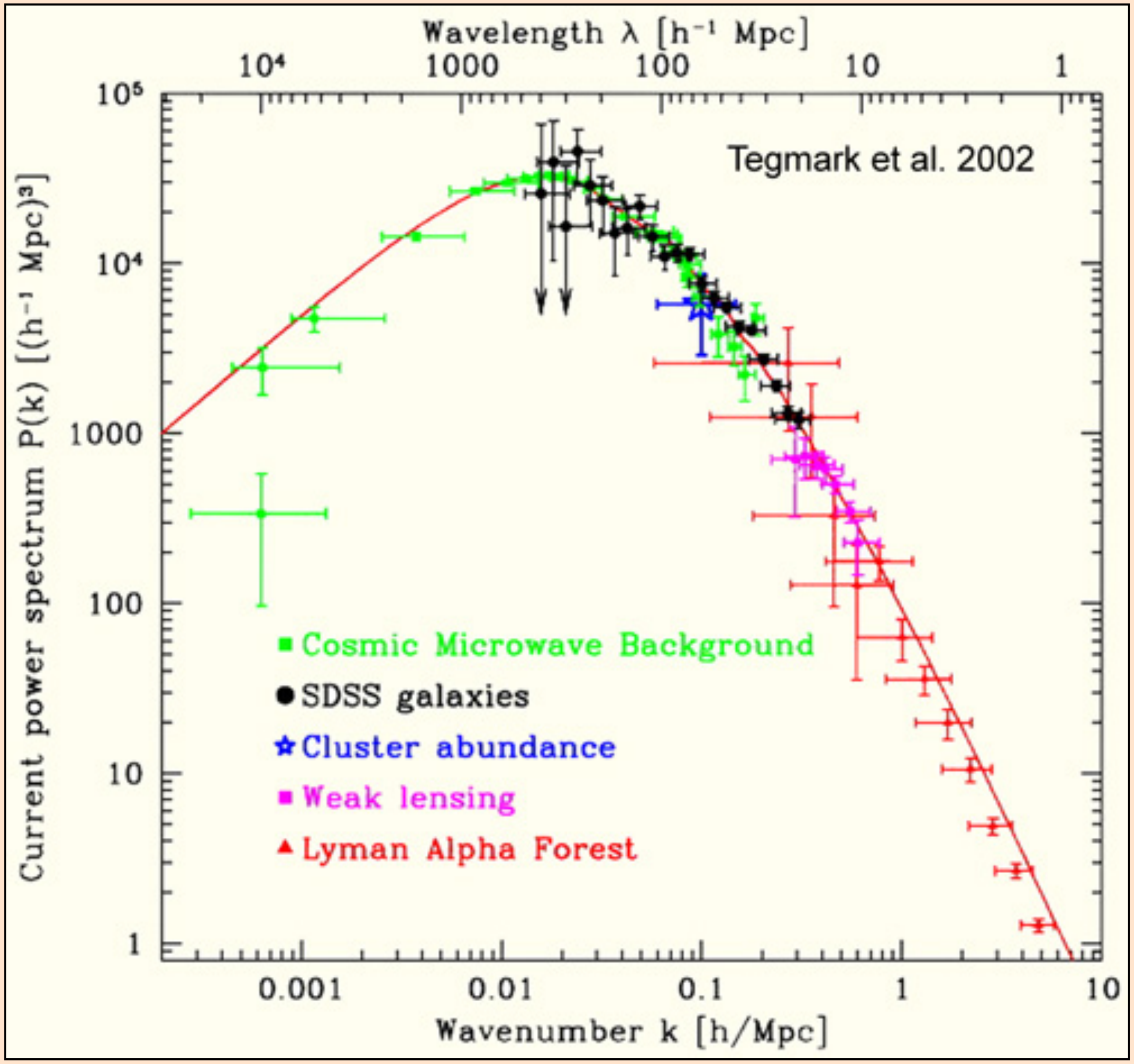
# Power Spectrum



# Power Spectrum



# Power Spectrum



# Power Spectrum

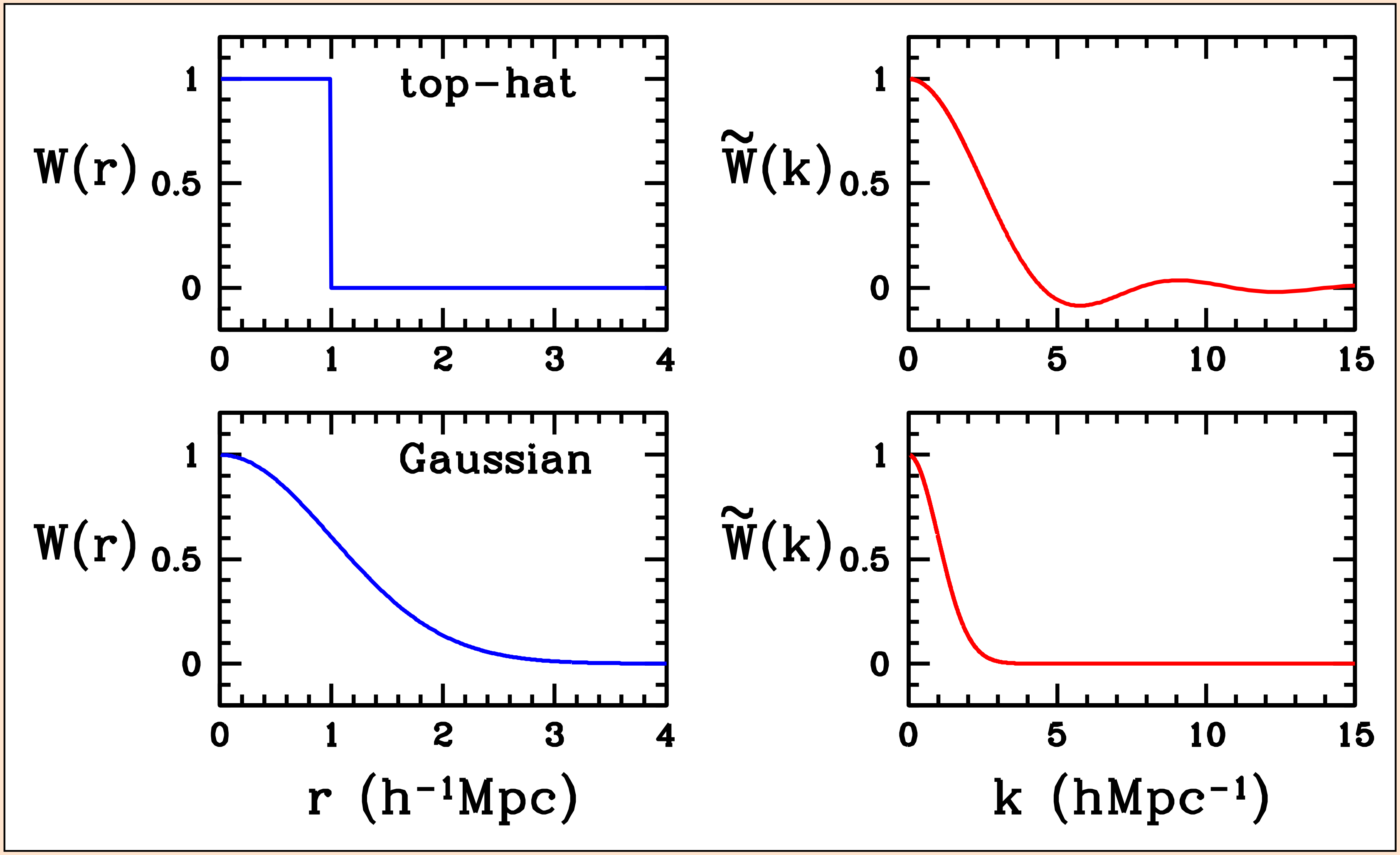
Window function: filter used to smooth density field

- Gaussian filter of scale *R* $W_R(r) = e^{-r^2/2R^2}$

- Top-hat filter of scale *R* $W_R(r) = \begin{cases} 1 & r < R \\ 0 & r > R \end{cases}$

In Fourier space: $\tilde{W}_R(k) = \int W_R(r) e^{i\vec{k}\cdot\vec{r}} d^3r$

# Power Spectrum



# Power Spectrum

Density field, smoothed with window function

$$\delta_R(\vec{x}) = \int \delta(\vec{x}') W_R(|\vec{x}' - \vec{x}|) d^3x'$$

Mean density, smoothed with window function

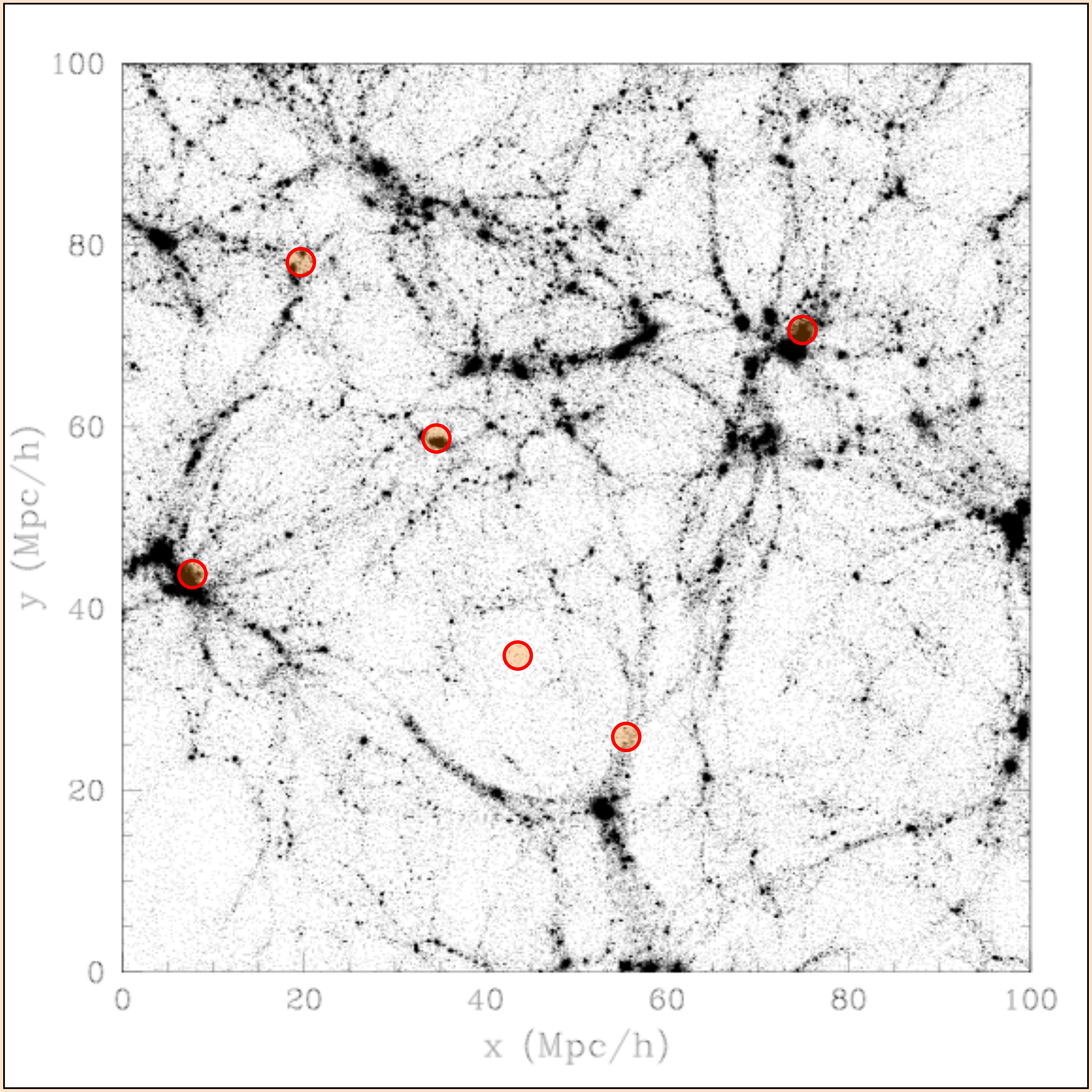
$$\bar{\delta}_R = \langle \delta_R(\vec{x}) \rangle = 0 \quad \text{since} \quad \delta = \frac{\rho - \bar{\rho}}{\bar{\rho}}$$

Variance of smoothed density field

$$\sigma_R^2 = \left\langle \delta_R(\vec{x})^2 \right\rangle$$

$$\sigma_R^2 = \int P(k) \tilde{W}_R^2(k) d^3k$$

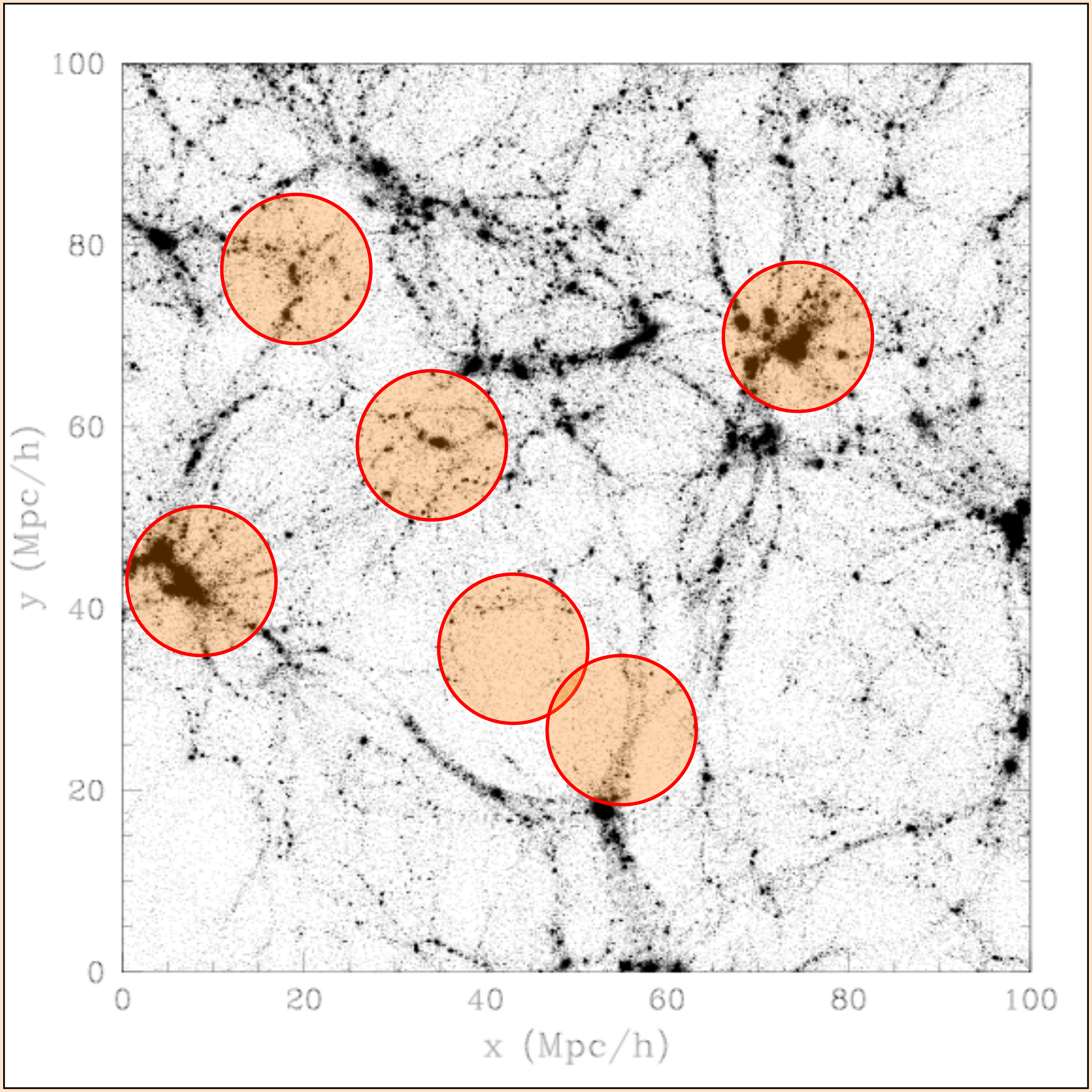
# Power Spectrum




R=2 Mpc/h

Top-hat filter

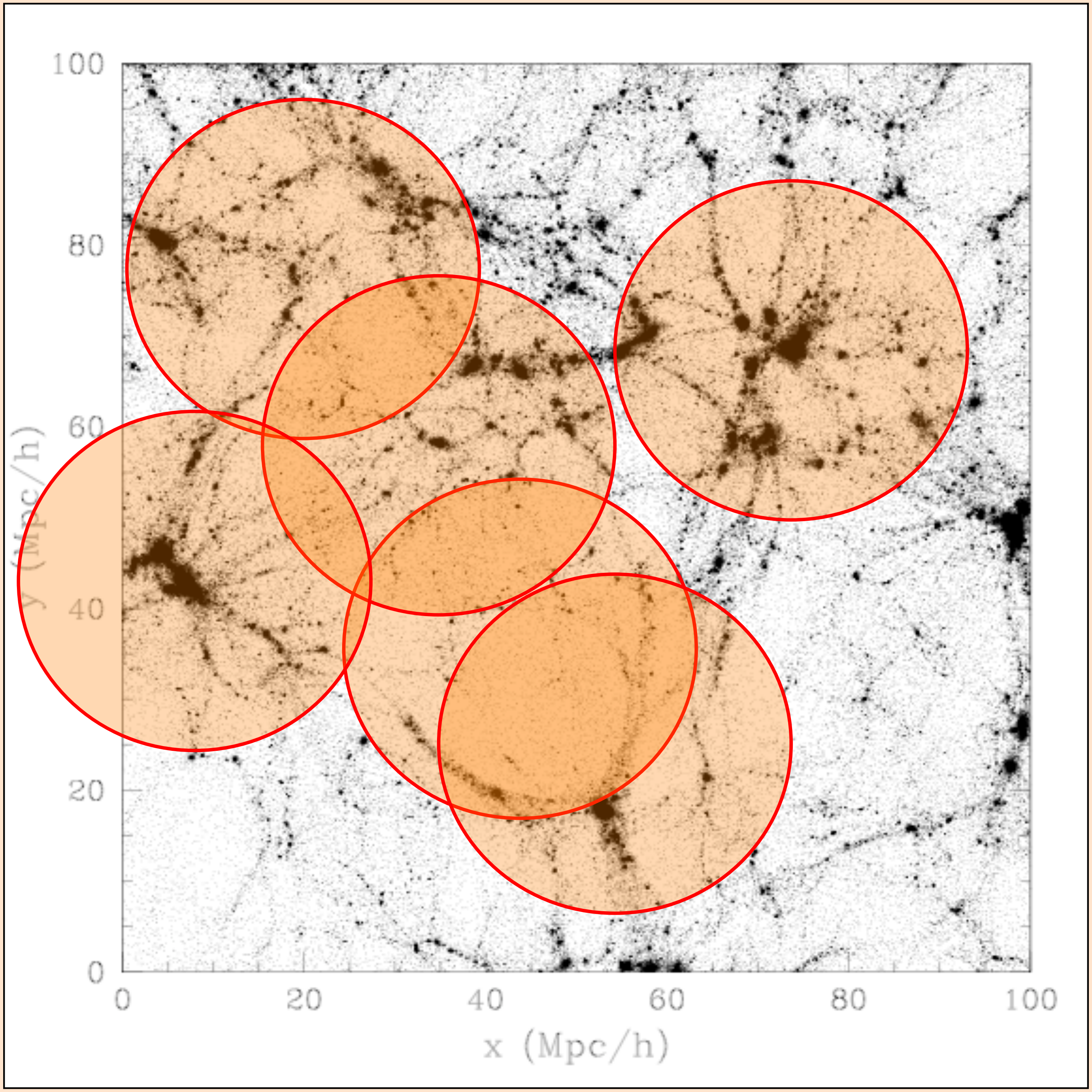
# Power Spectrum




R=8 Mpc/h

Top-hat filter

# Power Spectrum




R=20 Mpc/h

Top-hat filter

# Power Spectrum

$$\sigma_R^2 = \frac{1}{N}\sum\left(\delta_R - \cancelto{0}{\bar{\delta}_R}\right)^2 = \left\langle \delta_R^2 \right\rangle$$

The variance is large on small scales and approaches zero on large scales.

# Power Spectrum

$\sigma_R^2$ Is the variance of the matter density field

It also sets the amplitude of the matter power spectrum on scale *R*

$$\sigma_R^2 = \left\langle \delta_R(\vec{x})^2 \right\rangle$$

$$\sigma_R^2 = \int P(k)\tilde{W}_R^2(k)d^3k$$

For a power spectrum with a power-law shape $P(k) \sim k^n$, defining the variance on one scale sets the amplitude on all scales. Also, any window function will do.

We choose a top-hat filter of *R*=8 Mpc/h to describe the amplitude of P(k)