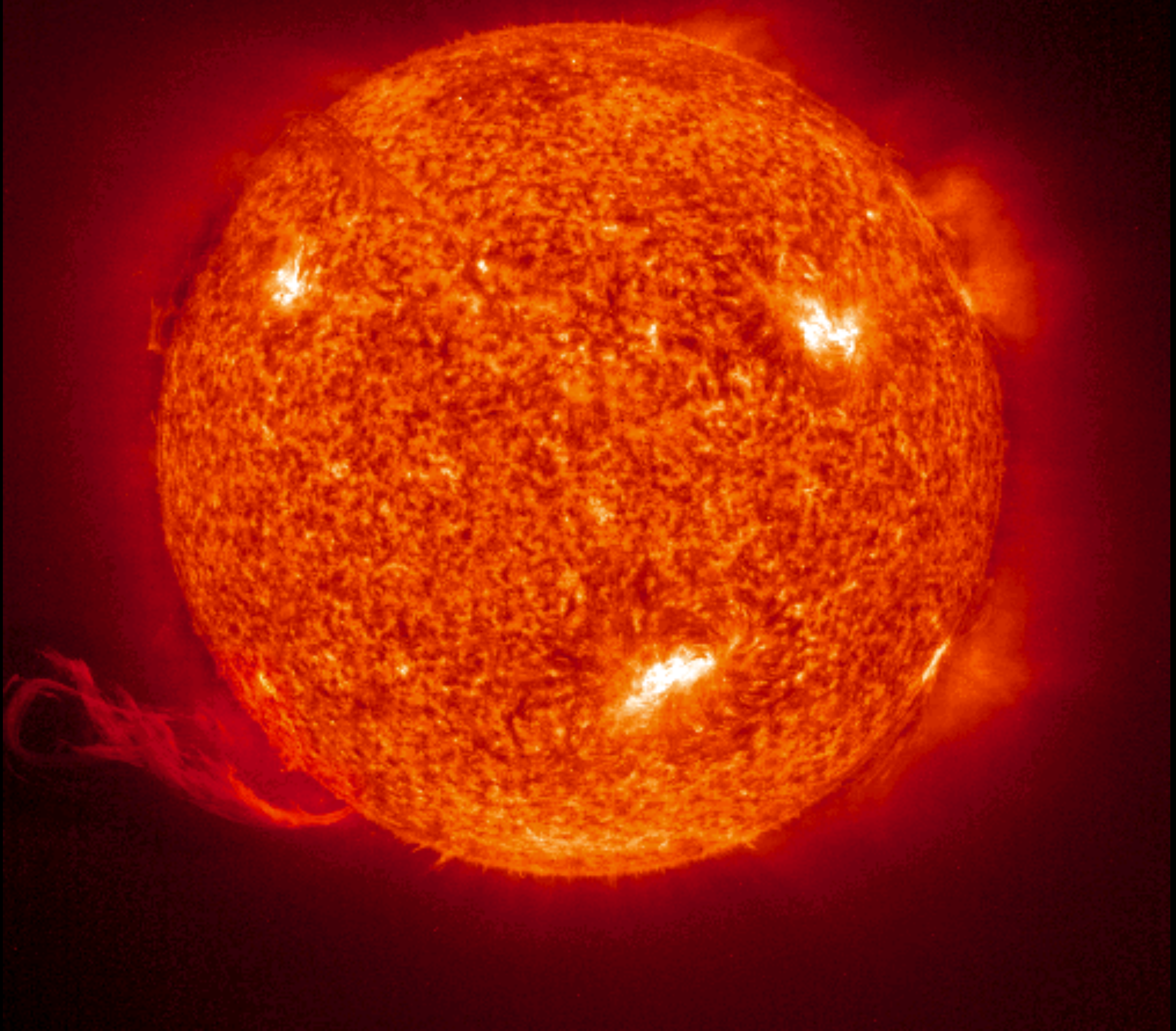
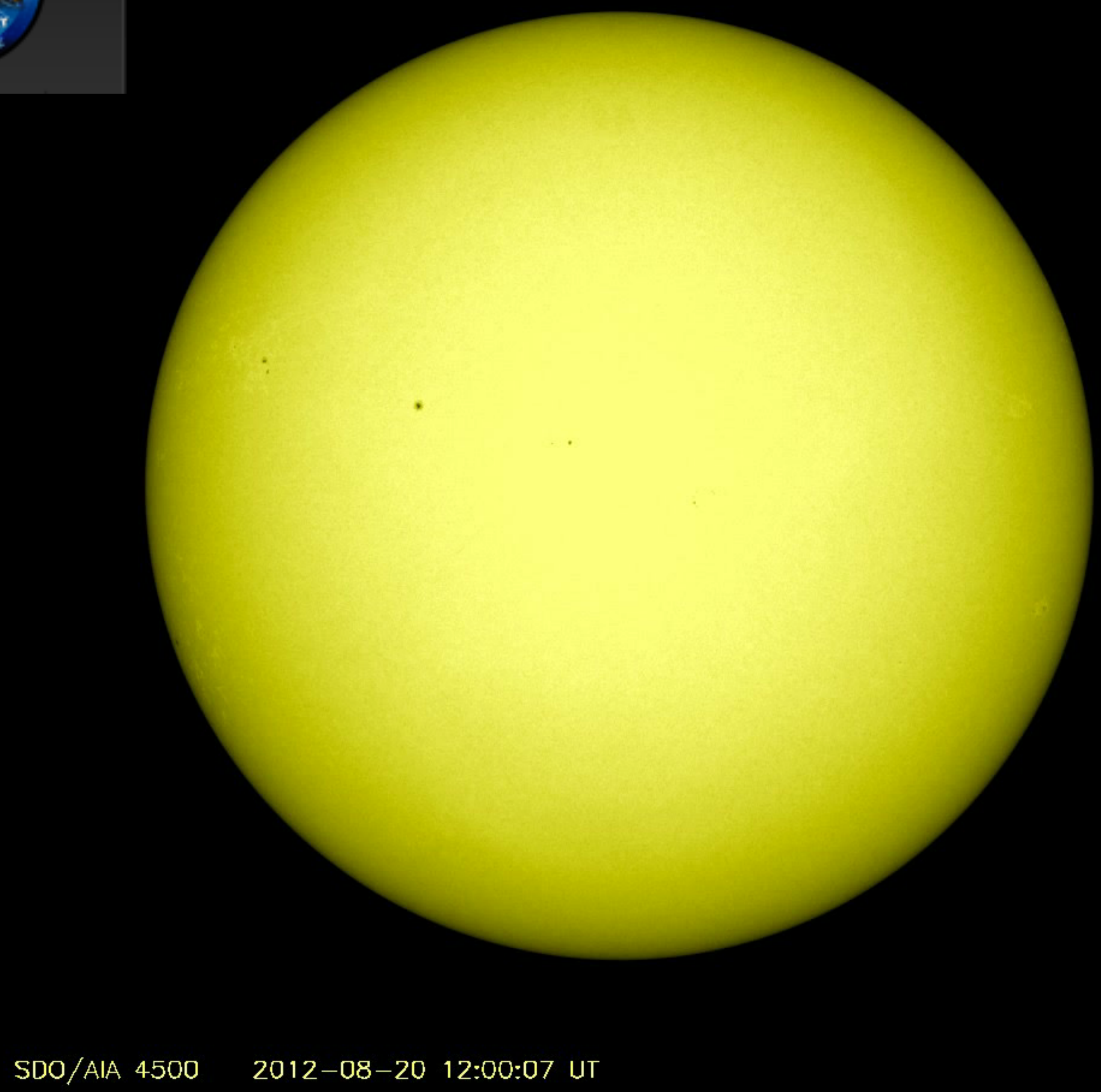
# ASTR 8030/3600 Stellar Astrophysics



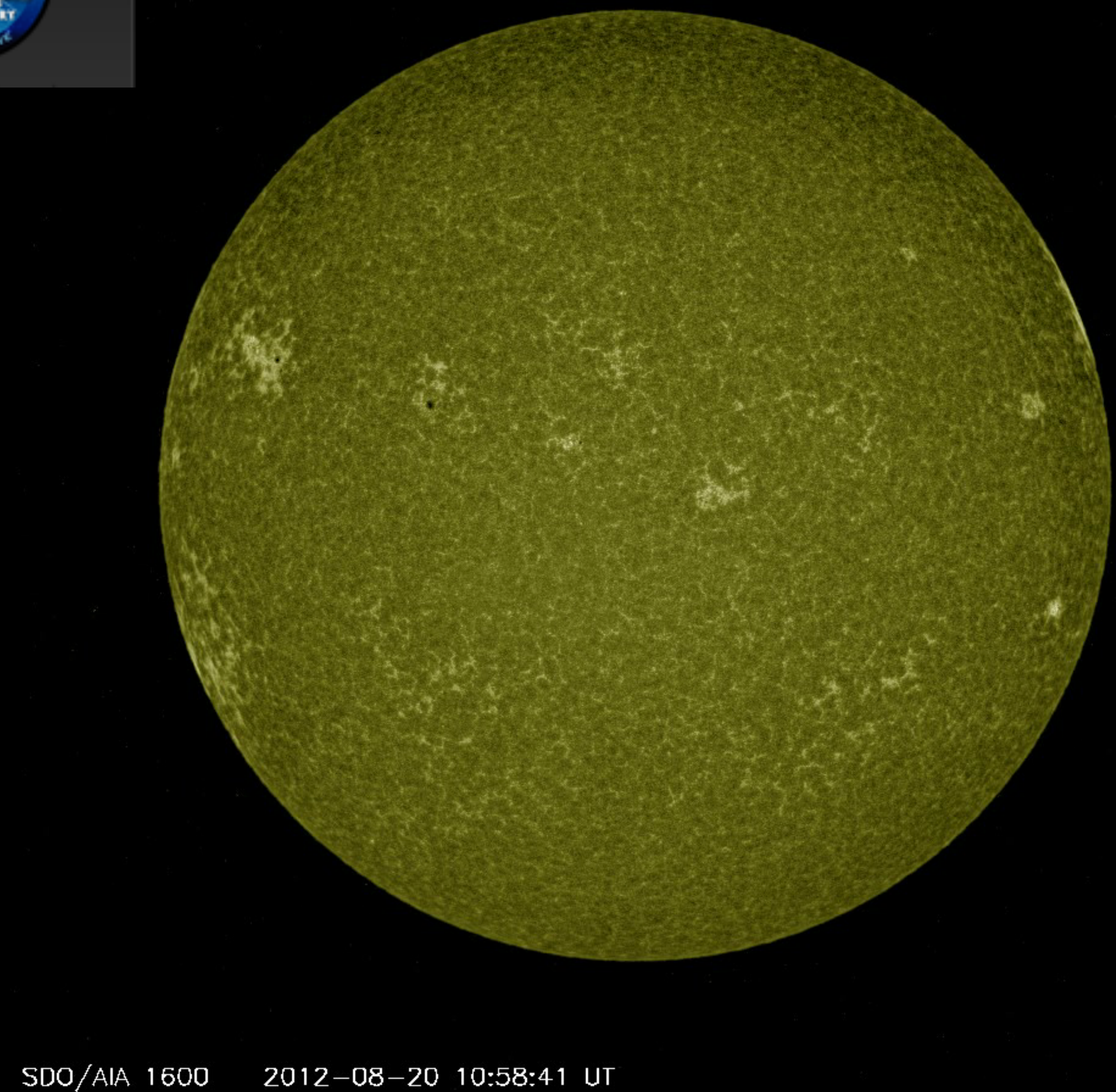
# SDO 4500 Angstroms: photosphere

T~5000K



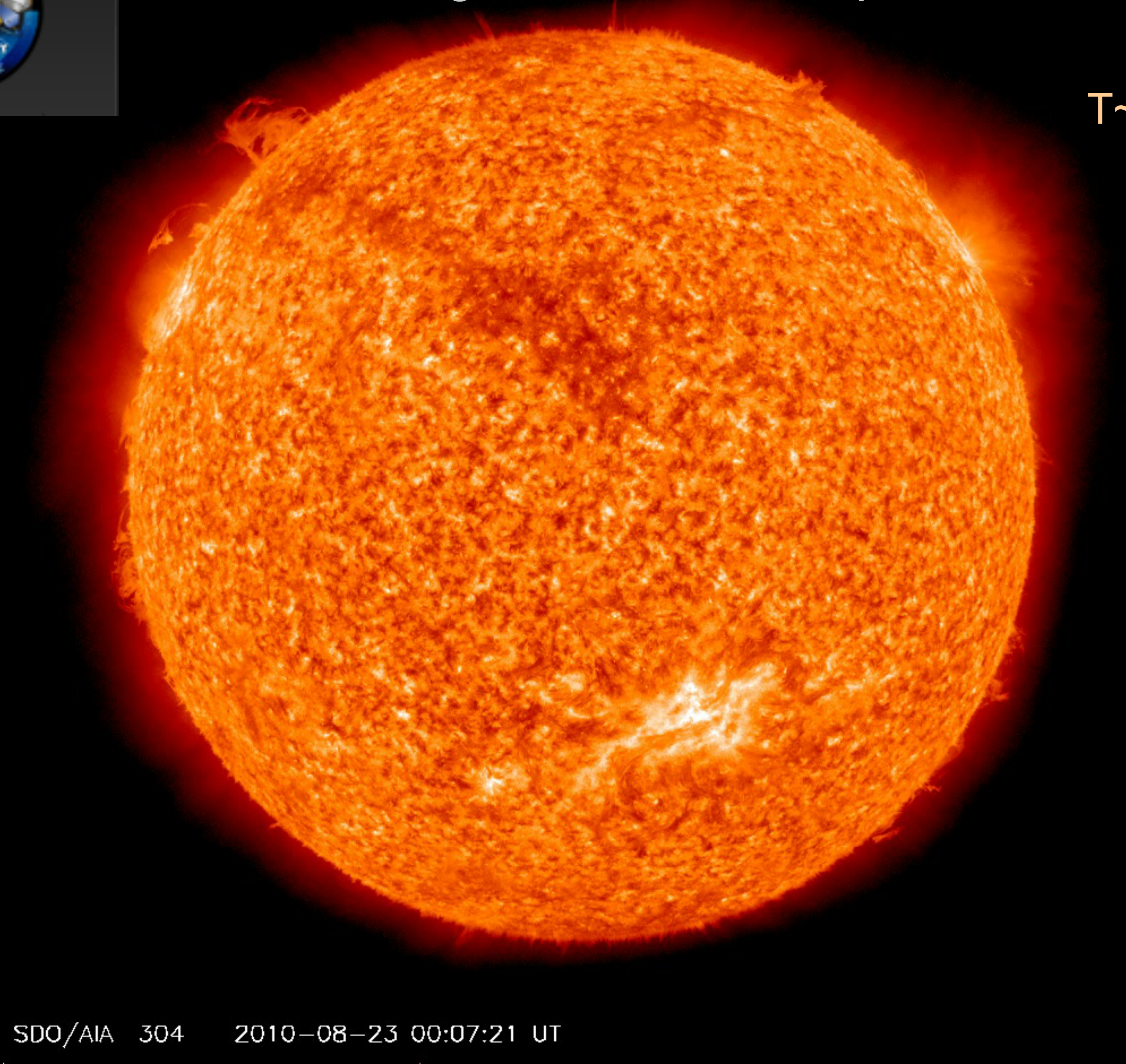
# SDO 1600 Angstroms: upper photosphere

T~5x10$^4$K



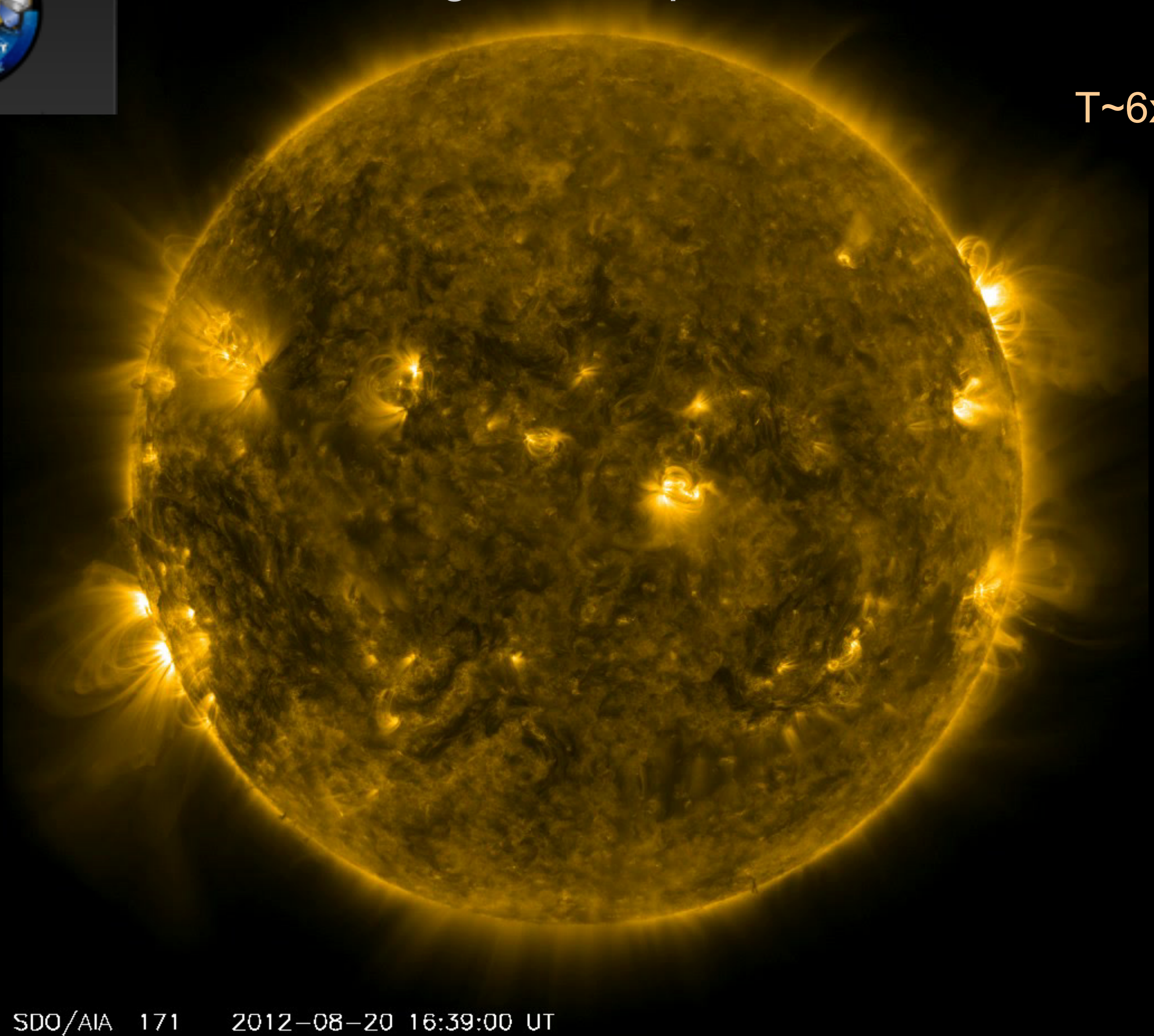
# SDO 304 Angstroms: chromosphere

$T \sim 10^5 K$



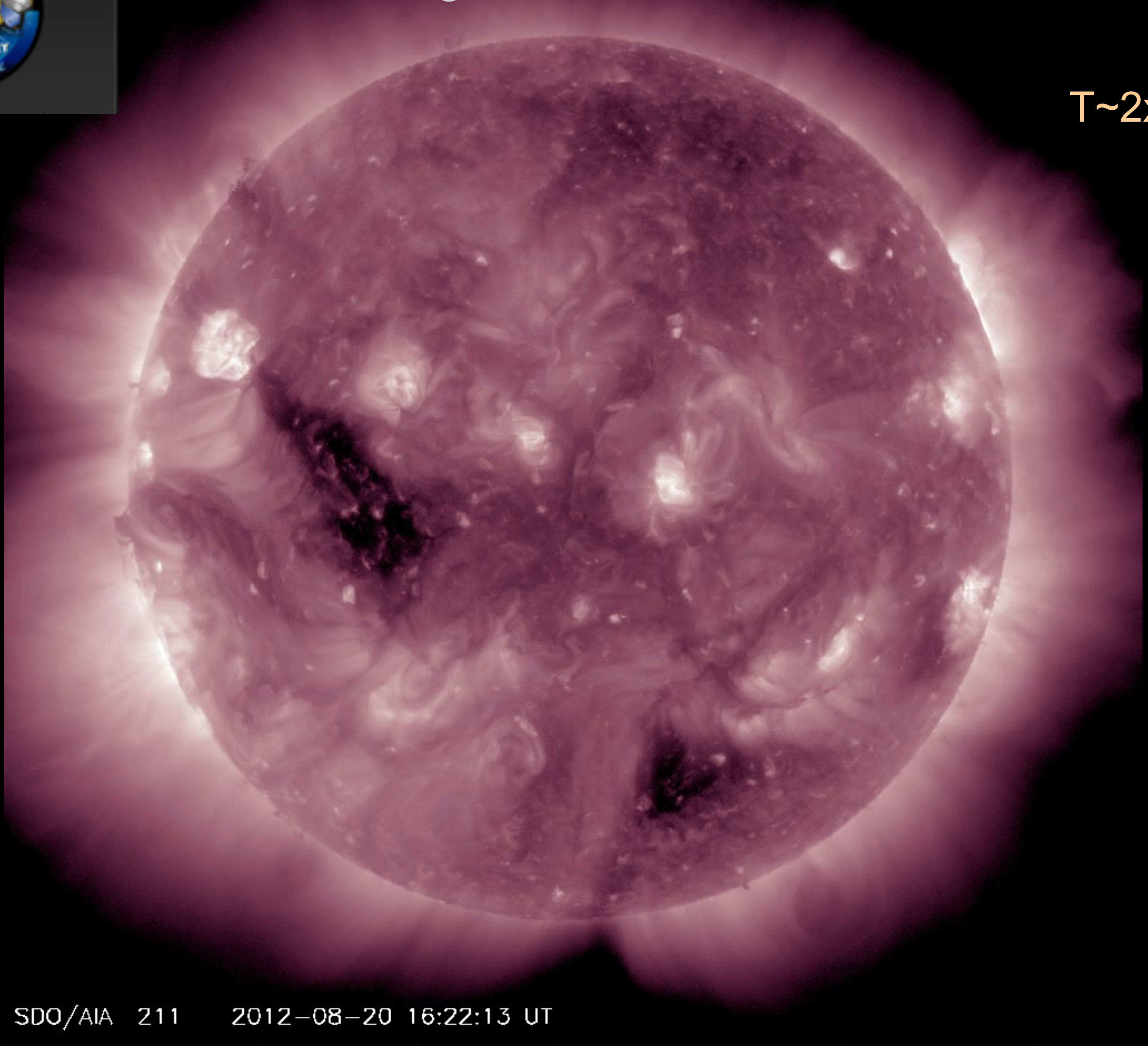
# SDO 171 Angstroms: quiet corona

$T \sim 6 \times 10^5 K$



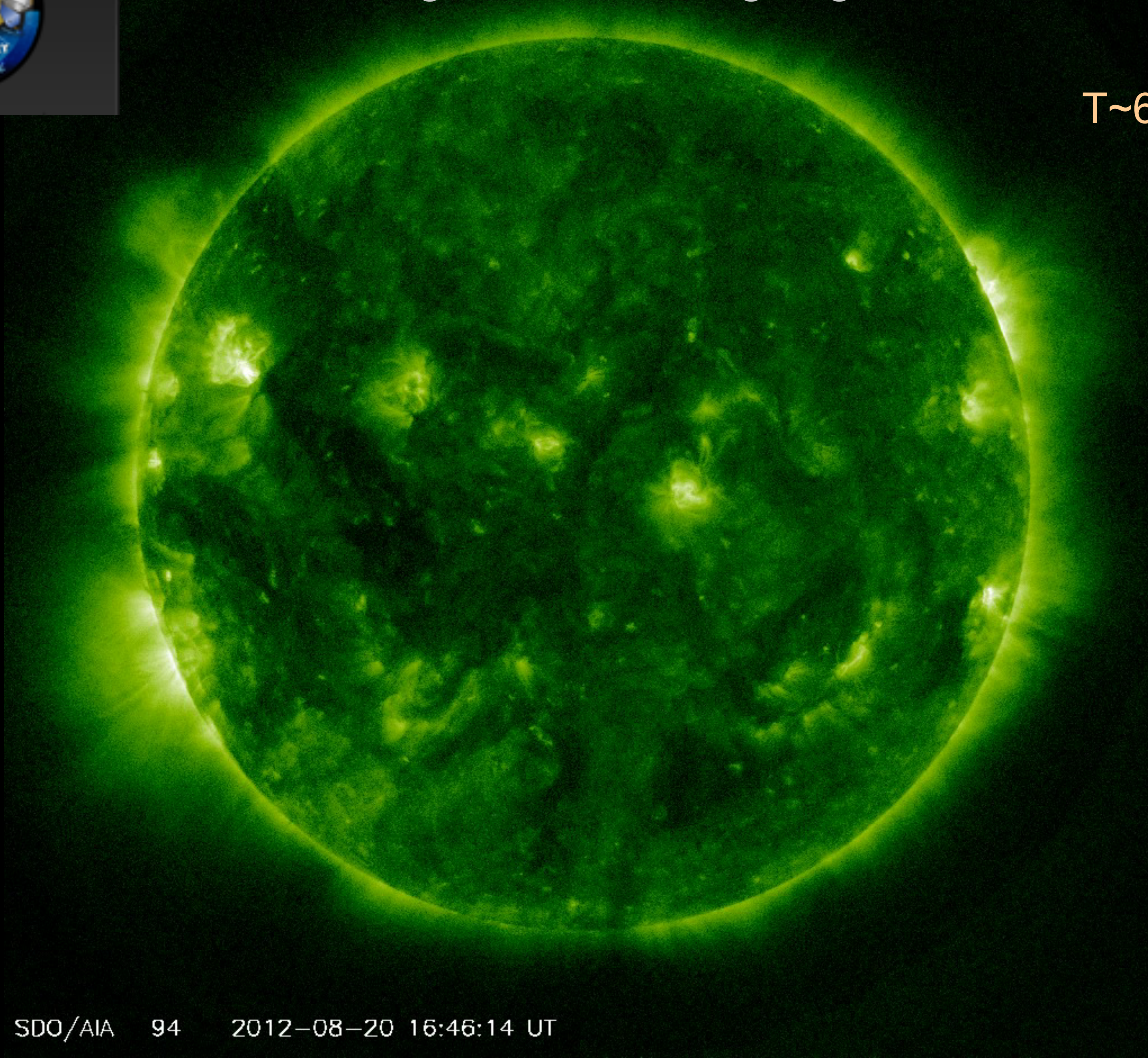
# SDO 211 Angstroms: active corona

T~2x$10^6$K

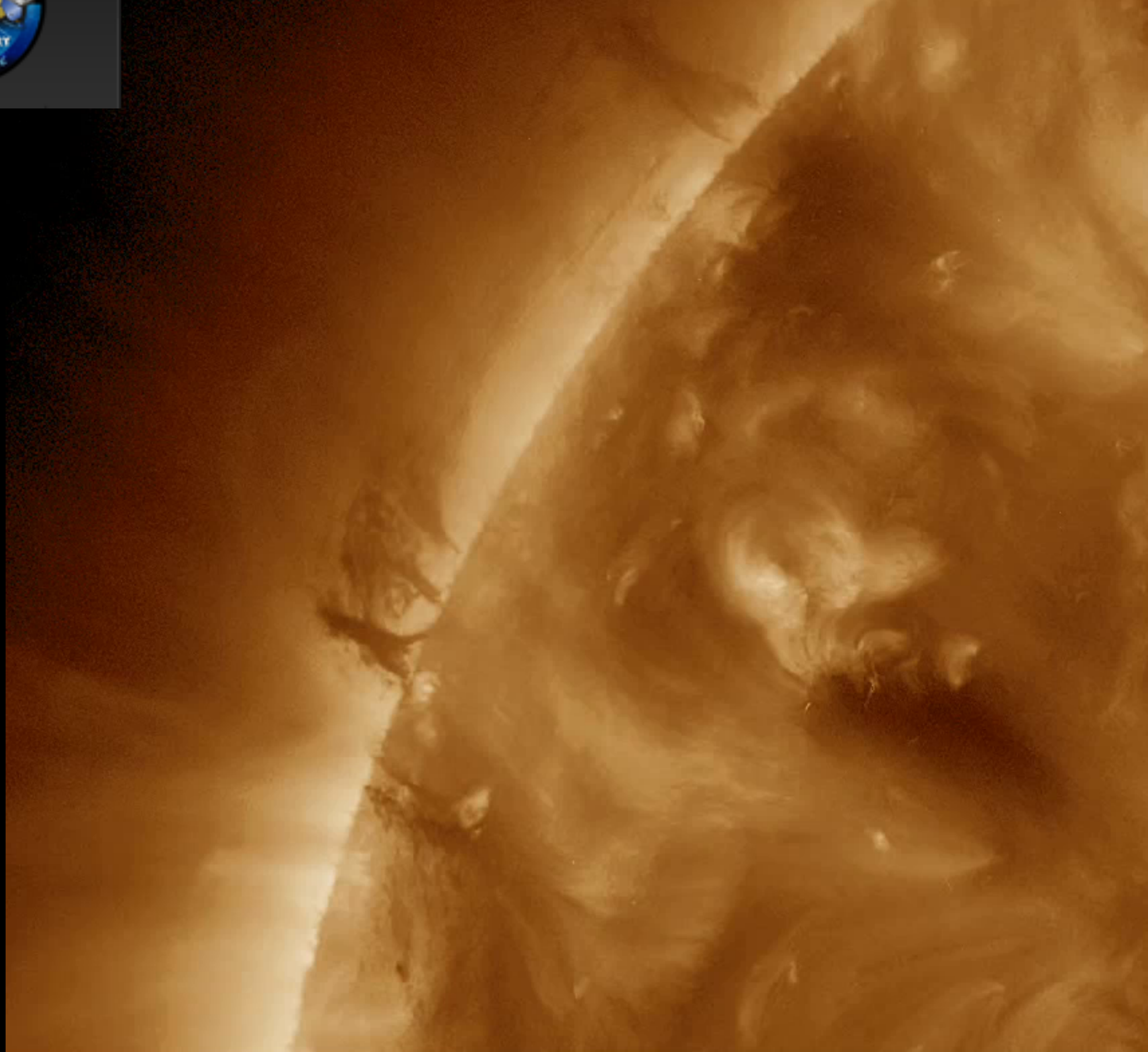


# SDO 94 Angstroms: flaring regions

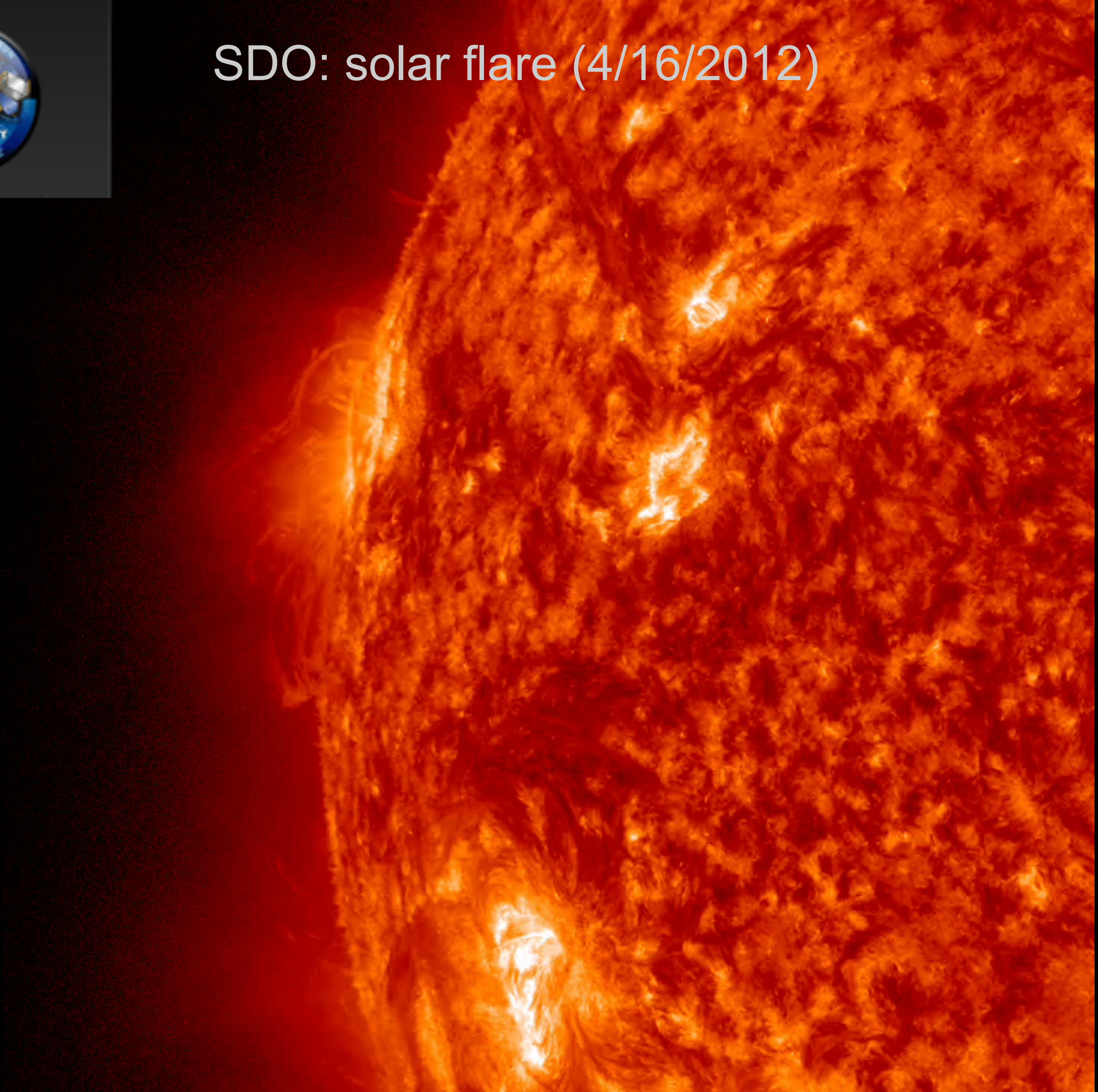
T~6x10$^6$K



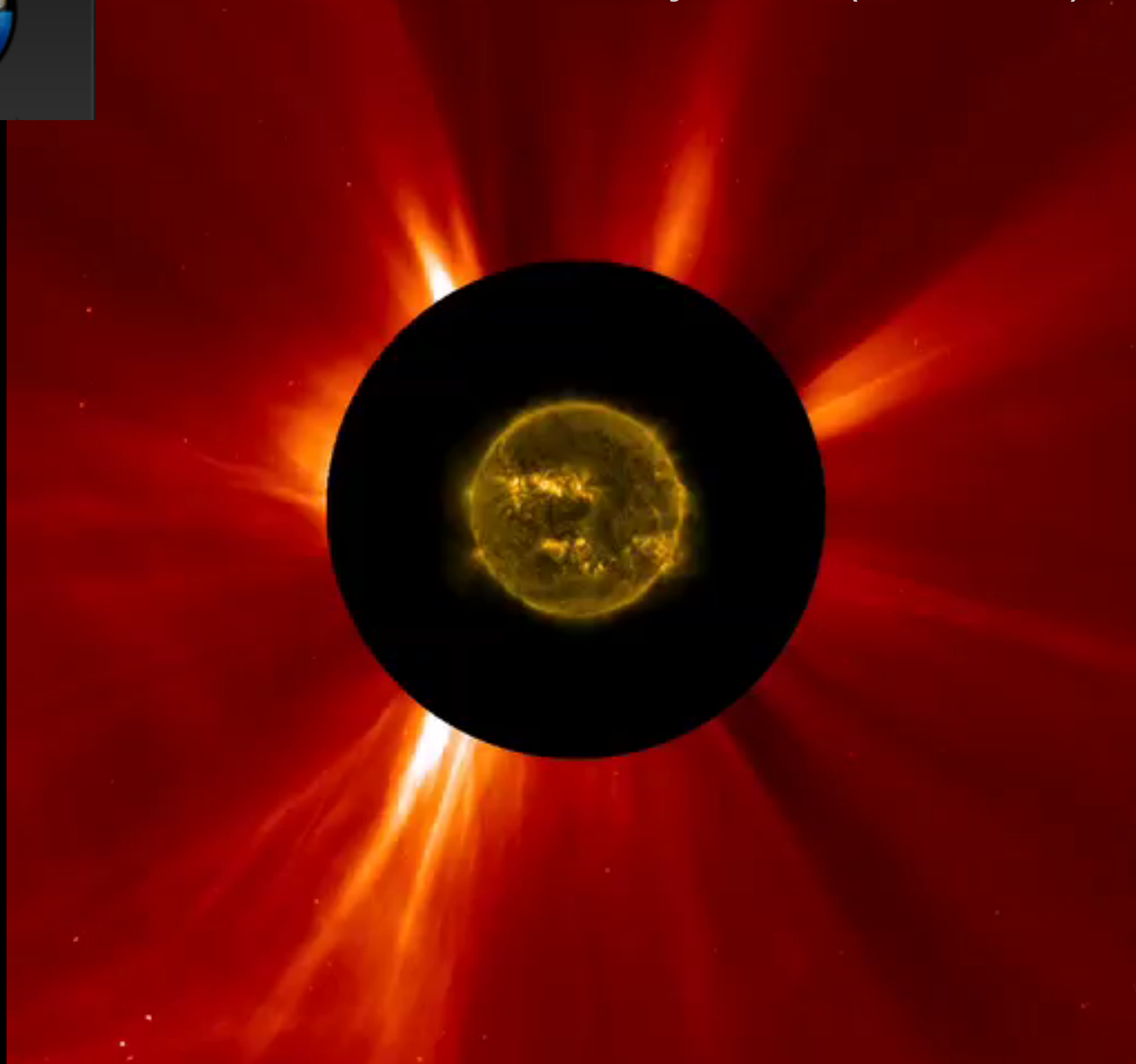
# SDO: dark plasma (3/27/2012)



# SDO: solar flare (4/16/2012)



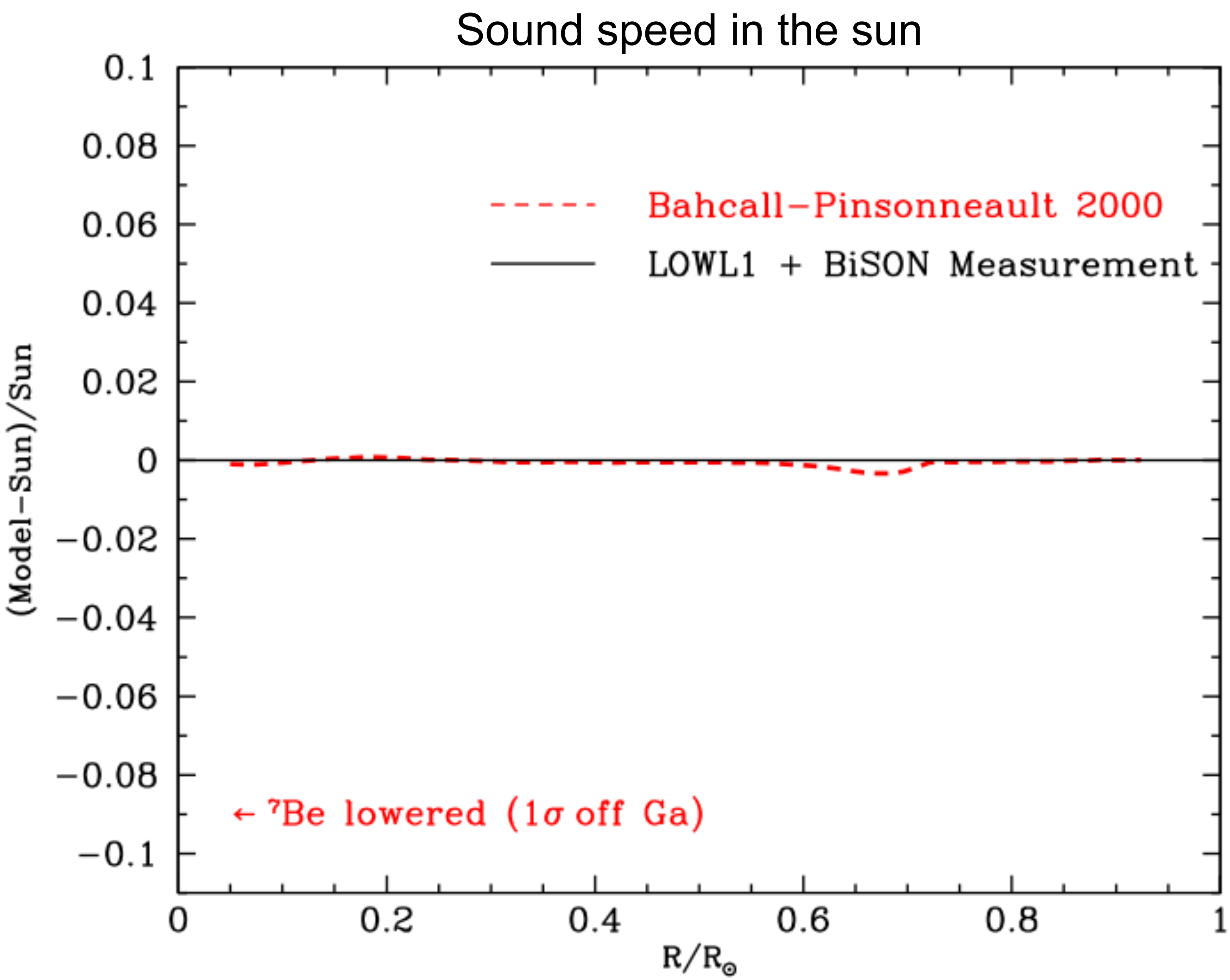
# SDO: coronal mass ejection (7/2/2012)



# Aims of the course

- Introduce the equations needed to model the internal structure of stars.
- Overview of how basic stellar properties are observationally measured.
- Study the microphysics relevant for stars: the equation of state, the opacity, nuclear reactions.
- Examine the properties of simple models for stars and consider how real models are computed.
- Survey (mostly qualitatively) how stars evolve, and the endpoints of stellar evolution.

# Stars are relatively simple physical systems

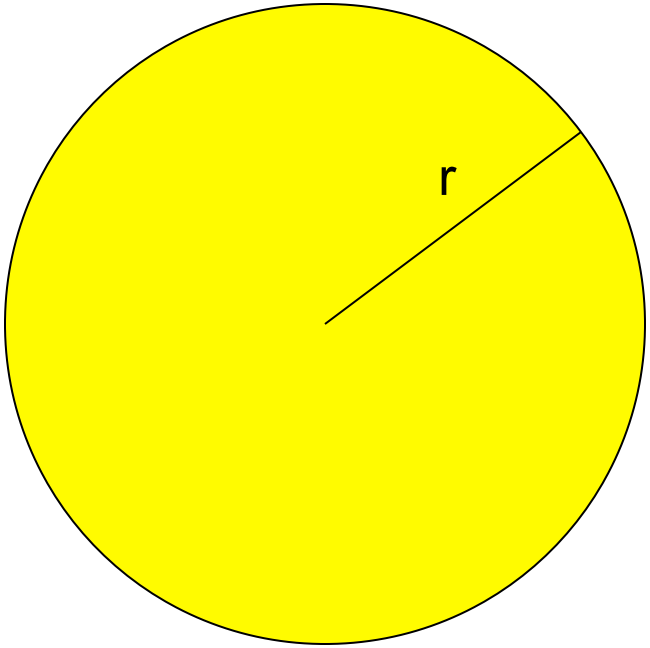


# Problem of Stellar Structure

We want to determine the structure (density, temperature, energy output, pressure as a function of radius) of an isolated mass M of gas with a given composition (e.g., H, He, etc.)

<u>Known:</u>

Mass
+
Composition




<u>Unknown:</u>

Density
Temperature
Energy
Pressure

# Simplifying assumptions

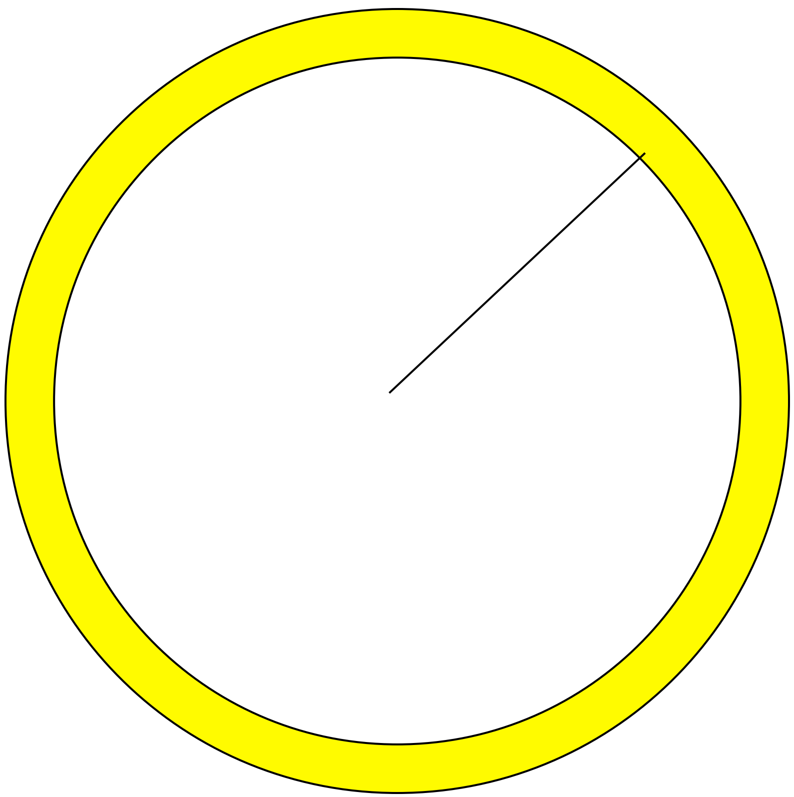
1. No rotation → spherical symmetry ✔
   For sun: rotation period at surface ~ 1 month
   orbital period at surface ~ few hours

2. No magnetic fields ✔
   For sun: magnetic field ~ 5G, ~ 1KG in sunspots
   equipartition field ~ 100 MG
   Some neutron stars have a large fraction of their energy in B fields

3. Static ✔
   For sun: convection, but no large scale variability
   Not valid for forming stars, pulsating stars and dying stars.

4. Newtonian gravity ✔
   For sun: escape velocity ~ 600 km/s << c
   Not true for neutron stars

# The Variables of Stellar Structure



| | |
|---|---|
| $r$ | radius |
| $m(r)$ | enclosed mass |
| $\rho(r)$ | mass density |
| $P(r)$ | Pressure |
| $g(r)$ | gravity |
| $T(r)$ | Temperature |
| $L_r(r)$ | Luminosity flow |
| $X_i(r)$ | Composition |

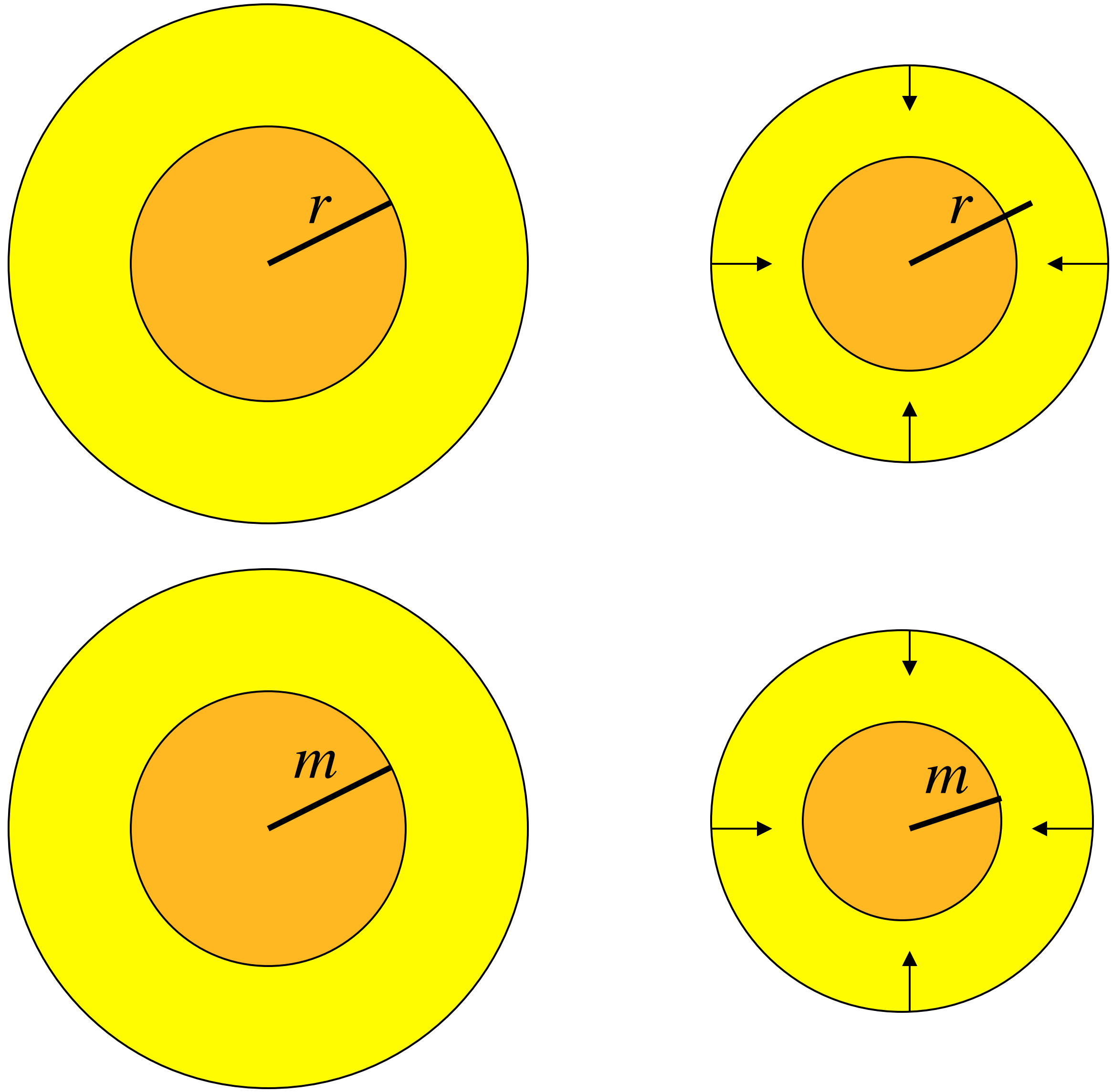
# Eulerian vs. Lagrangian

## Eulerian

Characterize quantities as a function of radius.

## Lagrangian

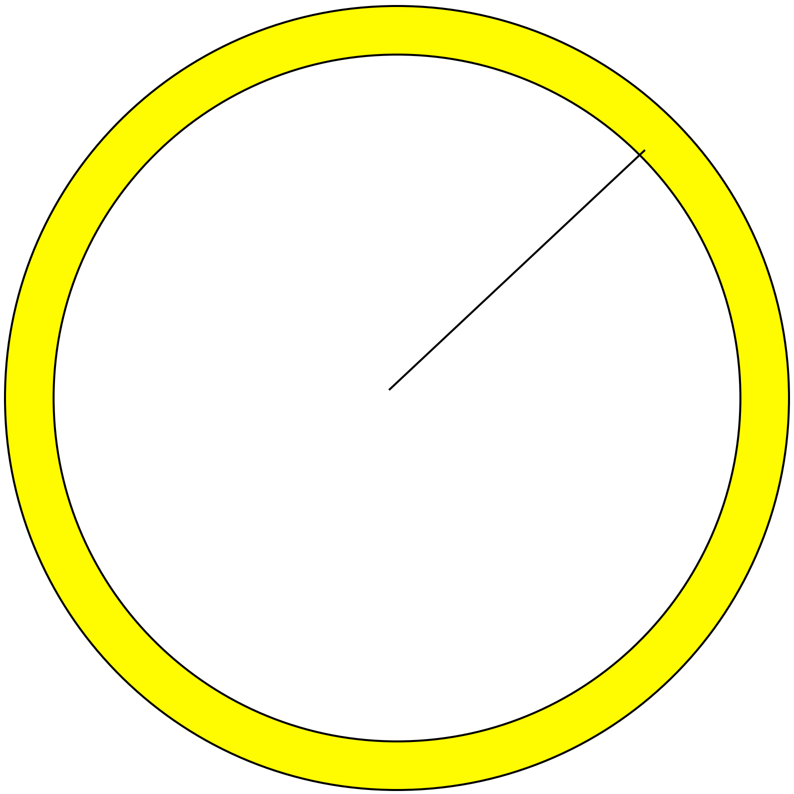
Characterize quantities as a function of enclosed mass.




time A

time B

# The Variables of Stellar Structure



| | |
|---|---|
| $m$ | enclosed mass |
| $r(m)$ | radius |
| $\rho(m)$ | mass density |
| $P(m)$ | Pressure |
| $g(m)$ | gravity |
| $T(m)$ | Temperature |
| $L_r(m)$ | Luminosity flow |
| $X_i(m)$ | Composition |

# Values for the Sun

$$\text{Mass} \equiv 1M_{\odot} \approx 2\times10^{33}\,\text{g}$$

$$\text{Radius} \equiv 1R_{\odot} \approx 7\times10^{10}\,\text{cm}$$

$$\text{Luminosity} \equiv 1L_{\odot} \approx 4\times10^{33}\,\text{erg/s}$$

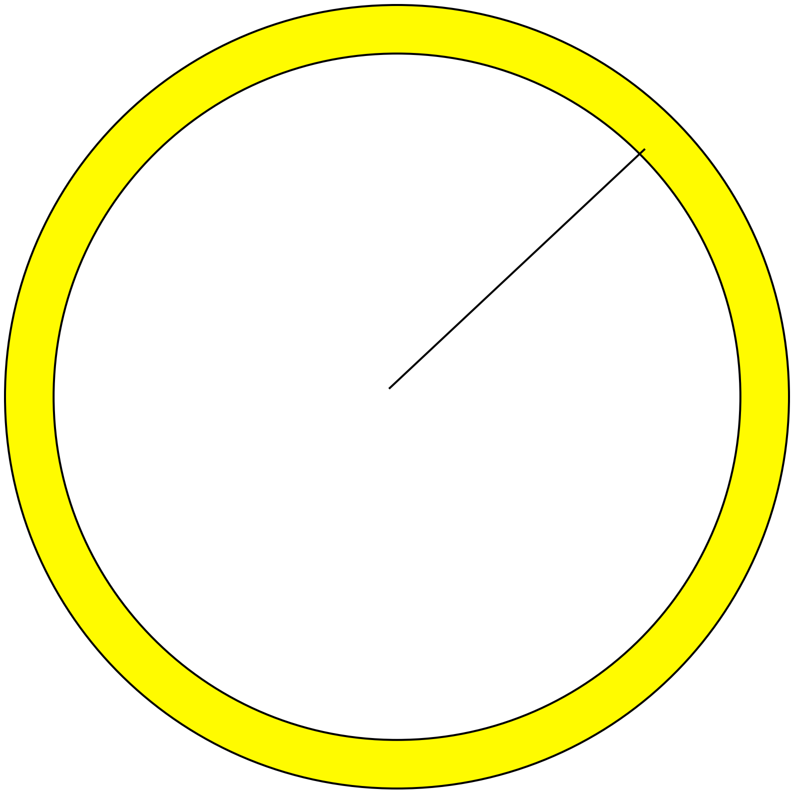
$$\text{Surface Temperature} \approx 5{,}800\ \text{K}$$

$$\text{Composition} \approx 70\%\ \text{H}\quad 28\%\ \text{He}\quad 2\%\ \text{metals}$$

# The Equations of Stellar Structure

mass in shell = density $\times$ volume of shell

$$dm = \rho \times 4\pi r^2 dr$$



# The Equations of Stellar Structure

## 1. Mass Conservation

$$\frac{dm}{dr} = 4\pi r^2 \rho$$

*eulerian*

$$\frac{dr}{dm} = \frac{1}{4\pi r^2 \rho}$$

*lagrangian*

# The Equations of Stellar Structure

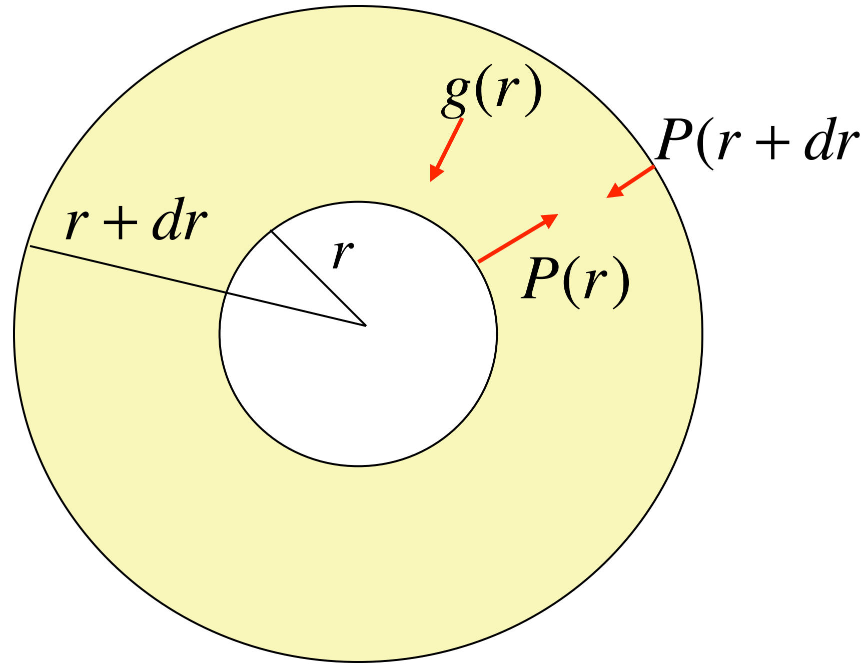
$$F = ma$$

Force pushing outwards:

$$P(r)4\pi r^2$$

Force pushing inwards:

$$-P(r+dr)4\pi r^2 - \frac{Gm}{r^2}dm$$




$$P(r)4\pi r^2 - P(r+dr)4\pi r^2 - \frac{Gm}{r^2}dm = \ddot{r}dm$$

$$\rightarrow -dP4\pi r^2 - \frac{Gm}{r^2}dm = \ddot{r}dm \quad \rightarrow -\frac{dP}{dm}4\pi r^2 - \frac{Gm}{r^2} = \ddot{r} \quad = 0$$

# The Equations of Stellar Structure

## 2. Hydrostatic Equilibrium

$$\frac{dP}{dr} = -\rho \frac{Gm}{r^2}$$

*eulerian*

$$\frac{dP}{dm} = -\frac{Gm}{4\pi r^4}$$

*lagrangian*

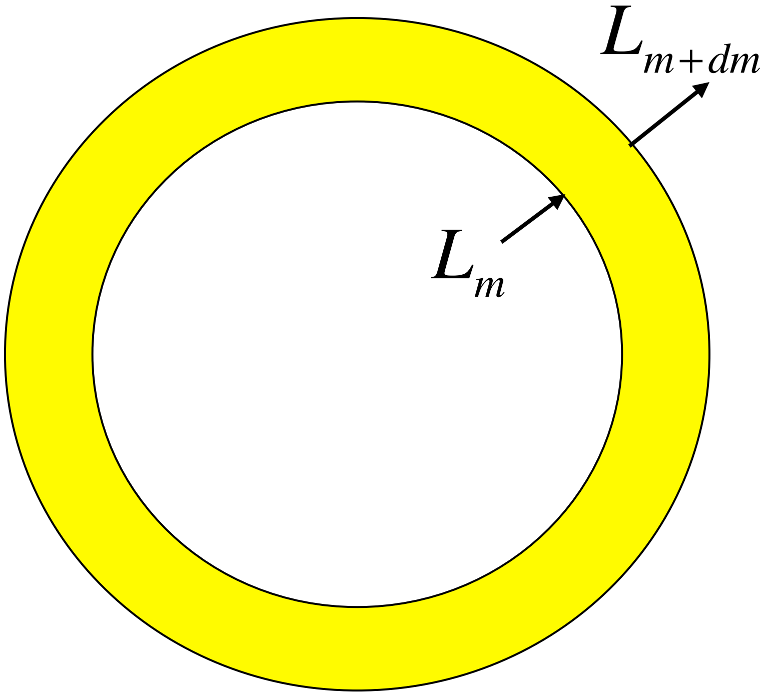
Need equation of state: $P = f\left(\rho, T, X_i\right)$

e.g., for an ideal gas: $P = \frac{R}{\mu}\rho T$

# The Equations of Stellar Structure

$\varepsilon$ = energy generation rate per unit mass (erg/s/g)

change in $L_m = \varepsilon \times dm$



# The Equations of Stellar Structure

## 3. Energy Generation

$$\frac{dL_r}{dr} = 4\pi r^2 \rho \varepsilon$$

*eulerian*

$$\frac{dL_m}{dm} = \varepsilon$$

*lagrangian*

Need nuclear physics: $\varepsilon = f\left(\rho, T, X_i\right)$

e.g., for the proton-proton chain: $\varepsilon \approx \varepsilon_0 \rho T^4$

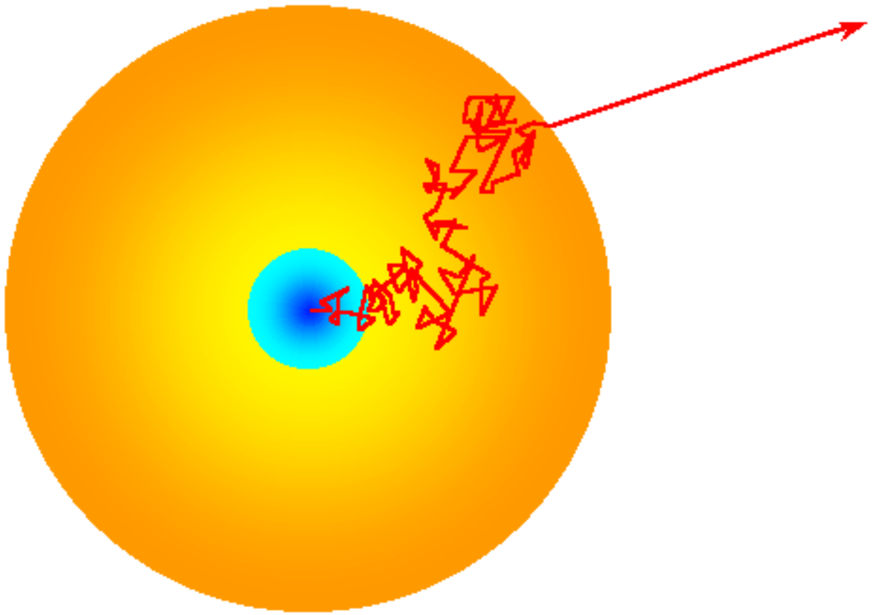
# The Equations of Stellar Structure

| | |
|---|---|
| <u>Energy flow</u>: | Depends on opacity $\kappa$ (area/mass) |
| • radiation | *low opacity* |
| • convection | *high opacity* |
| • conduction | *unimportant* |

<u>For radiation</u>:

radiation pressure decreases outward ➔
photons have net movement outward in their random walk.

# Random walk of photon through the sun.



Straight path: 2.3 seconds
Random walk: 30,000 years

# The Equations of Stellar Structure

## 4. Energy Flow (radiation)

$$\frac{dT}{dr} = -\frac{3\kappa\rho L_r}{16\pi acr^2 T^3}$$

*eulerian*

$$\frac{dT}{dm} = -\frac{3\kappa L_m}{64\pi^2 acr^4 T^3}$$

*lagrangian*

Need opacity: $\kappa = f(\rho, T, X_i)$

e.g., for electron Thomson scattering: $\kappa \approx \kappa_0$

# The Equations of Stellar Structure

## 4. Energy Flow (convection)

$$\frac{dT}{dr} = -\left(1 - \frac{1}{\Gamma_2}\right)\frac{Gm\rho T}{r^2 P}$$

*eulerian*

$$\frac{dT}{dm} = -\left(1 - \frac{1}{\Gamma_2}\right)\frac{GmT}{4\pi r^4 P}$$

*lagrangian*

Need adiabatic index: $\Gamma_2 = f\left(\rho, T, X_i\right)$

e.g., for a simple ideal gas: $\Gamma_2 = 5/3$

# Solving the Equations

Need microphysics: $P(\rho, T, X_i)$, $\varepsilon(\rho, T, X_i)$, $\kappa(\rho, T, X_i)$

$$\frac{dr}{dm} = \frac{1}{4\pi r^2 \rho}$$

$$\frac{dP}{dm} = -\frac{Gm}{4\pi r^4}$$

$$\frac{dL_m}{dm} = \varepsilon$$

$$\frac{dT}{dm} = -\frac{3\kappa L_m}{64\pi^2 acr^4 T^3}$$

Solve in 1 dimension from center to surface

# Boundary Conditions

$$\text{Center } (r=0): \quad m=0, \quad L_r=0, \quad P=P_c, \quad T=T_c$$

$$\text{Surface } (r=R): \quad m=M, \quad L_r=L, \quad P\sim 0, \quad T\sim 0$$

## Lagrangian

$$\text{Center } (m=0): \quad r=0, \quad L_r=0, \quad P=?, \quad T=?$$

$$\text{Surface } (m=M): \quad r=?, \quad L_r=?, \quad P\sim 0, \quad T\sim 0$$

Boundary conditions are incomplete at each end

# Complications

1. Stars are luminous
   radiate away energy → must change in time
   chemical composition is changing. Must account for $dX_i/dt$

2. Convection is very complicated and important

3. Convection can change chemical composition

4. Opacities are hard to calculate. And they matter!

# Homology Relations

To calculate the luminosity or radius of a star of mass *M*, we must solve these differential equations. However, we can get approximate scaling relations by using *Homology.*

Assume that each differential or local quantity simply scales with the global value of that quantity.

For example,

$$\frac{dr}{dm} \sim \frac{R}{M}$$

So,

$$\boxed{\frac{dr}{dm} = \frac{1}{4\pi r^2 \rho}}$$

becomes

$$\boxed{\frac{R}{M} \sim \frac{1}{R^2 \overline{\rho}}}$$

# Homology Relations

If we also know how pressure, opacity, and nuclear generation rate scale with density and temperature, we can solve all these equations to get scaling relations. Then we can turn these scaling relations into actual equations by normalizing to the Sun.

For example, suppose we find that $L \sim M^{\alpha}$

This means that $L = \text{const} \times M^{\alpha}$

For the Sun, this is $L_{\odot} = \text{const} \times M_{\odot}^{\alpha}$

Dividing the two equations we get

$$\frac{L}{L_{\odot}} = \left( \frac{M}{M_{\odot}} \right)^{\alpha}$$