Determining stellar properties

Measurements:

- position on sky
- flux in different bands
- spectrum
- time dependence of above

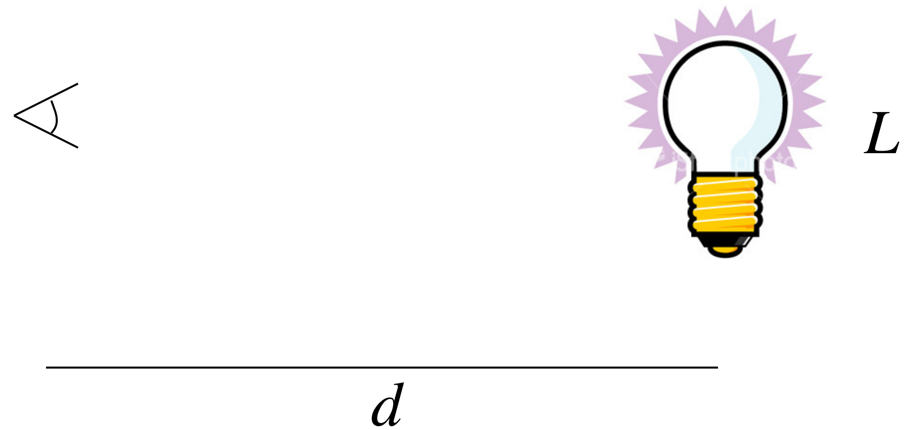
Physical parameters:

- distance d
- luminosity L
- temperature T_e
- radius R
- mass M
- age t
- chemical composition X_i

Determining distance

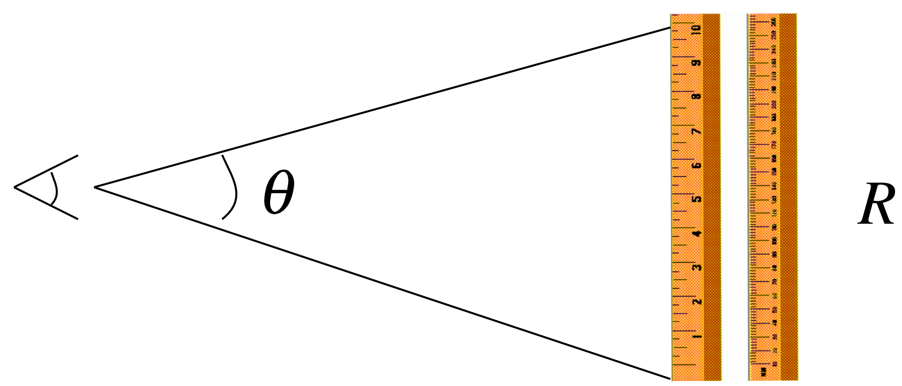
Standard candle

$$d = \left(\frac{L}{4\pi f} \right)^{\frac{1}{2}}$$



Standard ruler

$$d = \frac{R}{\theta}$$

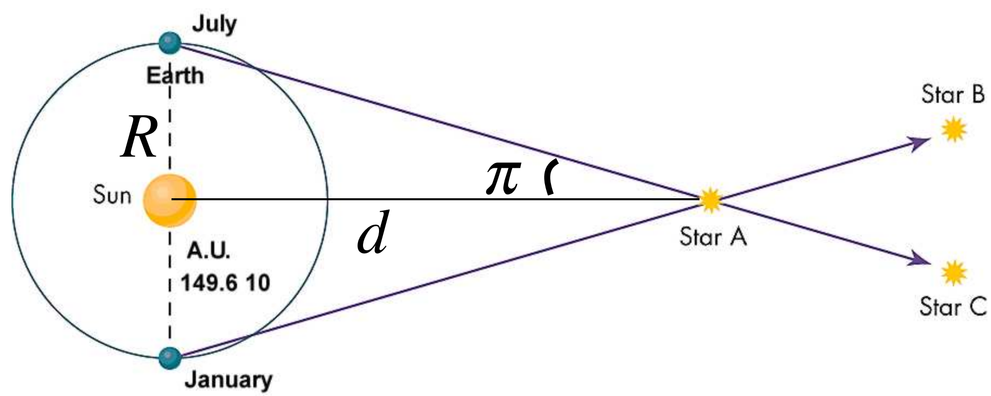


Determining distance: Parallax

RULER

$$\tan\pi = \frac{R}{d} \approx \pi$$

$$R = 1AU = 1.5\times 10^{13}\,cm$$

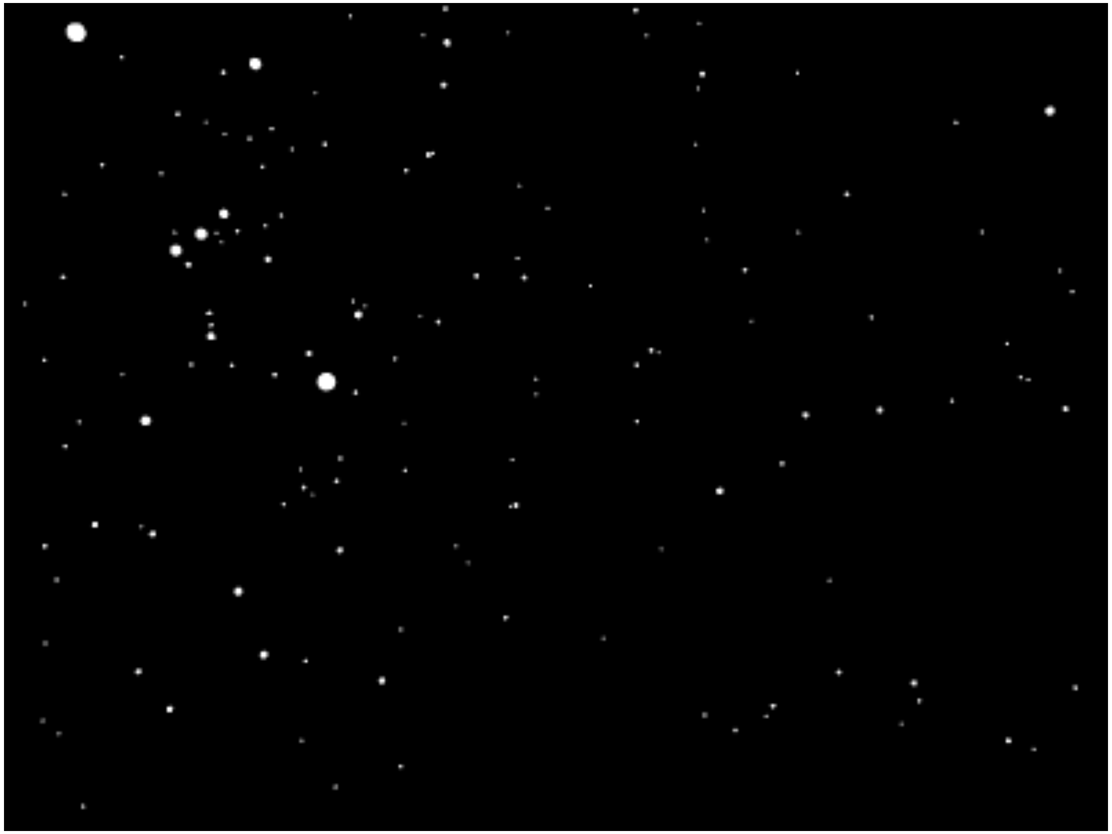


Define new distance unit: parsec (**par**allax-**sec**ond)

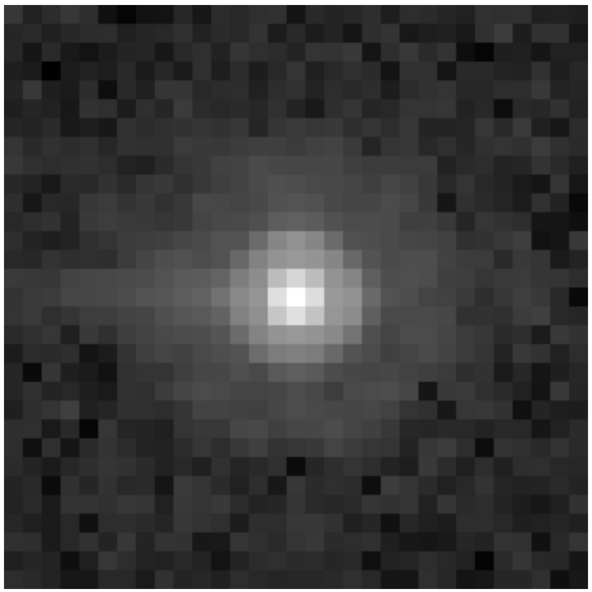
$$1pc = \frac{1AU}{\tan(1'')} = 206{,}265\,AU = 3.26\,ly$$

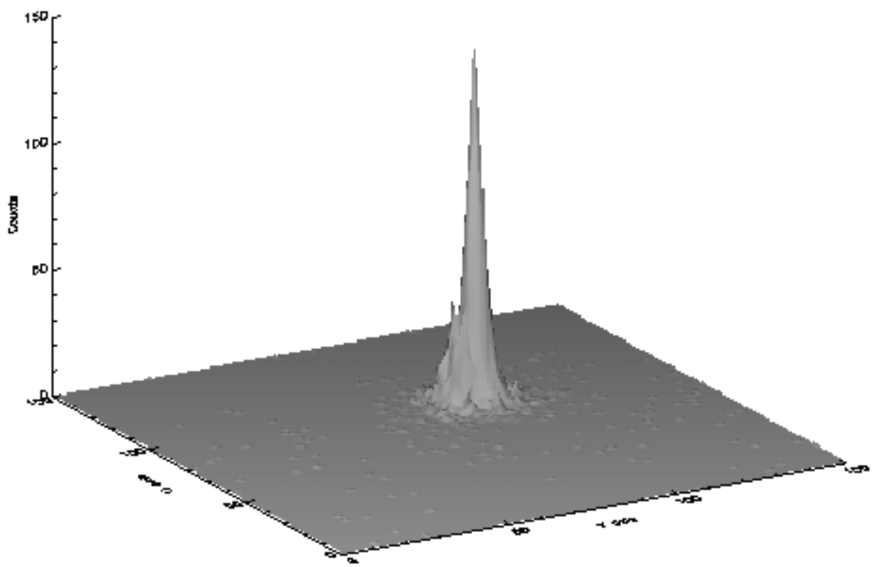
$$\boxed{\left(\frac{d}{1pc}\right) = \frac{1}{\pi''}}$$

Determining distance: Parallax



Determining distance: Parallax





Point spread function (PSF)

Determining distance: Parallax

Need high angular precision to probe far away stars.

$$\boxed{d = \frac{1}{\pi}}$$

Error propagation:

$$\sigma_d = \sqrt{\left(\frac{\partial d}{\partial \pi}\right)^2 \sigma_\pi^2} = \sqrt{\left(-\frac{1}{\pi^2}\right)^2 \sigma_\pi^2} = \frac{\sigma_\pi}{\pi^2} = \frac{\sigma_\pi}{\pi} d$$

$$\boxed{\frac{\sigma_d}{d} = \frac{\sigma_\pi}{\pi}}$$

At what distance do we get a given fractional distance error?

$$d = \left(\frac{\sigma_d}{d}\right)\frac{1}{\sigma_\pi}$$

Determining distance: Parallax

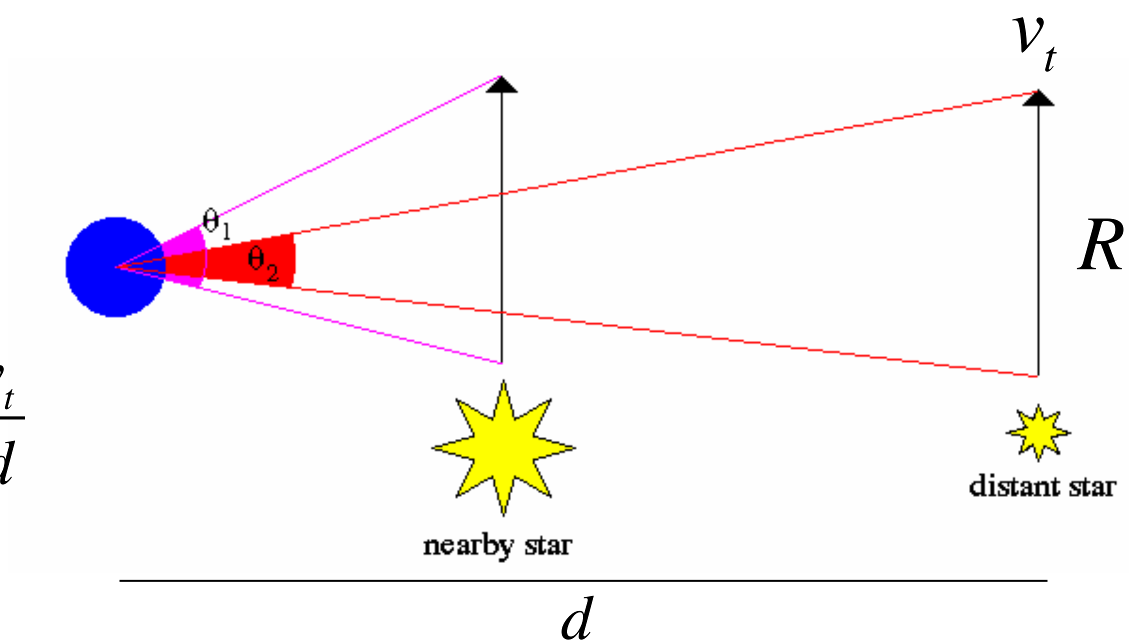
e.g., to get 10% distance errors

$$d_{\max} = \frac{0.1}{\sigma_\pi}$$

Mission	Dates	σ_π	$d_{\max}$
Earth telescope		$\sim 0.1\ as$	$1\ pc$
HST		$\sim 0.01\ as$	$10\ pc$
Hipparcos	1989-1993	$\sim 1\ mas$	$100\ pc$
Gaia	2013-2018	$\sim 20\ \mu as$	$5\ kpc$
SIM	cancelled	$\sim 4\ \mu as$	$25\ kpc$

Determining distance: moving cluster method

Proper motion



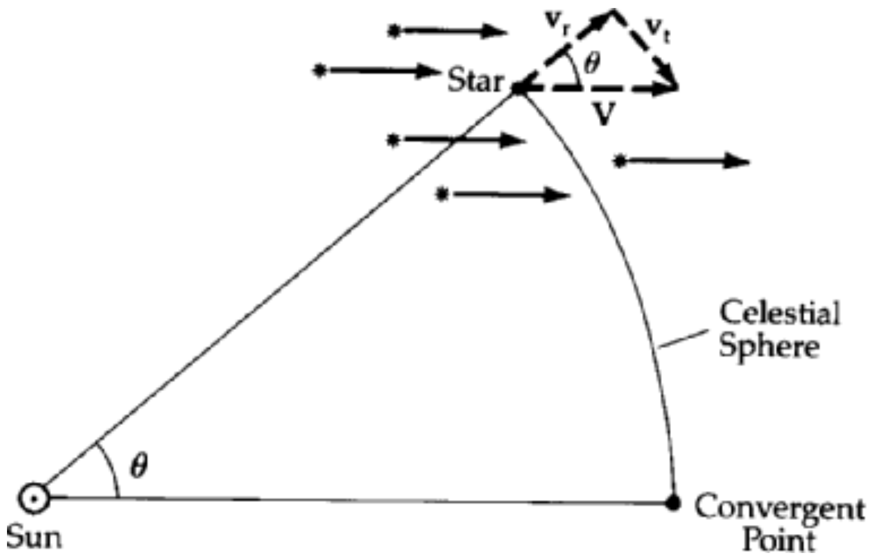
$$\mu = \frac{d\theta}{dt} = \frac{d}{dt}\left(\frac{R}{d}\right) = \frac{v_t}{d}$$

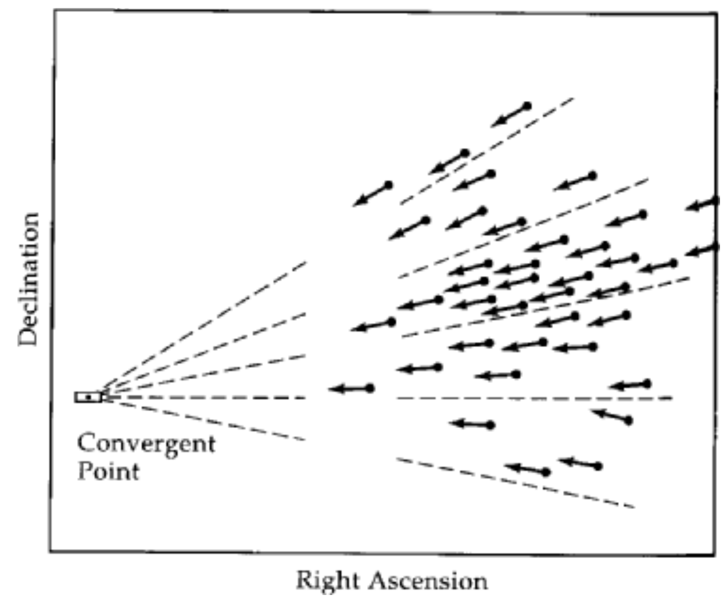
$$d = \frac{v_t}{\mu}$$

$$\left(\frac{d}{1pc}\right) = \frac{\left(v_t / 1\ \mathrm{kms}^{-1}\right)}{4.74\left(\mu / 1''\mathrm{yr}^{-1}\right)}$$

Determining distance: moving cluster method

RULER





1. Measure proper motions of stars in a cluster
2. Obtain convergent point
3. Measure angle between cluster and convergent point θ
4. Measure radial velocity of cluster v_r
5. Compute tangential velocity v_t
6. Use proper motion of cluster to get distance d

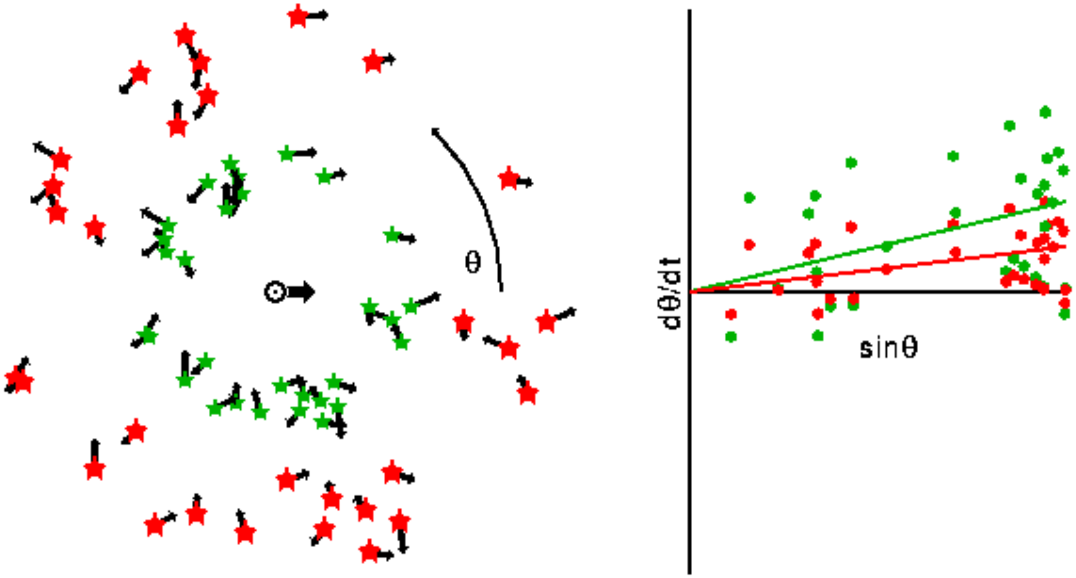
$$d = \frac{v_r \tan\theta}{4.74\mu}$$

Determining distance: secular parallax

- Parallax method is limited by 2AU baseline of earth's orbit
- Sun moves ~4AU/yr toward Vega relative to local rotation of Galactic disk
- Over a few years, this can build up to a large baseline
- Unfortunately, other stars are not at rest, rather have unknown motions
- However, if we average over many stars, their mean motion should be zero (relative to local rotation)
- Can therefore get the mean distance to a set of stars

Determining distance: secular parallax

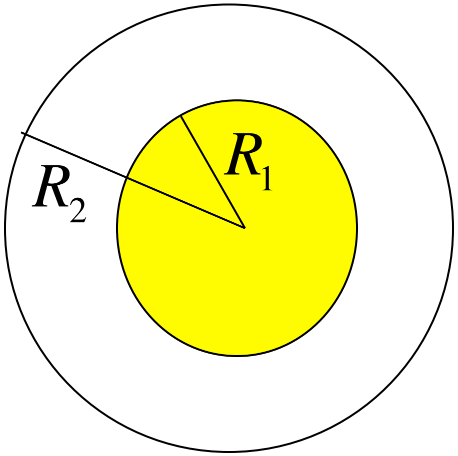
RULER



Determining distance: Baade-Wesselink (moving stellar atmosphere)

CANDLE

For pulsating stars, SN, novae, measure flux and effective temperature at two epochs, as well as the radial velocity and time between the epochs.



$$f_1, f_2, T_1, T_2, v_r(t), \Delta t$$

$$\frac{f_1}{f_2} = \frac{L_1}{L_2} = \left(\frac{R_1}{R_2}\right)^2 \left(\frac{T_1}{T_2}\right)^4 \rightarrow \frac{R_1}{R_2} = \left(\frac{f_1}{f_2}\right)^{\frac{1}{2}} \left(\frac{T_2}{T_1}\right)^2$$

$$R_2 - R_1 = \overline{v}_r \times \Delta t$$

Solve for R_1 and R_2. R and T $\longrightarrow$ L $\longrightarrow$ d

Determining distance: spectroscopic parallax

CANDLE

- Stellar spectra alone can give us the luminosity class + spectral class ➔ position on HR diagram
- This gives the absolute magnitude, which gives the distance modulus.
- Basically, compare a star's spectrum to an identical spectrum of another star with known luminosity.
- This method is poor in accuracy (+/- 1 magnitude error in absolute magnitude ➔ 50% error in distance)
- But it can be applied to all stars

Determining distance: main sequence fitting

CANDLE

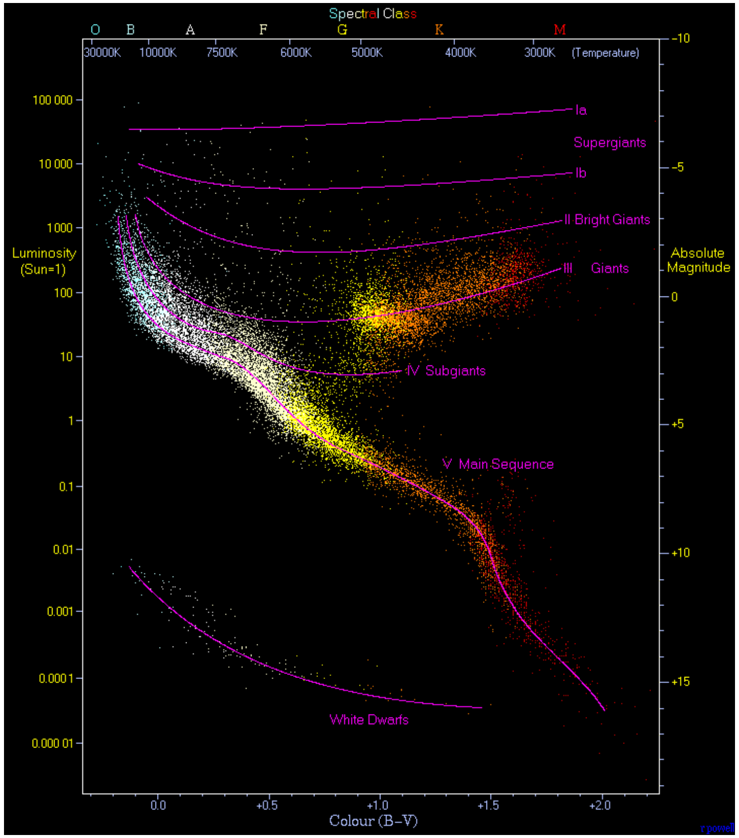
- Measure the colors and magnitudes of stars in a cluster (e.g., *r* and *g-r*)

- Plot the HR diagram: *r* vs. *g-r*

- Compare the the HR diagram of another cluster of known distance: M_r vs. *g-r*

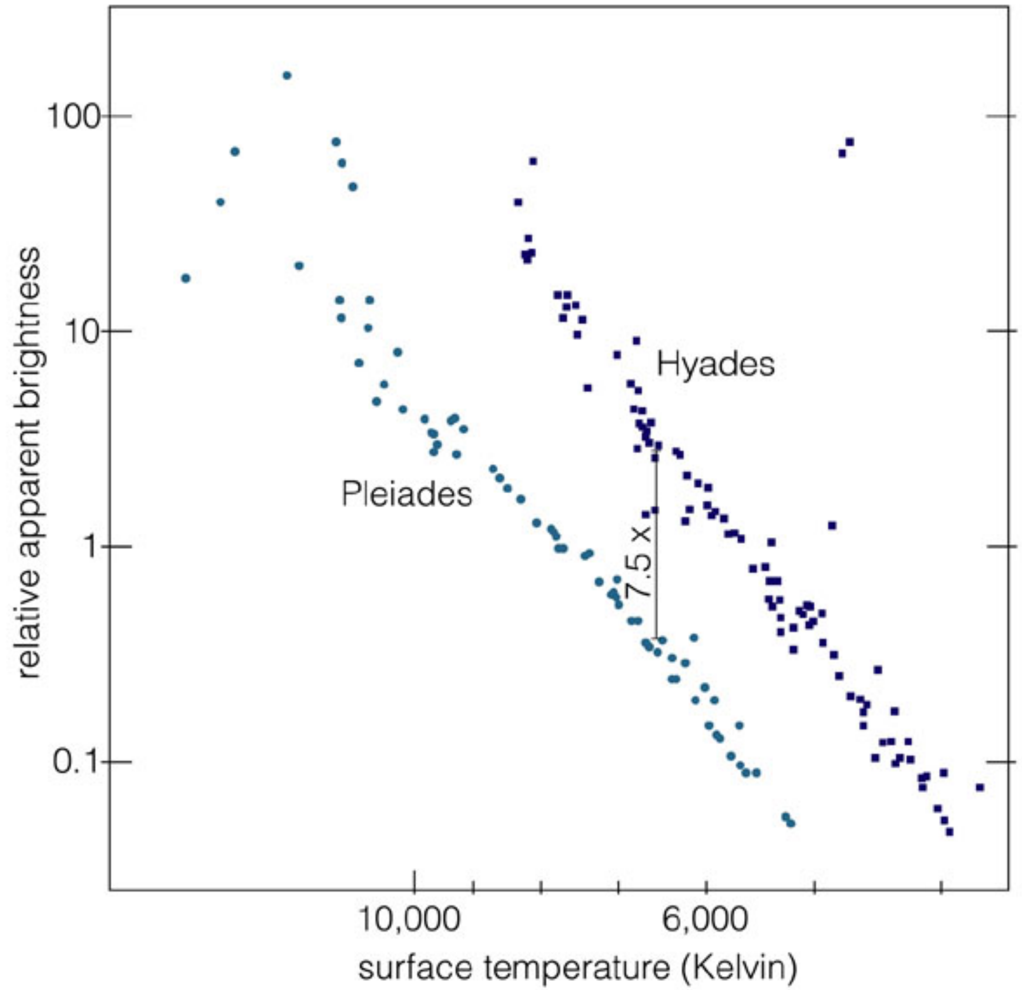
- Find the vertical offset in HR diagram between the main sequences of the two clusters ➔ distance modulus

$$m - M = 5\log d - 5$$



Determining distance: main sequence fitting

CANDLE



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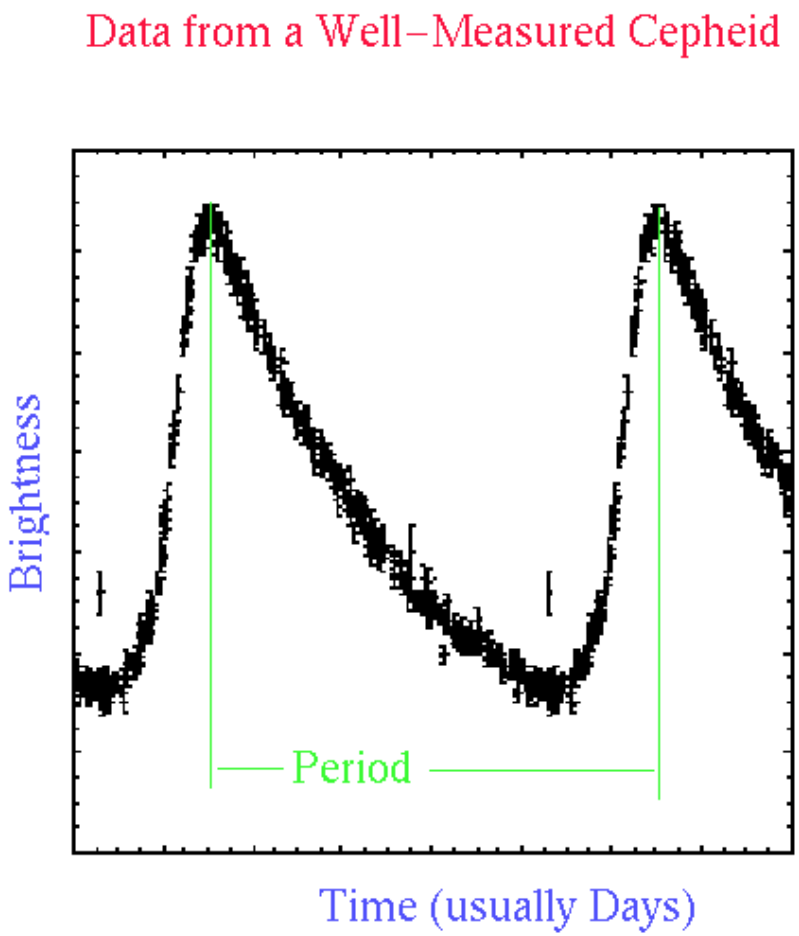
Determining distance: variable stars

<u>Cepheid variables</u>:
Pop I giants, M ~ 5-20 M_{sun}

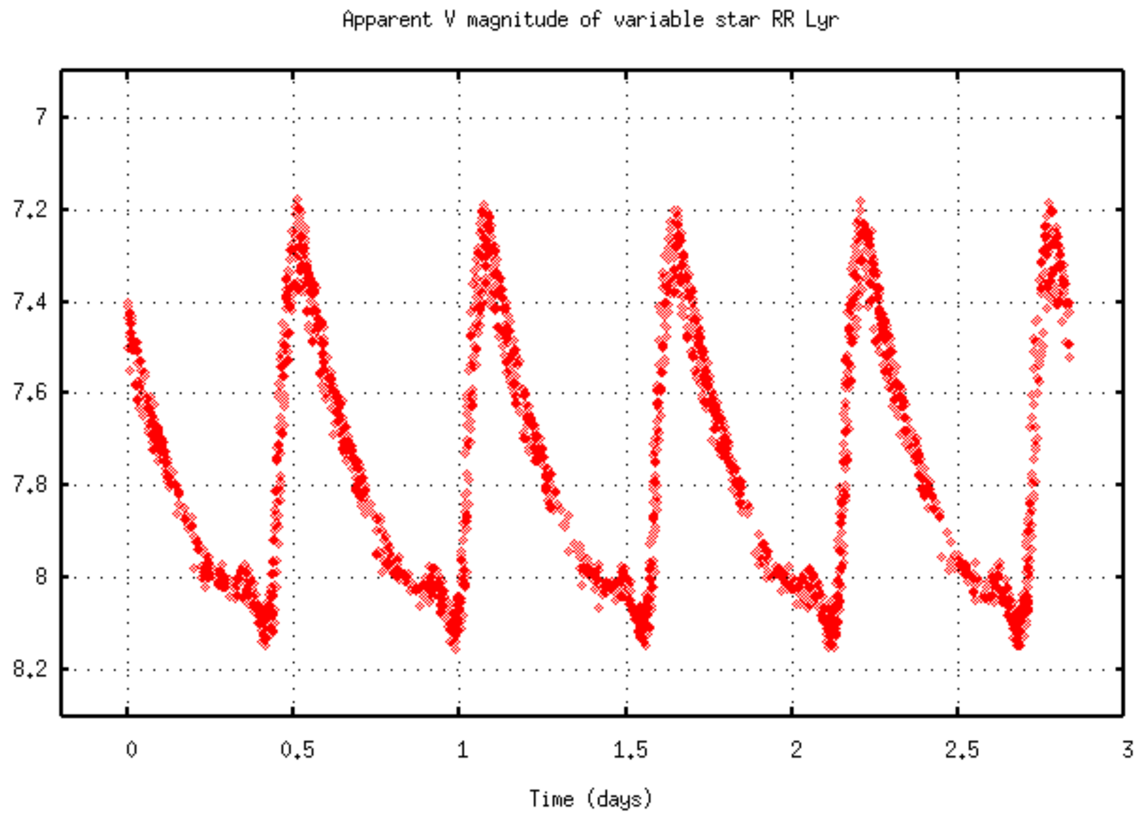
Pulsation due to feedback loop:

An increase in T
➔ HeIII (doubly ionized He)
➔ high opacity
➔ radiation can't escape
➔ even higher T and P
➔ atmosphere expands
➔ low T
➔ HeII (singly ionized He)
➔ low opacity
➔ atmosphere contracts
➔ rinse and repeat…



Determining distance: variable stars

RR-Lyrae variables:
Pop II dwarfs, $M \sim 0.5\ M_{sun}$

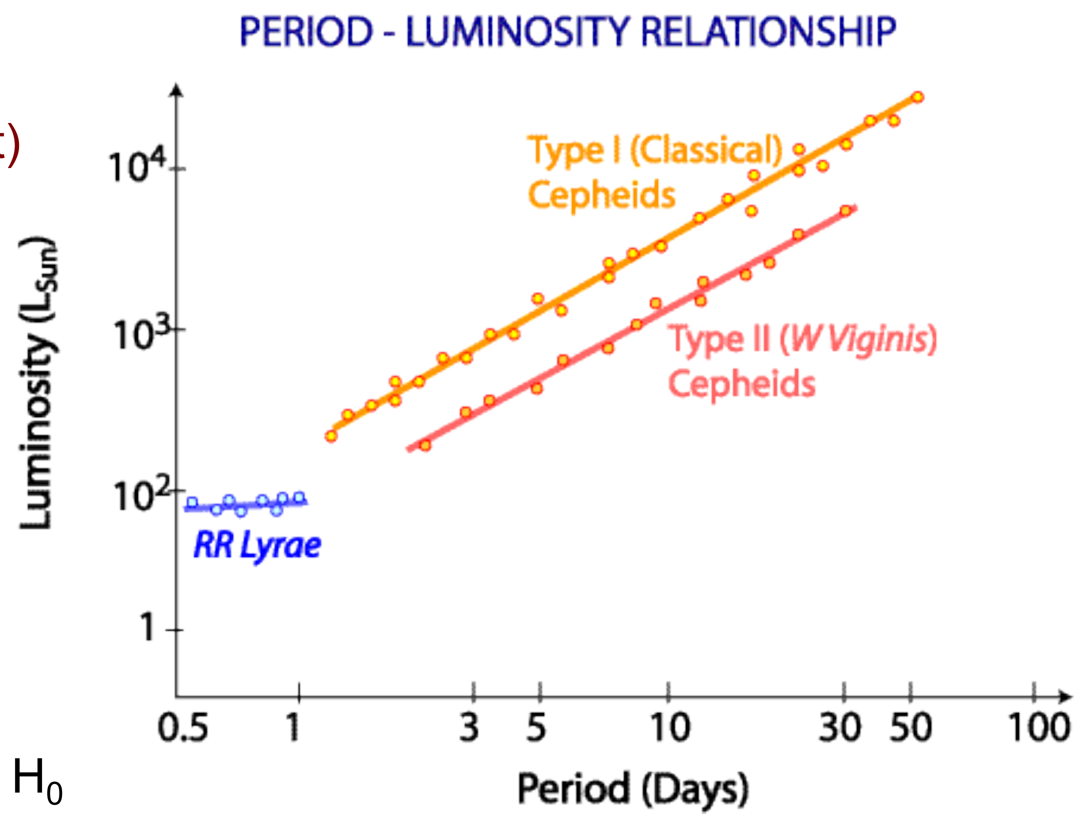


Determining distance: variable stars

CANDLE

Variable stars have a tight period-luminosity relation

- Measure lightcurves: flux(t)
- Get period P
- From P-L relation, get L
- Use L to get distance



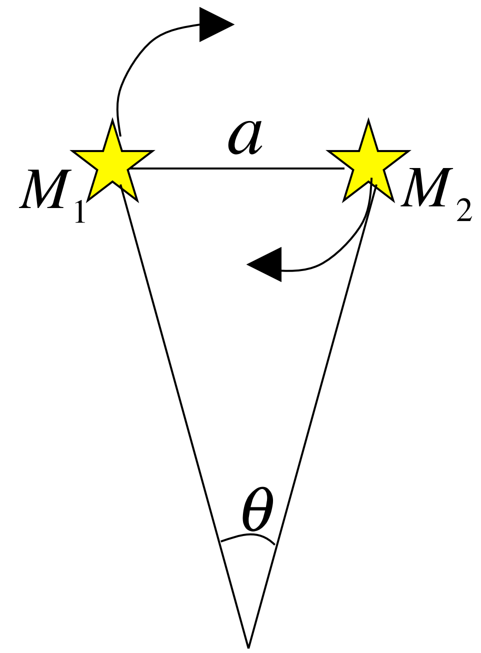
Very powerful method. Cepheids can be seen very far away. Used to measure H_0

P-L relation is calibrated on local variables with parallax measurements

Determining distance: dynamical parallax

RULER

For binary star systems on main sequence



- Measure: period of orbit P, angular separation θ, fluxes f_1 and f_2
- Kepler's 3rd law: $P^2 = \frac{4\pi^2}{G(M_1 + M_2)} a^3$
- Assume $M_1 = M_2 = M_\odot$
- Use Kepler to get a
- Use θ and a to get preliminary distance
- Use distance and fluxes to get luminosities L_1, L_2
- Use mass-luminosity relation for main sequence to get better masses
- Iterate until convergence

Determining Luminosity

1. Measure flux or magnitude

   Measure distance

$$L = 4\pi d^2 f$$

$$M = m - 5\log d - 5$$

2. Measure spectrum

   line ratios and widths (i.e., compare the star to a star of identical spectrum that has a known L)

$$\longrightarrow \quad T, L$$

Determining Temperature

1. Spectral lines → spectral type

2. Colors
   (cheap and accurate)

3. Blackbody fitting or Wien's law: $h\nu \sim 2.8kT$
   (not good for very hot or cool stars)

4. Stellar atmosphere modeling
   (uncertain)

5. Measure angular size and flux
   (need very high resolution imaging)

$$R = \theta_R d \qquad L = 4\pi d^2 f$$

$$L = 4\pi R^2 \sigma T^4 \rightarrow T = \left(\frac{L}{4\pi R^2 \sigma}\right)^{\frac{1}{4}} \rightarrow T = \left(\frac{4\pi d^2 f}{4\pi (\theta_R d)^2 \sigma}\right)^{\frac{1}{4}} \qquad \boxed{T = \left(\frac{f}{\sigma \theta_R^2}\right)^{\frac{1}{4}}}$$

Determining Radius

- Very difficult because angular sizes are tiny.
  The sun at 1pc distance has an angular radius of ~0.5mas

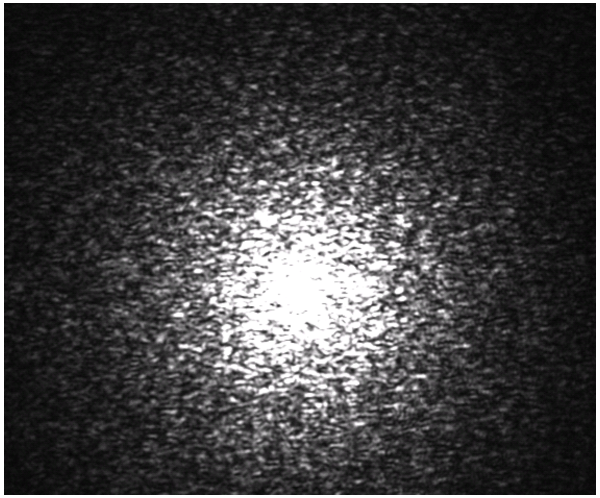
- Important because of:
  - surface gravity $g \sim \frac{GM}{R^2}$
  - density $\rho \sim \frac{M}{R^3}$
  - Temperature $T \sim \left(L/R^2\right)^{1/4}$
  - testing models

Determining Radius: Interferometry

Interferometry yields high resolution images

1. <u>Speckle interferometry</u>

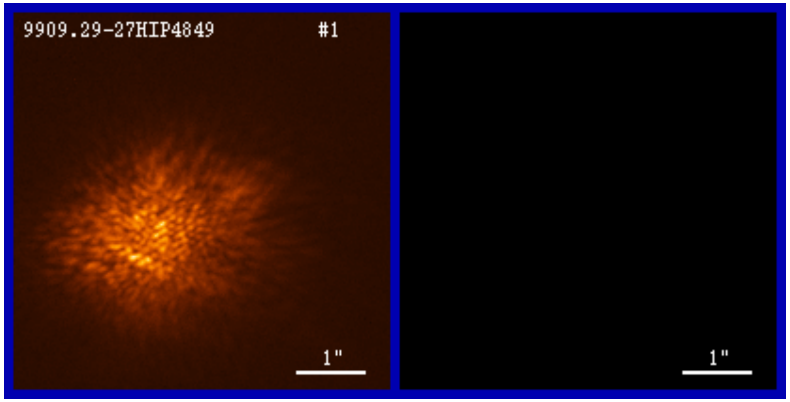
   ~ 0.02 as



Diffraction limited:

$$\theta = \frac{1.22\lambda}{D_T}$$

e.g., for HST (D_T=2.4m), θ =0.05 as



Determining Radius: Interferometry

2. <u>Phase interferometry</u>

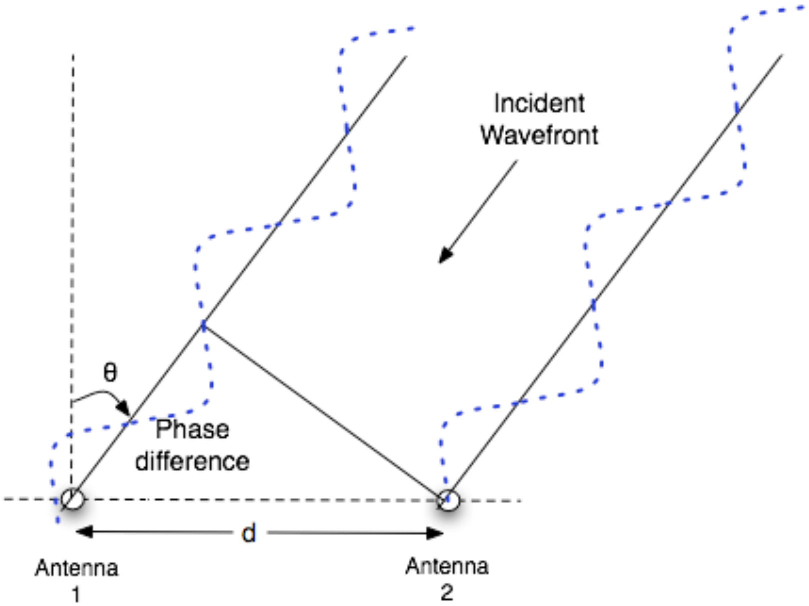
   ~ 0.01 as

Fringes disappear when

$$d \times \theta = \lambda/2$$

3. <u>Intensity Interferometry</u>

   ~ 0.5 mas



The sun's radius could be measured out to ~1-10pc

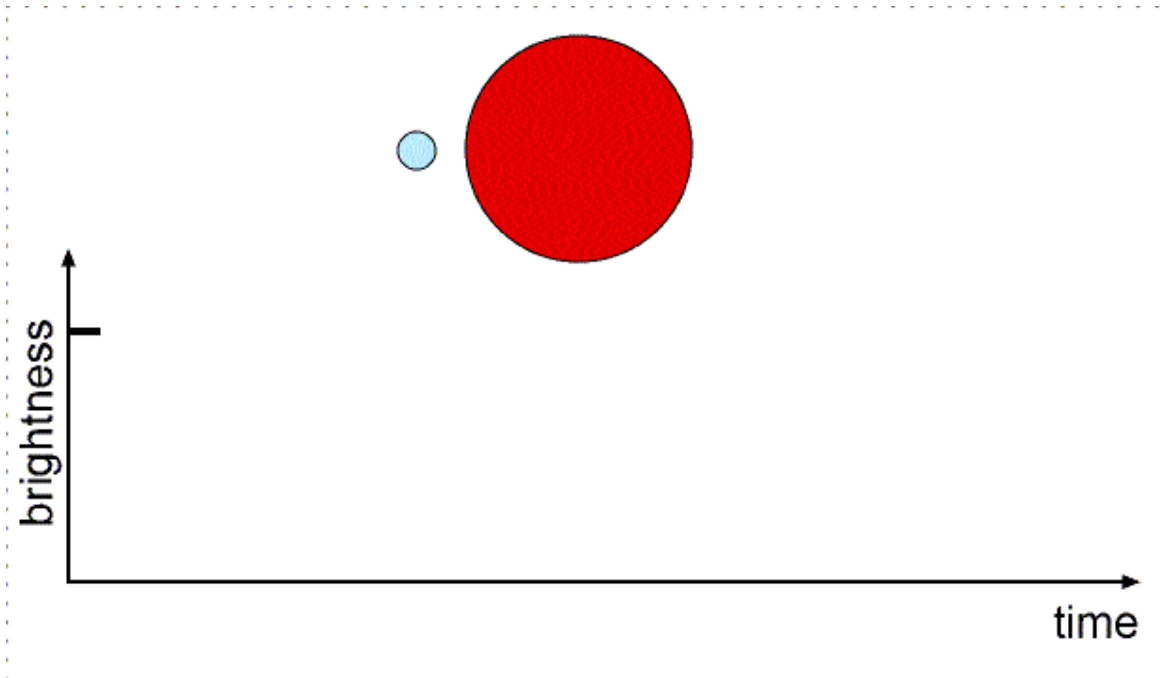
Determining Radius

- Luminosity + Temperature → Radius
- Baade-Wesselink (moving atmosphere)
- Lunar Occultation

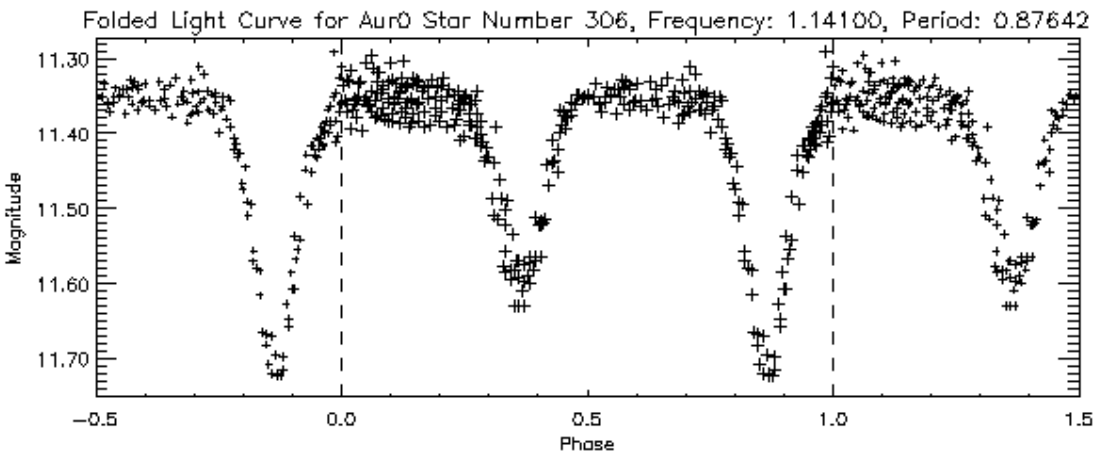
  As a star disappears behind the moon, a Fresnel diffraction pattern is created that depends slightly on θ_R

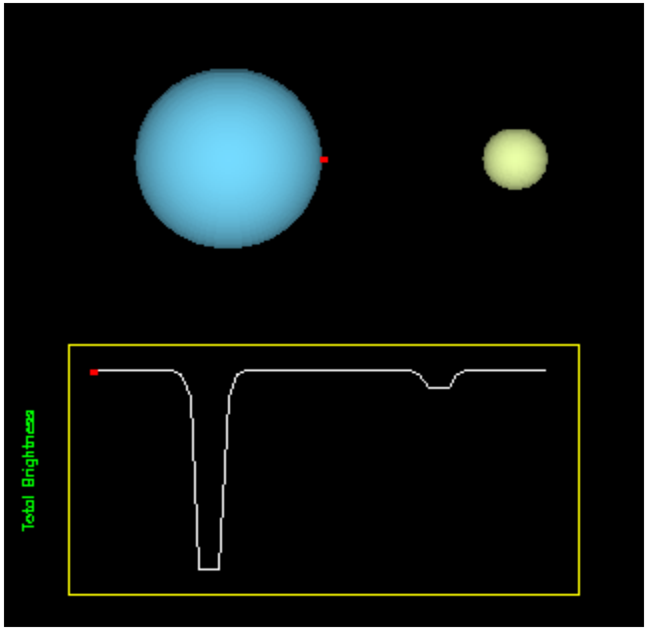
  ~ 2 mas, only good for stars in ecliptic.

Determining radius: eclipsing binaries



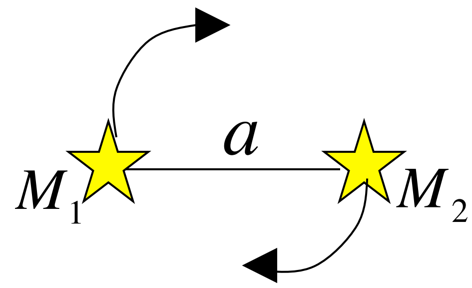
Determining radius: eclipsing binaries





Determining Mass

Only possible to measure accurately for binaries.



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Kepler's 3rd law: $G\left(M_1 + M_2\right)P^2 = 4\pi^2 a^3$

For Sun-Earth system:

$$M_1 + M_2 \approx M_\odot$$

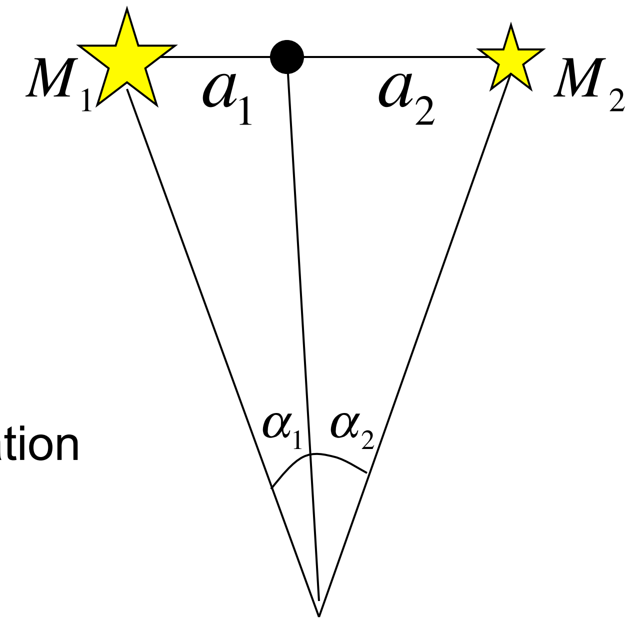
$$P = 1\mathrm{yr} \longrightarrow GM_\odot\left(1\mathrm{yr}\right)^2 = 4\pi^2\left(1\mathrm{AU}\right)^3$$

$$\mathrm{a{=}1AU}$$

$$\boxed{\left(\frac{M_1 + M_2}{M_\odot}\right)\left(\frac{P}{1\mathrm{yr}}\right)^2 = \left(\frac{a}{1\mathrm{AU}}\right)^3}$$

Determining Mass: visual binaries

Visual binaries are binaries where the angular separation is detectable and the orbit can be traced out.



- Find the center of mass of the system (c.o.m. must move with constant velocity)

$$M_1 a_1 = M_2 a_2 \rightarrow \boxed{\frac{M_1}{M_2} = \frac{a_2}{a_1} = \frac{\alpha_2}{\alpha_1}}$$

- Measure the period, semi-major angular separation and distance.

$$\alpha = \alpha_1 + \alpha_2$$

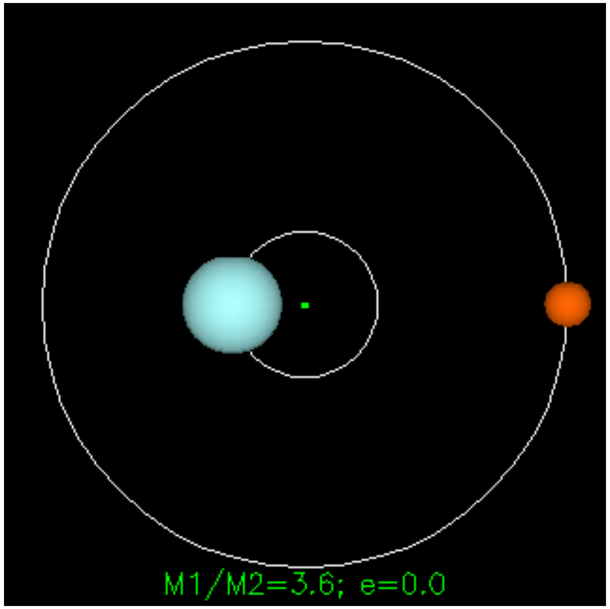
$$a = d\alpha$$

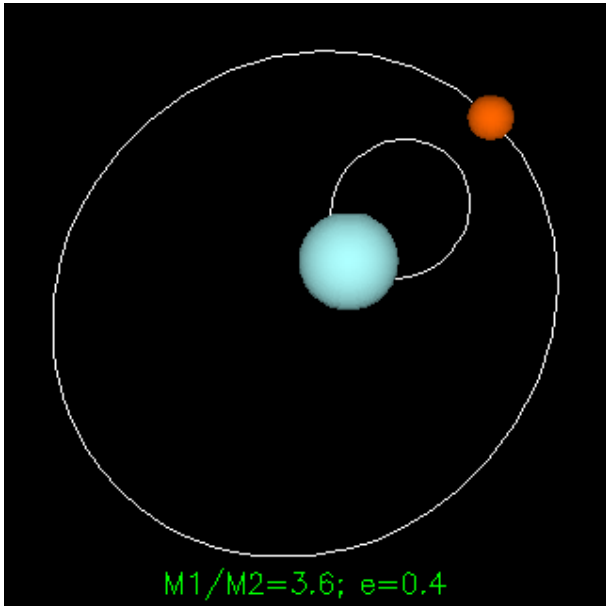
$$\boxed{M_1 + M_2 = \frac{(d\alpha)^3}{P^2}}$$

- Solve for individual masses. Very sensitive to distance errors: $M \sim d^3$

- If distance is not known, radial velocity data is sufficient to get a
  e.g., for a circular orbit: $v = 2\pi a / P$

Determining Mass: visual binaries



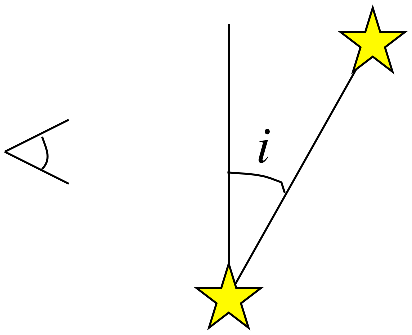


Determining Mass: visual binaries

If the orbit is inclined, then we must know the inclination angle i

$$\frac{M_1}{M_2} = \frac{a_2}{a_1} = \frac{\alpha_2 \cos i}{\alpha_1 \cos i} = \frac{\alpha_2}{\alpha_1}$$

Don't need *i*



$$M_1 + M_2 = \left(\frac{\alpha}{\cos i}\right)^3 \frac{d^3}{P^2}$$

Need *i*

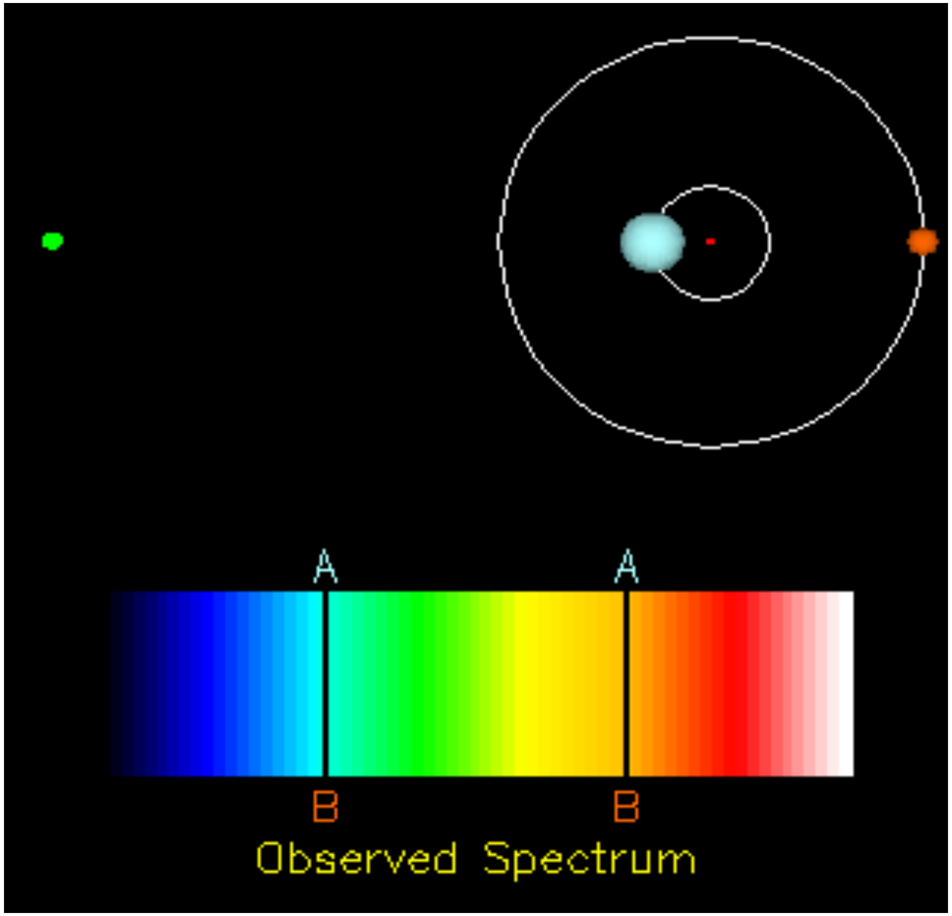
In an inclined orbit, the center of mass does not lie at the ellipse focus.
Find the projection that fixes this ➔ cos *i*

Determining Mass: spectroscopic binaries

Spectroscopic binaries are unresolved binaries that are only detected via doppler shifts in their spectrum.

- Single line binaries: single set of shifting spectral lines
- Double line binaries: two sets of spectral lines (one from each star)
- Two sets of lines, but for different spectral types

Determining mass: spectroscopic binaries



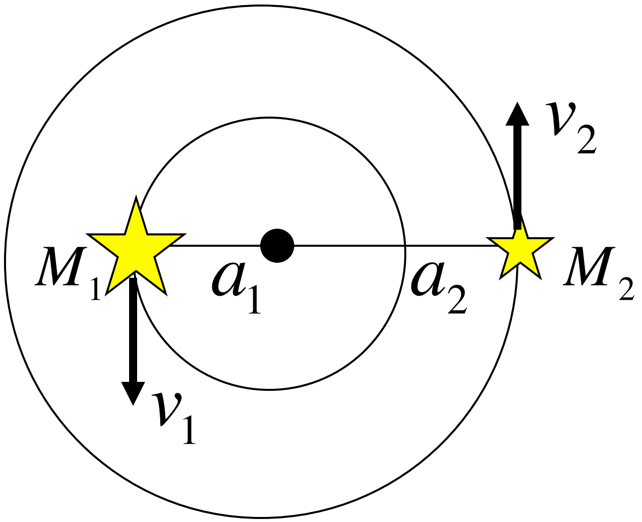
Determining Mass: spectroscopic binaries

- Measure velocities and period

$$a_1 = \frac{v_1 P}{2\pi}$$

$$a_2 = \frac{v_2 P}{2\pi}$$

$$\rightarrow \frac{a_1}{a_2} = \frac{v_1}{v_2} \rightarrow \boxed{\frac{M_1}{M_2} = \frac{v_2}{v_1}}$$



$$a = a_1 + a_2 = \frac{P}{2\pi}\left(v_1 + v_2\right)$$

$$M_1 + M_2 = \frac{a^3}{P^2} = \frac{P\left(v_1 + v_2\right)^3}{\left(2\pi\right)^3}$$

$$v_{r,1} + v_{r,2} = \left(v_1 + v_2\right)\sin i$$

$$\rightarrow \boxed{M_1 + M_2 = \frac{P\left(v_{r,1} + v_{r,2}\right)^3}{\left(2\pi \sin i\right)^3}}$$

- Can get a lower limit on M_1+M_2 since sin *i* <1

Determining Mass: spectroscopic binaries

Need to measure both radial velocities - only possible with double line binaries.

Need to know sin *i*. This is possible when:

- also an eclipsing binary: $\sin i = 1$

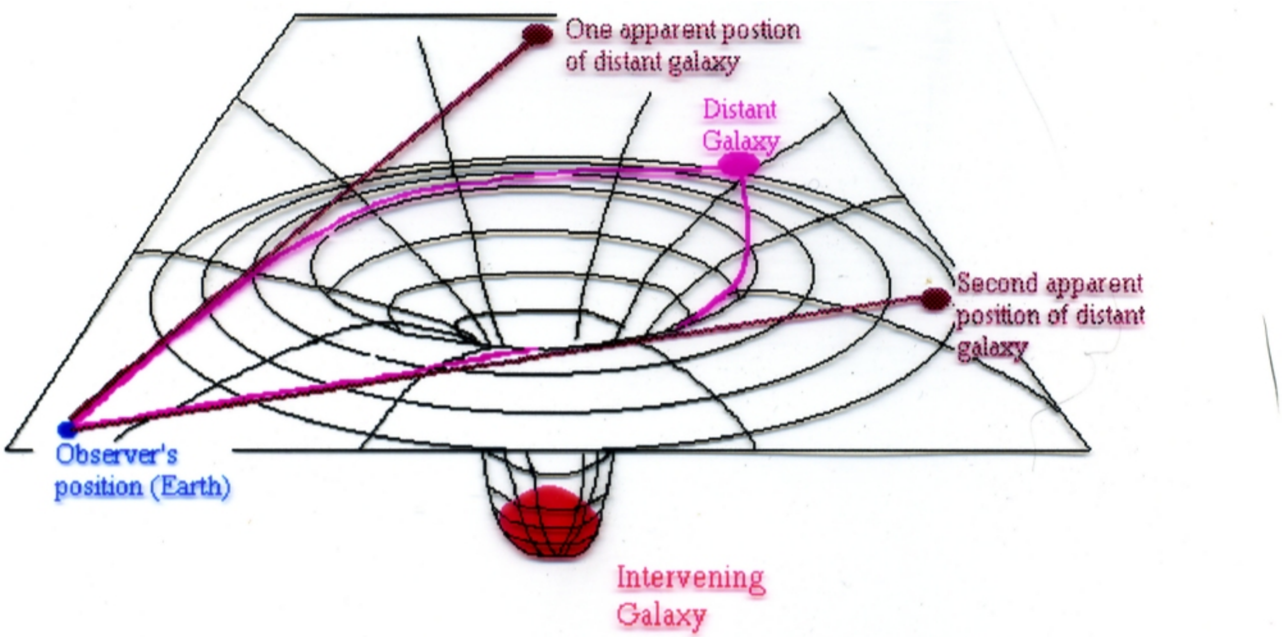
- also a visual binary: measure orbit

- compute M_1+M_2 for a statistical sample where $<\sin^3 i>$ is known
  (For an isotropic distribution, $<\sin^3 i>=0.42$. However, no Doppler shift will be observed if $i=0$, so there is a selection effect that favors high inclinations)

Determining Mass: surface gravity

- Pressure broadening of spectral lines ➔ surface gravity $$g = \frac{GM}{R^2}$$
- Measure radius R ➔ $$\boxed{M = \frac{gR^2}{G}}$$
- Sensitive to errors in R

Determining Mass: Gravitational lensing



Discovery of Widest-separation Quasar Gravitational Lens

Suprime-Cam (g',r',i')
December 17, 2003

Subaru Telescope, National Astronomical Observatory of Japan

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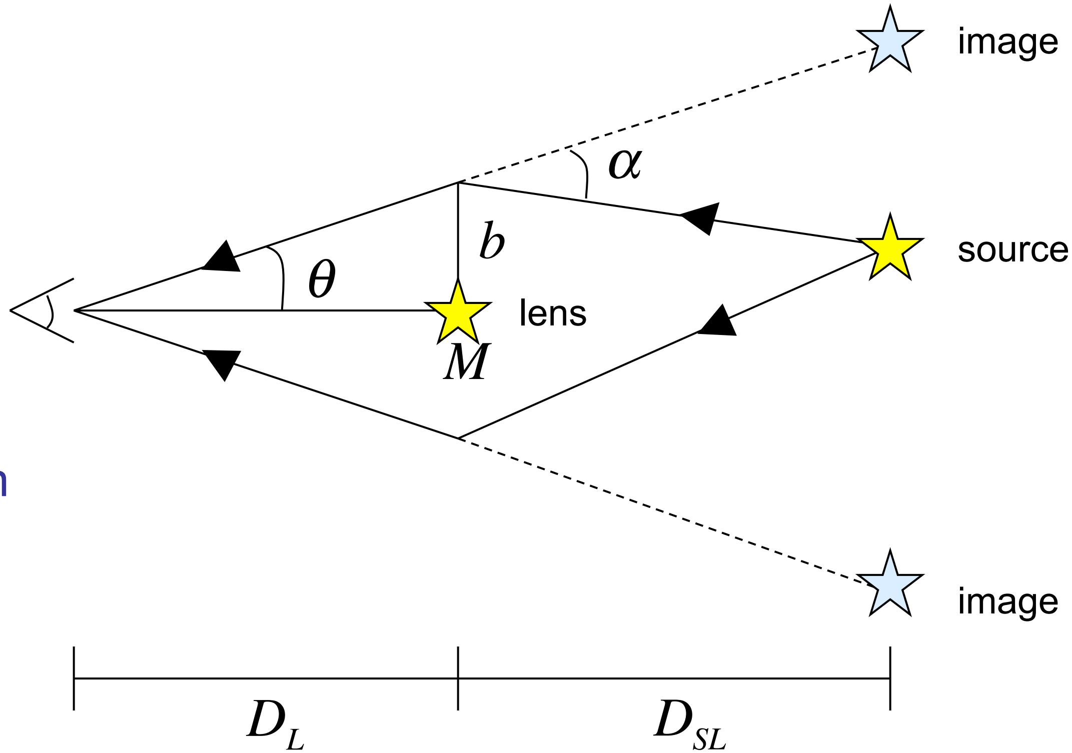
Determining Mass: Gravitational lensing

Determining Mass: gravitational microlensing

Case of lensing when multiple images are unresolved and we only detect an increase in the flux of one image.

General Relativity:

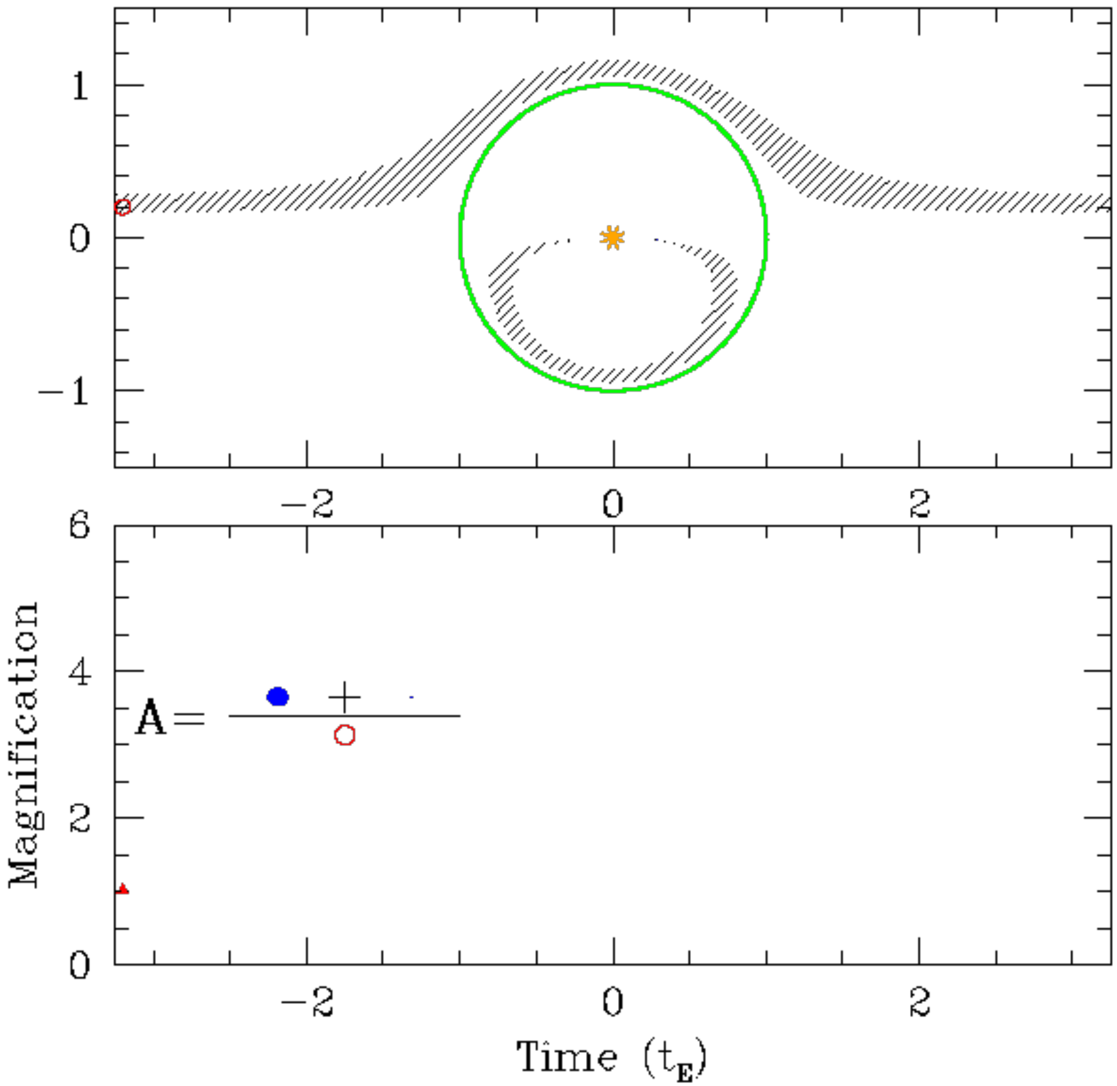
$$\alpha = \frac{4GM}{c^2 b}$$



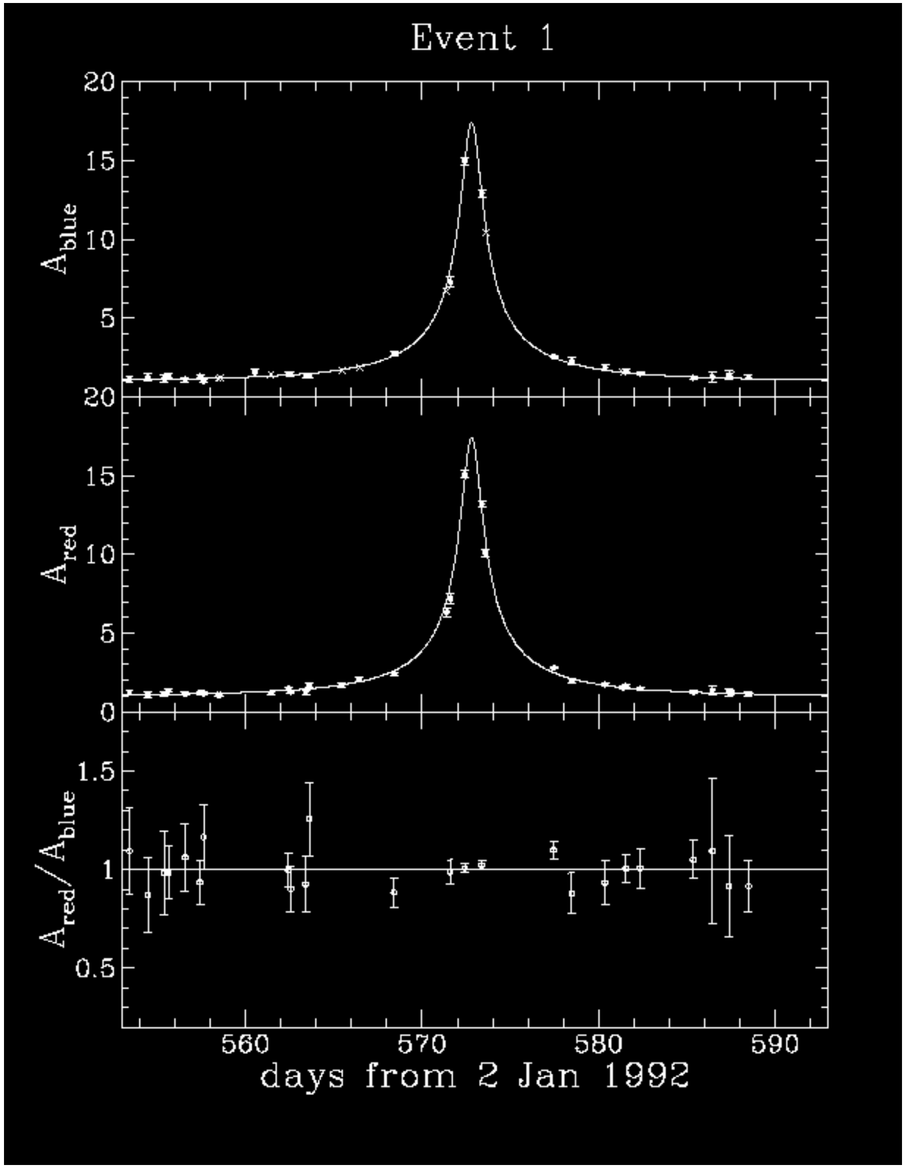
Can compute the amplification of light due to multiple images

$$A = f\left(M, D_L, D_{SL}, \theta\right)$$

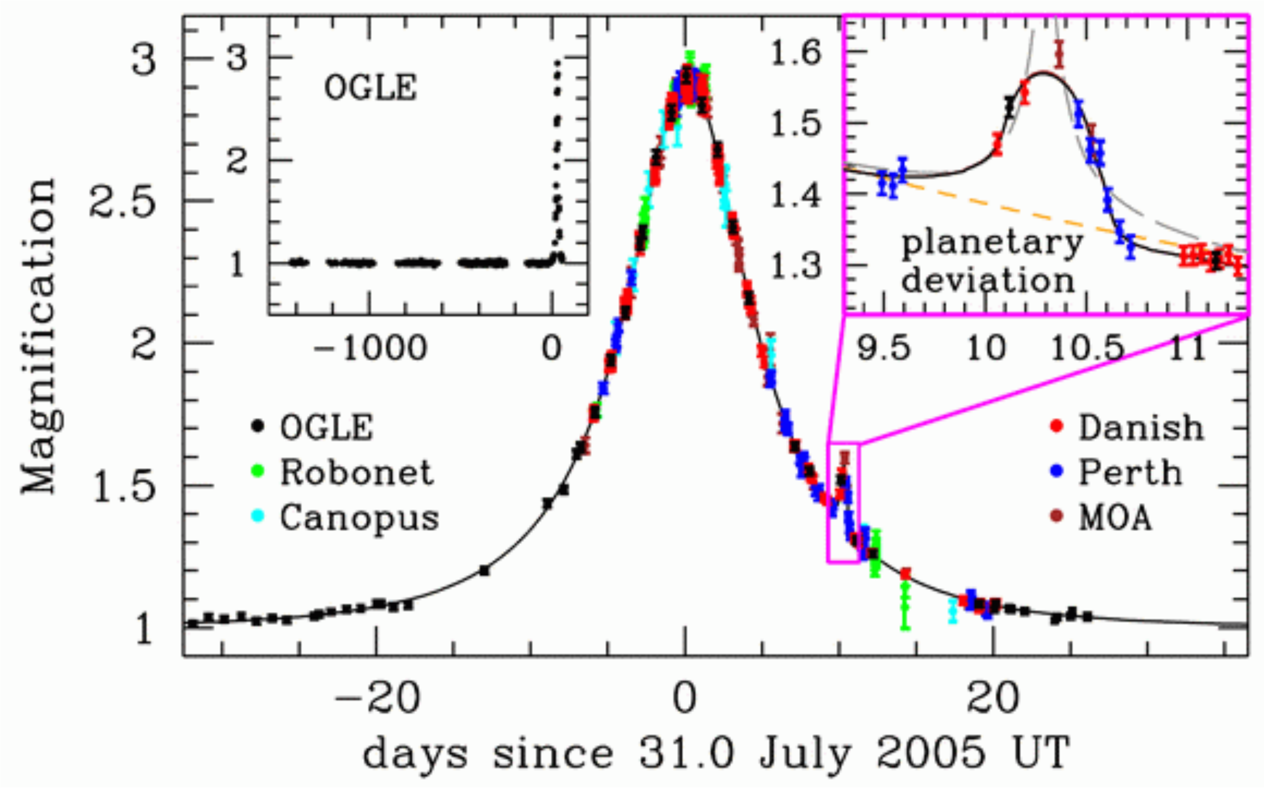
Determining mass: gravitational microlensing



Determining mass: gravitational microlensing



Determining mass: gravitational microlensing



Determining Chemical Composition

Theoretical: fractional abundances by mass

X: Hydrogen Y: Helium Z: metals   Sun: (0.7, 0.28, 0.02)

Observational: number density relative to Hydrogen, normalized to sun.

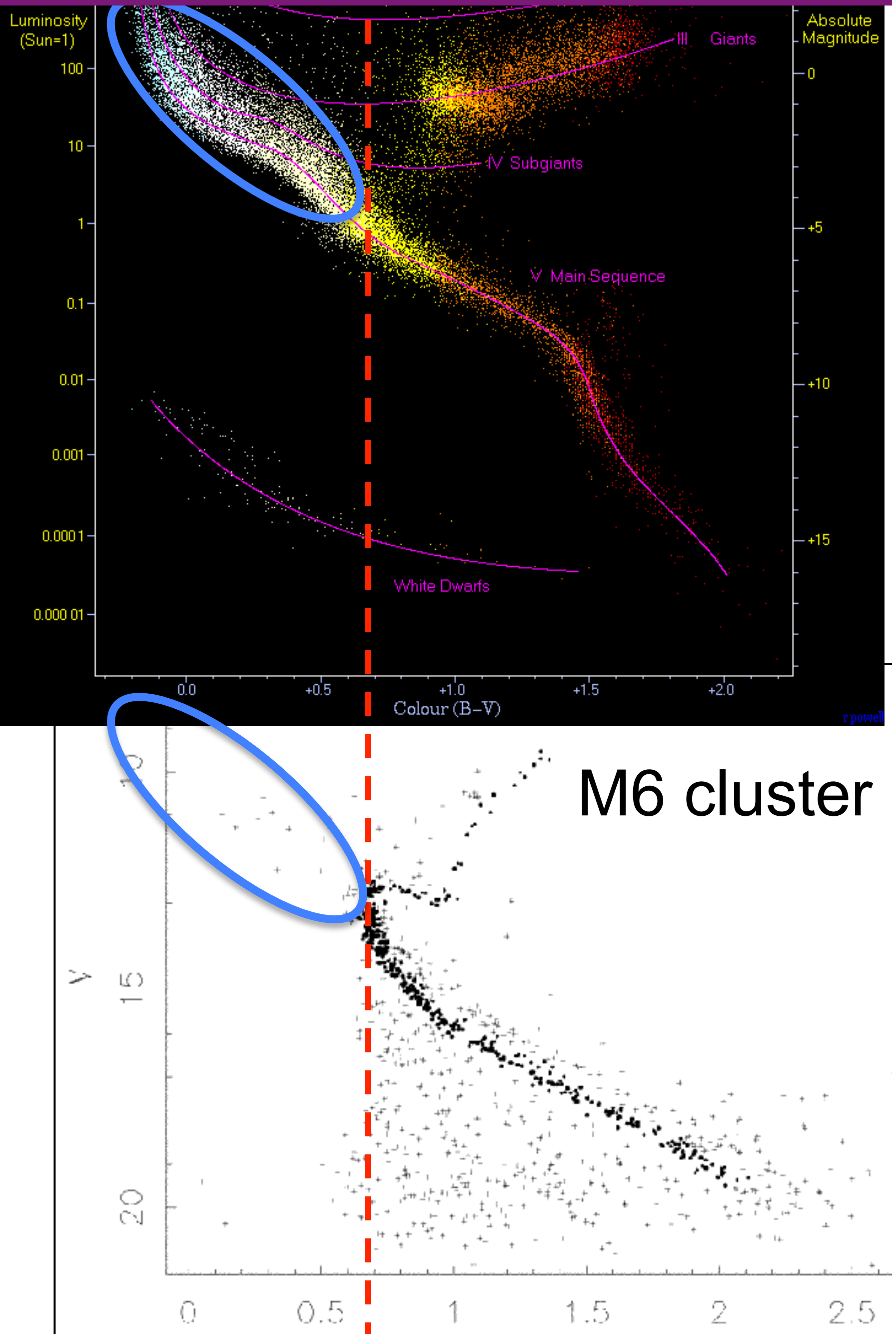
e.g., iron abundance $$\left[\mathrm{Fe}/\mathrm{H}\right] \equiv \log\left(\frac{n(\mathrm{Fe})}{n(\mathrm{H})}\right)_{*} - \log\left(\frac{n(\mathrm{Fe})}{n(\mathrm{H})}\right)_{\odot}$$

So, for solar abundance: $\left[\mathrm{Fe}/\mathrm{H}\right] = 0$, 10 times more iron: $\left[\mathrm{Fe}/\mathrm{H}\right] = +1$

In our galaxy, $\left[\mathrm{Fe}/\mathrm{H}\right]$ ranges from -4.5 to +1.

Need high resolution spectra + stellar atmosphere models to get abundances via fitting.

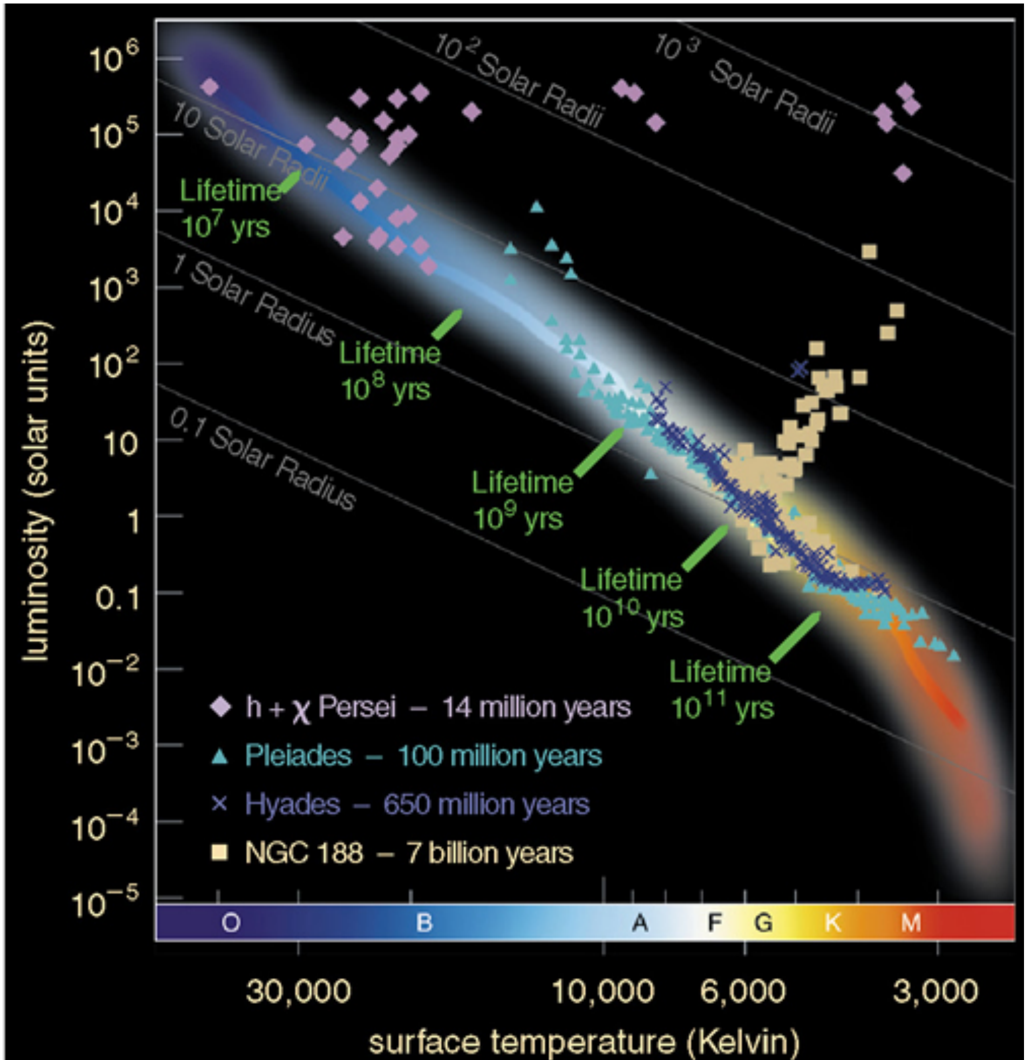
Determining age: Main Sequence turn-off



The lack of blue dwarfs tells us that the age of M6 is sufficiently old that the massive blue stars have died.

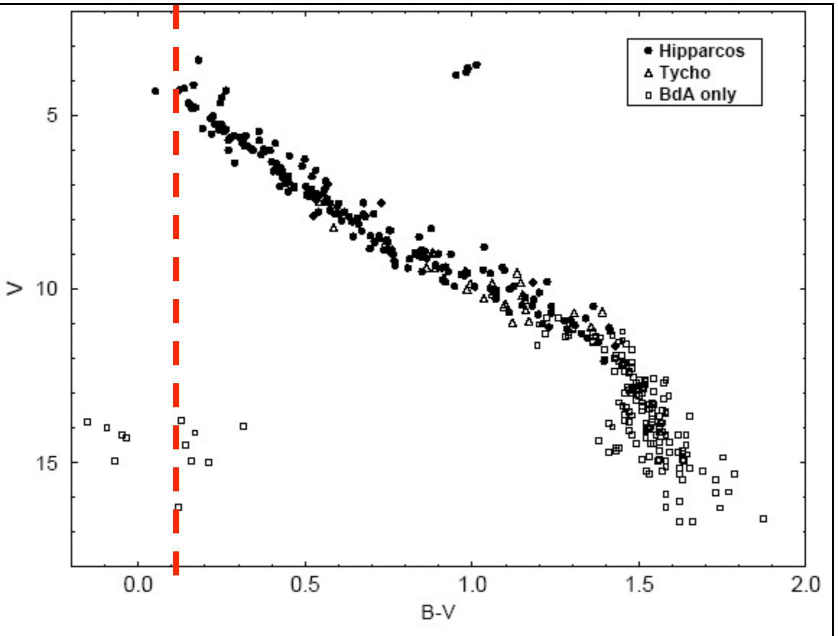
Yellow and red stars are still present because they live longer lives.

Determining age: Main Sequence turn-off

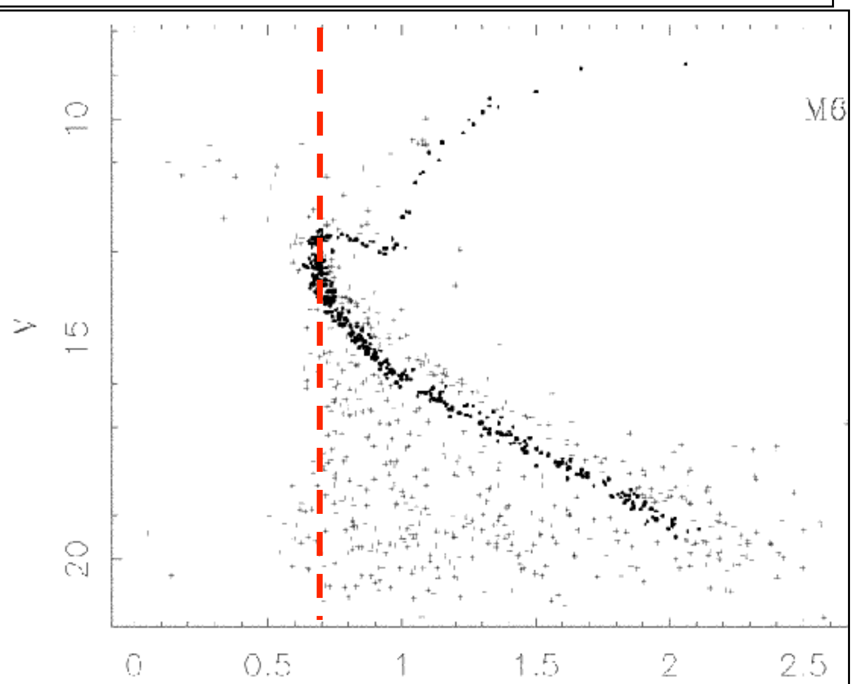


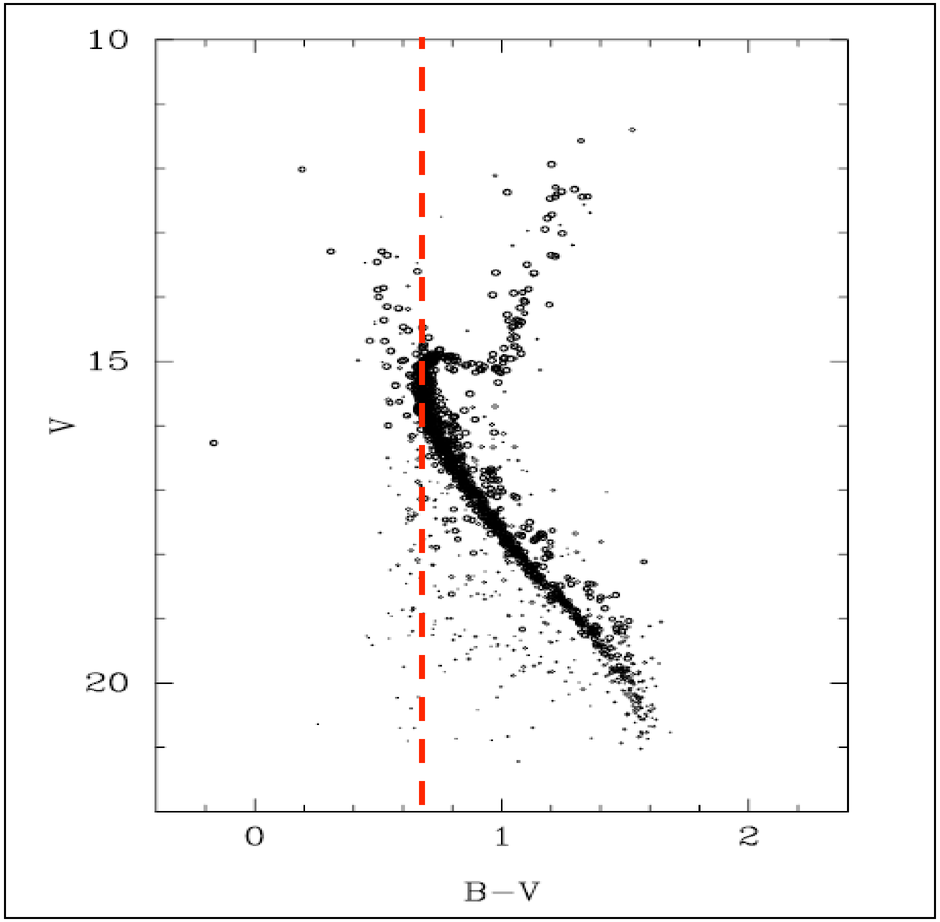
- All stars arrived on the main-sequence (MS) at about the same time.
- The massive stars on the top left of MS are the first to go.
- The cluster is as old as the most luminous star that remains on the MS.
- The position of the hottest, brightest star on a cluster's MS is called the *main-sequence turnoff point*.

Determining age: Main Sequence turn-off

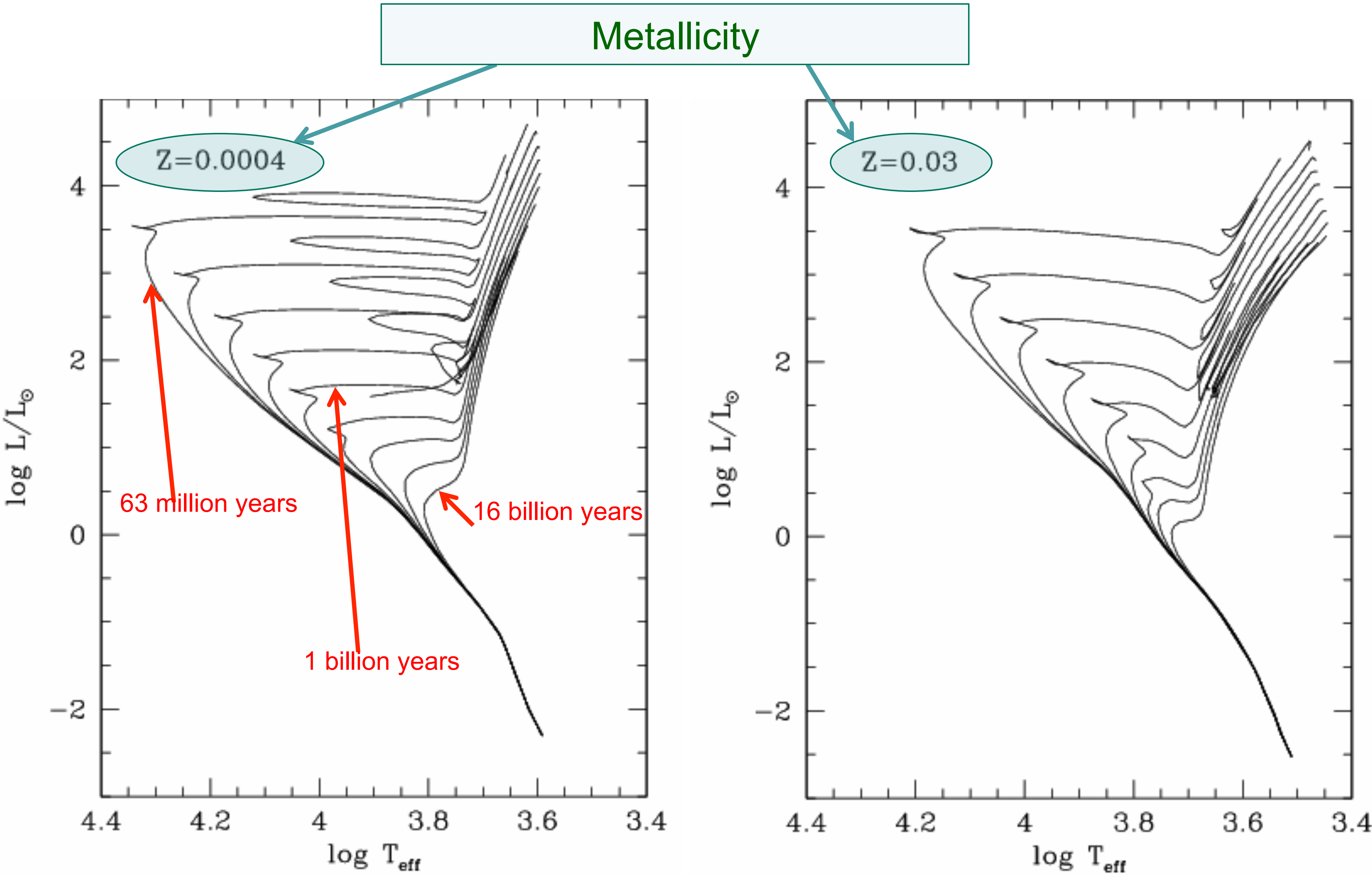


Which cluster is oldest?



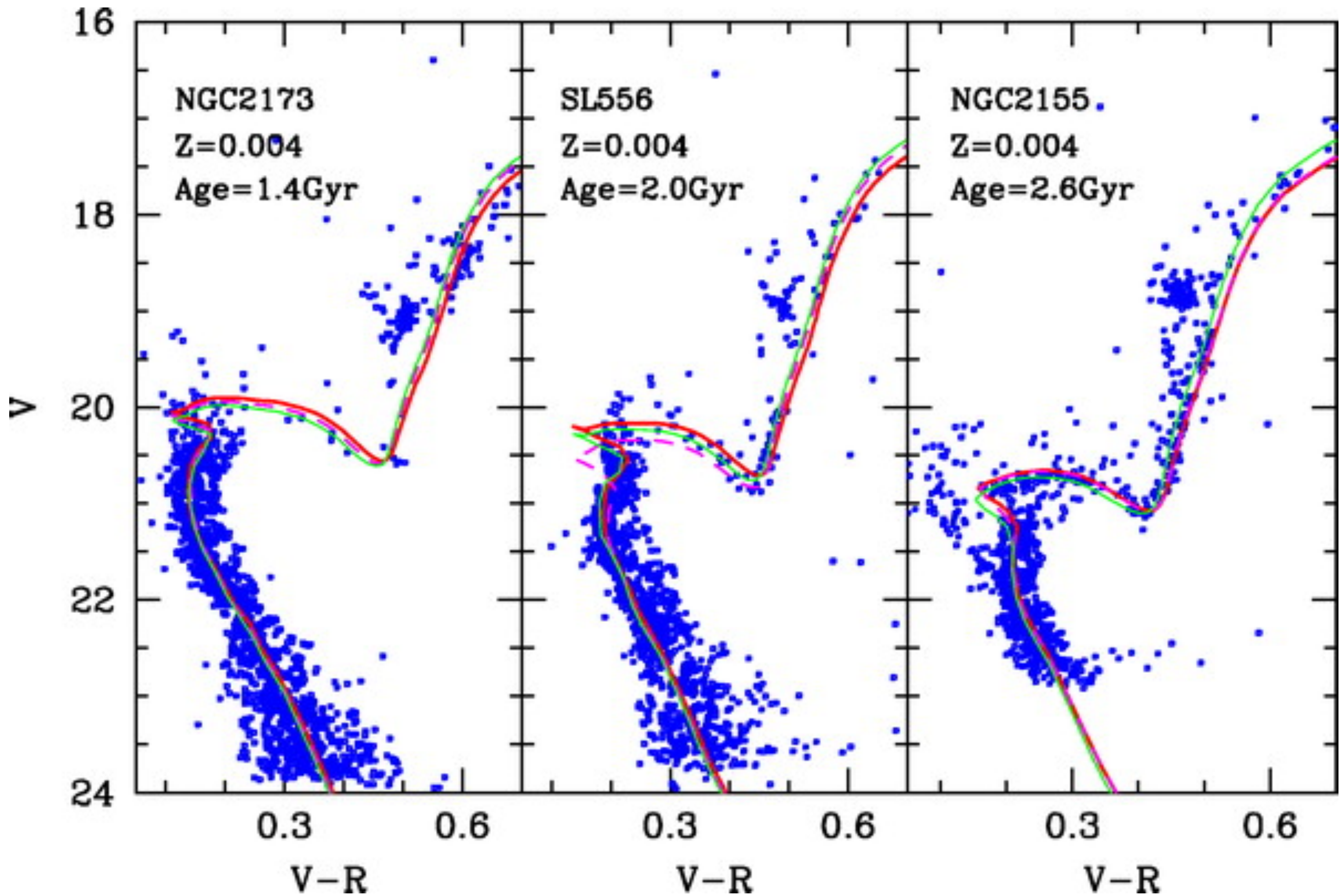


Determining age: Isochrone Fitting



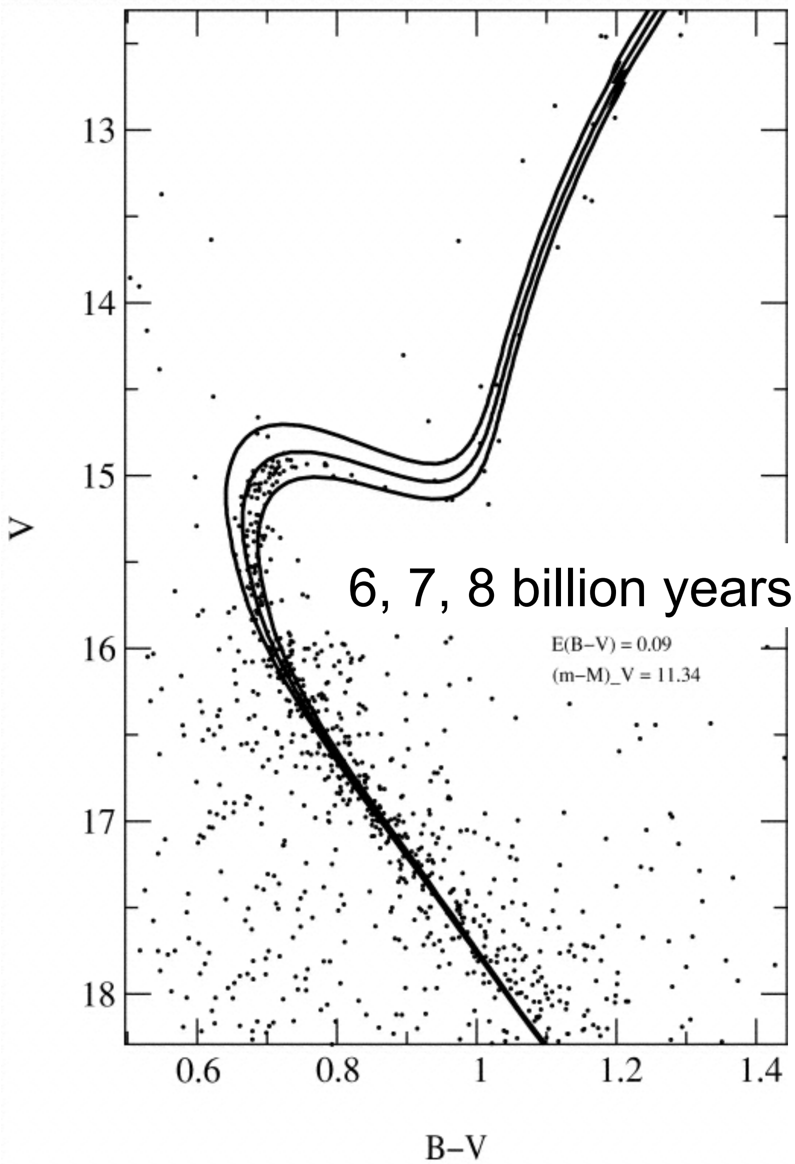
Determining age: Isochrone Fitting

Open clusters



1.4 billion years 2 billion years 2.6 billion years

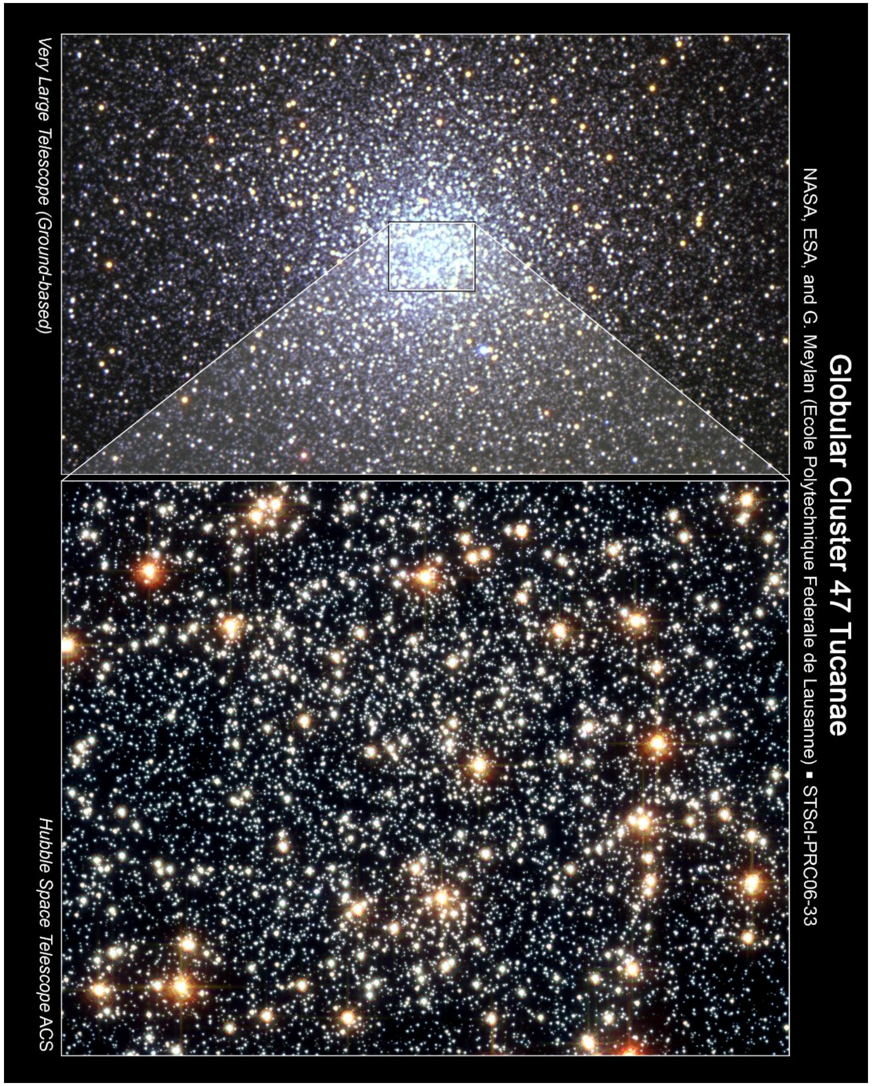
Determining age: Isochrone Fitting



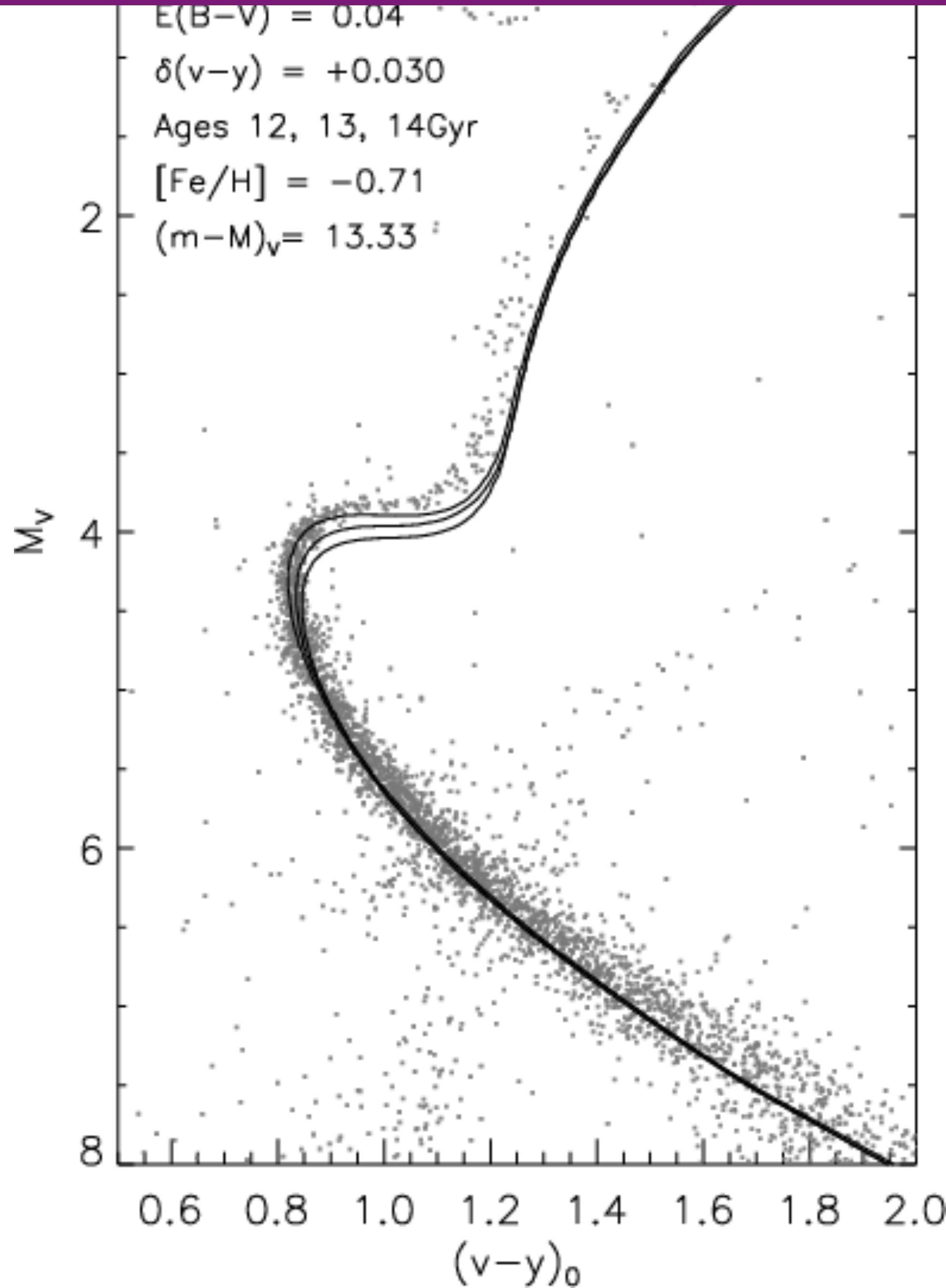
6, 7, 8 billion years

NGC 188: 7 billion years

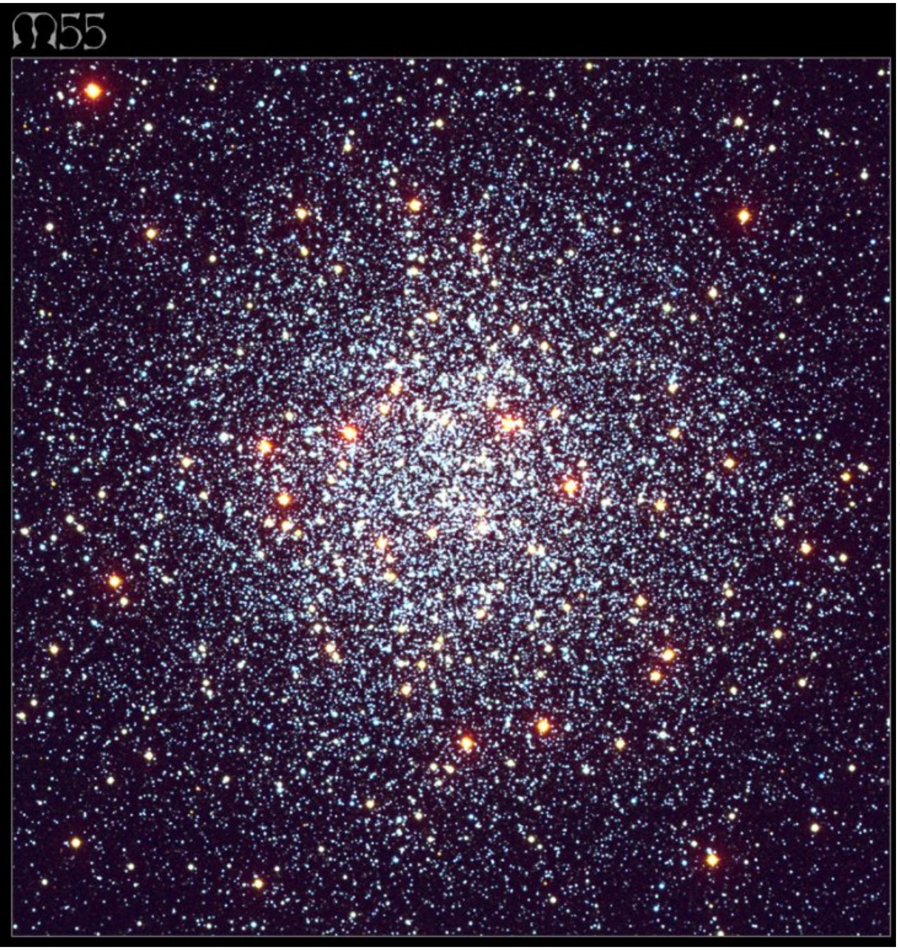
Determining age: Isochrone Fitting



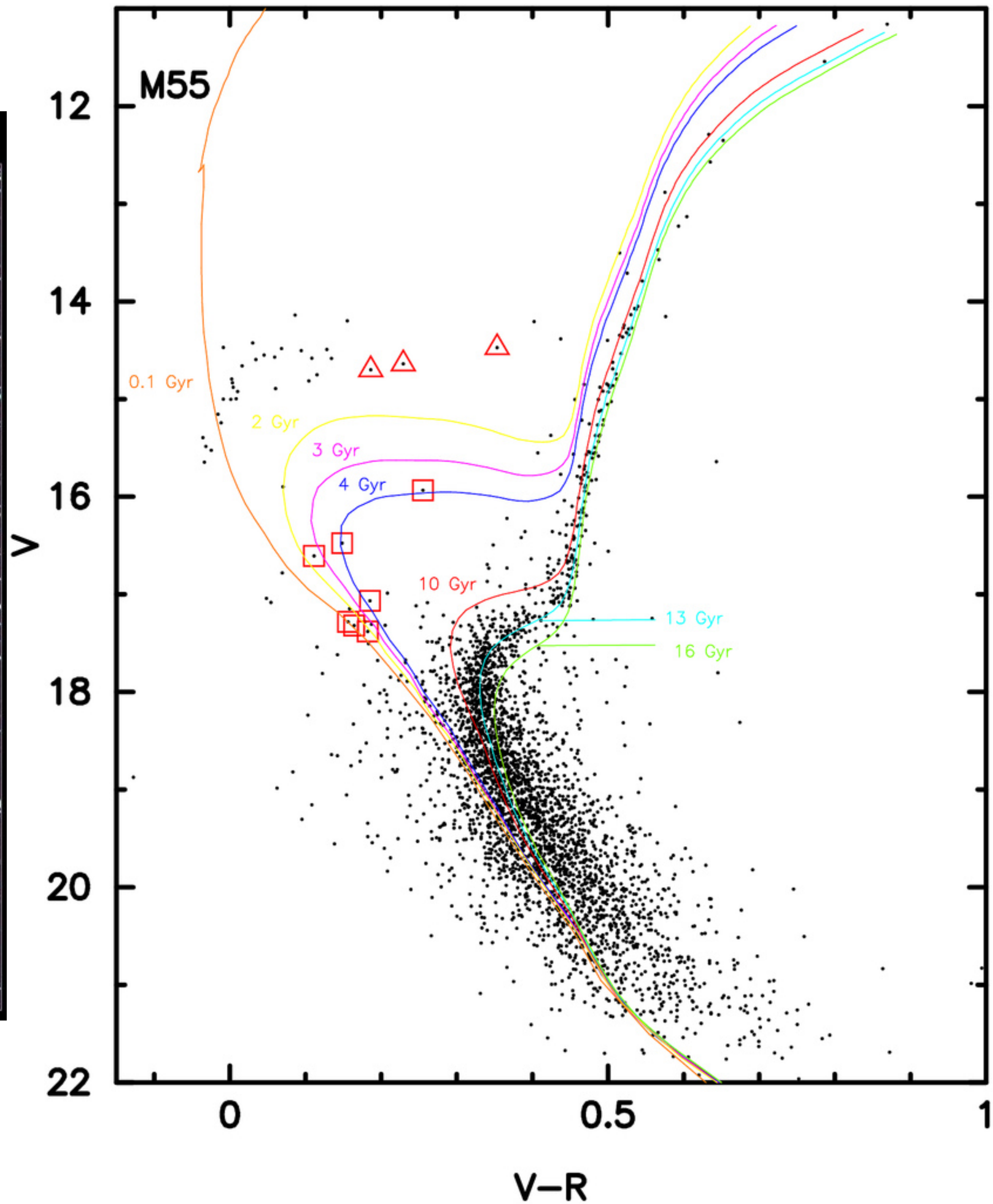
47 Tuc: 12 billion years



Determining age: Isochrone Fitting

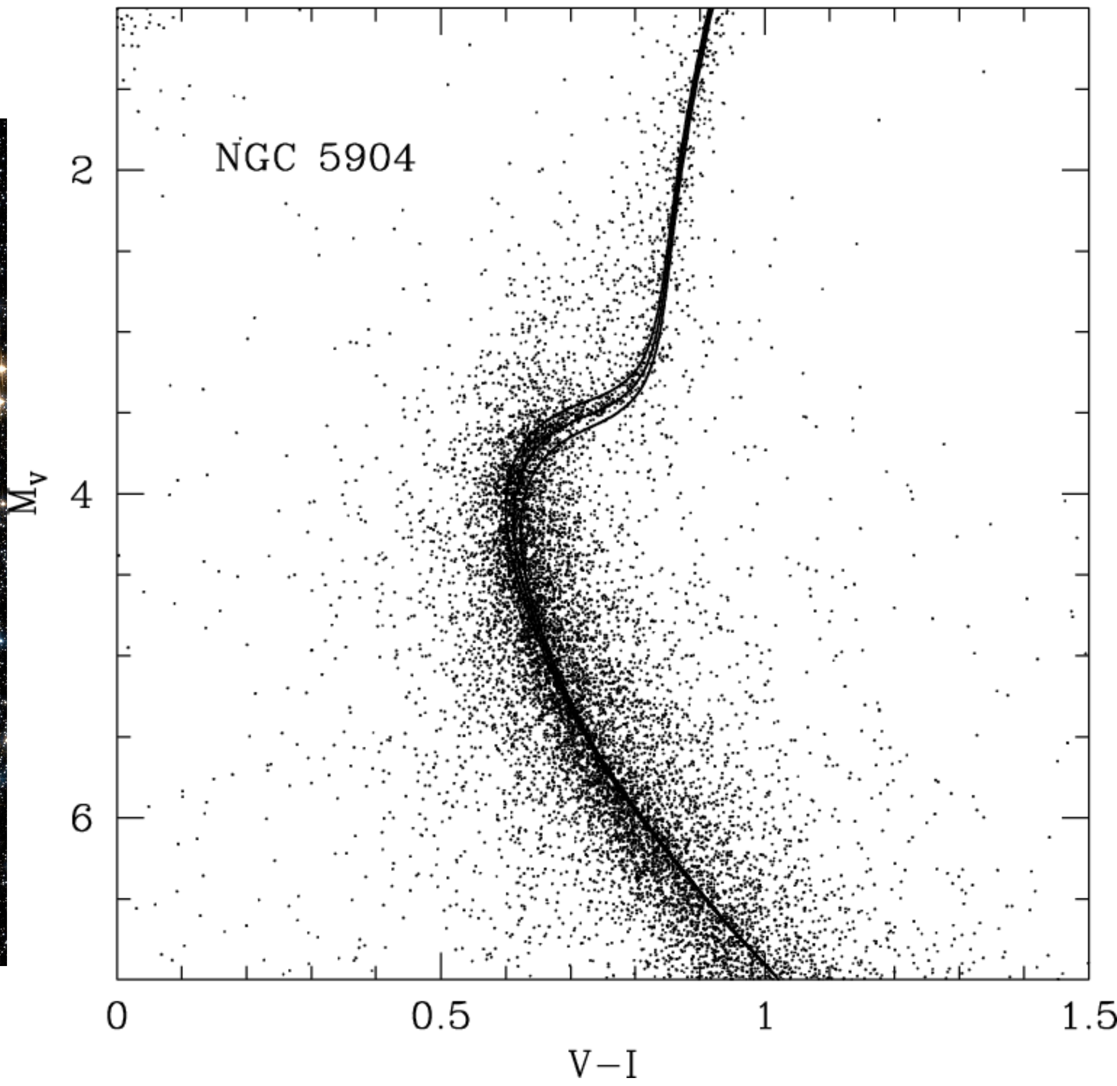


M55: 12.5 billion years



Determining age: Isochrone Fitting

M5: 13-14 billion years



Determining age: Isochrone Fitting

Measurement Errors
- errors in brightness/colors
- errors in distance/luminosity
- errors in cluster membership

Theoretical Errors
- uncertain chemical abundances
- uncertain amount of dust
- uncertain stellar physics

Typical error < 1 billion years