Adiabatic Exponents

$Q:$ specific heat $\left(\mathrm{erg\ g^{-1}}\right)$

$V_\rho:$ specific volume $\left(\mathrm{cm^3 g^{-1}}\right) = \dfrac{1}{\rho}$

$E:$ specific internal energy $\left(\mathrm{erg\ g^{-1}}\right)$

First law of thermodynamics

$$dQ = dE + PdV_\rho \quad = dE + Pd\left(\frac{1}{\rho}\right) \quad = dE - \frac{P}{\rho^2}d\rho$$

$$\Gamma_1 = \left(\frac{\partial \ln P}{\partial \ln \rho}\right)_{\mathrm{ad}} \quad = -\left(\frac{\partial \ln P}{\partial \ln V_\rho}\right)_{\mathrm{ad}}$$

$$\frac{\Gamma_2}{\Gamma_2 - 1} = \left(\frac{\partial \ln P}{\partial \ln T}\right)_{\mathrm{ad}} \quad = \frac{1}{\nabla_{\mathrm{ad}}}$$

$$\Gamma_3 - 1 = \left(\frac{\partial \ln T}{\partial \ln \rho}\right)_{\mathrm{ad}} \quad = -\left(\frac{\partial \ln T}{\partial \ln V_\rho}\right)_{\mathrm{ad}}$$

$$\nabla_{\mathrm{ad}} = \left(\frac{\partial \ln T}{\partial \ln \rho}\right)_{\mathrm{ad}}\left(\frac{\partial \ln \rho}{\partial \ln P}\right)_{\mathrm{ad}} = \frac{\Gamma_3 - 1}{\Gamma_1} = \frac{\Gamma_2 - 1}{\Gamma_2}$$

Adiabatic Exponents

- Γ_1 describes how pressure responds to compression
  (relevant for dynamical processes like pulsations)

- Γ_2 describes how temperature responds to changes in pressure
  (relevant for determining whether convection takes place)

- Γ_3 describes how temperature responds to compression

<u>Ideal Gas:</u> $$\Gamma_1 = \Gamma_2 = \Gamma_3 = \frac{5}{3}$$

<u>Radiation:</u> $$\Gamma_1 = \Gamma_2 = \Gamma_3 = \frac{4}{3}$$

Γ_3 is equivalent to the γ in the γ-law equation of state $P = (\gamma - 1)\rho E$

For mixtures of gas and radiation, as well as for cases where chemical reaction happen, the adiabatic exponents can differ from each other.

Adiabatic Exponents: Methodology

$$P = f\left(T, V_\rho\right)$$

$$E = g\left(T, V_\rho\right)$$

Adiabatic change
$dQ = 0$

$$dQ = dE + PdV_\rho = 0$$

$$= \left(\frac{\partial E}{\partial T}\right)_{V_\rho} dT + \left(\frac{\partial E}{\partial V_\rho}\right)_T dV_\rho + PdV_\rho = 0$$

$$dP = \left(\frac{\partial P}{\partial T}\right)_{V_\rho} dT + \left(\frac{\partial P}{\partial V_\rho}\right)_T dV_\rho$$

$$= \left(\frac{\partial P}{\partial T}\right)_{ad} dT = \left(\frac{\partial P}{\partial V_\rho}\right)_{ad} dV_\rho$$

$$= \left(\frac{\Gamma_2}{\Gamma_2 - 1}\right) dT = \left(-\Gamma_1\right) dV_\rho$$

Adiabatic Exponents: Ideal gas + Radiation

Pressure

$$P = P_{\text{gas}} + P_{\text{rad}} = \frac{N_A k}{\mu}\rho T + \frac{1}{3}aT^4$$

$$= \frac{N_A k}{\mu}\frac{T}{V_\rho} + \frac{1}{3}aT^4$$

Specific Internal Energy

$$E = E_{\text{gas}} + E_{\text{rad}} = \frac{3}{2}\frac{N_A k}{\mu}T + a\frac{T^4}{\rho}$$

$$= \frac{3}{2}\frac{N_A k}{\mu}T + aT^4 V_\rho$$

Adiabatic Exponents: Ideal gas + Radiation

$$P = \frac{N_A k}{\mu}\frac{T}{V_\rho} + \frac{1}{3}aT^4$$

$$E = \frac{3}{2}\frac{N_A k}{\mu}T + aT^4 V_\rho$$

Adiabatic change
$dQ = 0$

$$dQ = \left(\frac{\partial E}{\partial T}\right)_{V_\rho} dT + \left(\frac{\partial E}{\partial V_\rho}\right)_T dV_\rho + P dV_\rho = 0$$

$$= \left(\frac{3N_A k}{2\mu} + 4aT^3 V_\rho\right) dT + \left(aT^4\right) dV_\rho + \left(\frac{N_A k}{\mu}\frac{T}{V_\rho} + \frac{1}{3}aT^4\right) dV_\rho = 0$$

$$= \left(\frac{3N_A k}{2\mu} + 4aT^3 V_\rho\right) dT + \left(\frac{N_A k}{\mu}\frac{T}{V_\rho} + \frac{4}{3}aT^4\right) dV_\rho = 0$$

$$= \left(\frac{3N_A k}{2\mu}\frac{T}{V_\rho} + 4aT^4\right)\frac{V_\rho}{T} dT + \left(\frac{N_A k}{\mu}\frac{T}{V_\rho} + \frac{4}{3}aT^4\right) dV_\rho = 0$$

$$= \left(\frac{3}{2}P_{\text{gas}} + 12P_{\text{rad}}\right)\frac{dT}{T} + \left(P_{\text{gas}} + 4P_{\text{rad}}\right)\frac{dV_\rho}{V_\rho} = 0$$

Adiabatic Exponents: Ideal gas + Radiation

$$P = \frac{N_A k}{\mu}\frac{T}{V_\rho} + \frac{1}{3}aT^4$$

$$E = \frac{3}{2}\frac{N_A k}{\mu}T + aT^4 V_\rho$$

Equation of state

$$dP = \left(\frac{\partial P}{\partial T}\right)_{V_\rho} dT + \left(\frac{\partial P}{\partial V_\rho}\right)_T dV_\rho = \left(\frac{N_A k}{\mu}\frac{1}{V_\rho} + \frac{4}{3}aT^3\right)dT + \left(-\frac{N_A k}{\mu}\frac{T}{V_\rho^2}\right)dV_\rho$$

$$= \left(\frac{N_A k}{\mu}\frac{T}{V_\rho} + \frac{4}{3}aT^4\right)\frac{1}{T}dT + \left(-\frac{N_A k}{\mu}\frac{T}{V_\rho}\right)\frac{1}{V_\rho}dV_\rho$$

$$\frac{\Gamma_2}{\Gamma_2 - 1}$$

$$= \left(P_{\mathrm{gas}} + 4P_{\mathrm{rad}}\right)\frac{dT}{T} - P_{\mathrm{gas}}\frac{dV_\rho}{V_\rho} \quad = dP = \left(\frac{\partial P}{\partial T}\right)_{\mathrm{ad}} dT \quad = \frac{P}{T}\left(\frac{\partial \ln P}{\partial \ln T}\right)_{\mathrm{ad}} dT$$

$$\rightarrow \left(P_{\mathrm{gas}} + 4P_{\mathrm{rad}}\right)\frac{dT}{T} - P_{\mathrm{gas}}\frac{dV_\rho}{V_\rho} - \frac{\Gamma_2}{\Gamma_2 - 1}\left(P_{\mathrm{gas}} + P_{\mathrm{rad}}\right)\frac{dT}{T} = 0$$

$$= \left(P_{\mathrm{gas}} + 4P_{\mathrm{rad}} - \frac{\Gamma_2}{\Gamma_2 - 1}\left(P_{\mathrm{gas}} + P_{\mathrm{rad}}\right)\right)\frac{dT}{T} - P_{\mathrm{gas}}\frac{dV_\rho}{V_\rho} = 0$$

Adiabatic Exponents: Ideal gas + Radiation

1st Law of Thermodynamics

$$\left(\frac{3}{2}P_{\text{gas}} + 12P_{\text{rad}}\right)\frac{dT}{T} + \left(P_{\text{gas}} + 4P_{\text{rad}}\right)\frac{dV_\rho}{V_\rho} = 0$$

Equation of State

$$\left(P_{\text{gas}} + 4P_{\text{rad}} - \frac{\Gamma_2}{\Gamma_2 - 1}\left(P_{\text{gas}} + P_{\text{rad}}\right)\right)\frac{dT}{T} - P_{\text{gas}}\frac{dV_\rho}{V_\rho} = 0$$

$$\left.\begin{aligned} Ax + By &= 0 \\ Cx + Dy &= 0 \end{aligned}\right\} \rightarrow \frac{A}{C} = \frac{B}{D}$$

$$\frac{P_{\text{gas}} + 4P_{\text{rad}} - \frac{\Gamma_2}{\Gamma_2 - 1}\left(P_{\text{gas}} + P_{\text{rad}}\right)}{\frac{3}{2}P_{\text{gas}} + 12P_{\text{rad}}} = -\frac{P_{\text{gas}}}{P_{\text{gas}} + 4P_{\text{rad}}}$$

$$P_{\text{gas}} = \beta P$$
$$P_{\text{rad}} = (1-\beta)P$$

$$\frac{\beta \cancel{P} + 4(1-\beta)\cancel{P} - \frac{\Gamma_2}{\Gamma_2 - 1}\cancel{P}}{\frac{3}{2}\beta \cancel{P} + 12(1-\beta)\cancel{P}} = -\frac{\beta \cancel{P}}{\beta \cancel{P} + 4(1-\beta)\cancel{P}}$$

Adiabatic Exponents: Ideal gas + Radiation

$$\frac{\beta + 4(1-\beta) - \frac{\Gamma_2}{\Gamma_2 - 1}}{\frac{3}{2}\beta + 12(1-\beta)} = -\frac{\beta}{\beta + 4(1-\beta)} \quad \rightarrow \frac{4 - 3\beta - \frac{\Gamma_2}{\Gamma_2 - 1}}{12 - \frac{21}{2}\beta} = -\frac{\beta}{4 - 3\beta}$$

$$\rightarrow \left(4 - 3\beta - \frac{\Gamma_2}{\Gamma_2 - 1}\right)(4 - 3\beta) = (-\beta)\left(12 - \frac{21}{2}\beta\right)$$

$$\rightarrow 16 - 12\beta - 12\beta + 9\beta^2 - \frac{\Gamma_2}{\Gamma_2 - 1}(4 - 3\beta) = -12\beta + \frac{21}{2}\beta^2$$

$$\rightarrow \frac{\Gamma_2}{\Gamma_2 - 1}(4 - 3\beta) = 16 - 12\beta - \frac{3}{2}\beta^2 \quad \rightarrow \frac{\Gamma_2 - 1}{\Gamma_2} = \frac{4 - 3\beta}{16 - 12\beta - \frac{3}{2}\beta^2}$$

$$\rightarrow \frac{1}{\Gamma_2} = 1 - \frac{4 - 3\beta}{16 - 12\beta - \frac{3}{2}\beta^2} = \frac{16 - 12\beta - \frac{3}{2}\beta^2 - 4 + 3\beta}{16 - 12\beta - \frac{3}{2}\beta^2} = \frac{12 - 9\beta - \frac{3}{2}\beta^2}{16 - 12\beta - \frac{3}{2}\beta^2}$$

Adiabatic Exponents: Ideal gas + Radiation

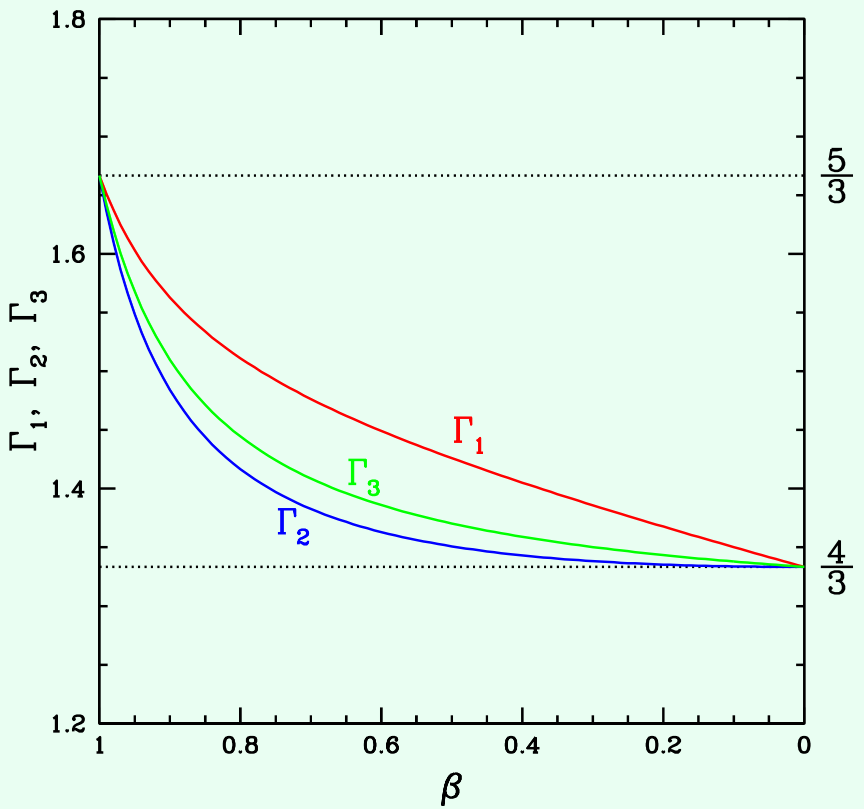
$$\frac{1}{\Gamma_2} = \frac{12 - 9\beta - \frac{3}{2}\beta^2}{16 - 12\beta - \frac{3}{2}\beta^2} \rightarrow \Gamma_2 = \frac{16 - 12\beta - \frac{3}{2}\beta^2}{12 - 9\beta - \frac{3}{2}\beta^2} \rightarrow \Gamma_2 = \frac{32 - 24\beta - 3\beta^2}{24 - 18\beta - 3\beta^2}$$

Similarly, we get:

$$\Gamma_1 = \frac{32 - 24\beta - 3\beta^2}{24 - 21\beta}$$

$$\Gamma_2 = \frac{32 - 24\beta - 3\beta^2}{24 - 18\beta - 3\beta^2}$$

$$\Gamma_3 = \frac{32 - 27\beta}{24 - 21\beta}$$



Adiabatic Exponents: Ionization Zones

Thermodynamics of partially ionized gas is different because

- The number of free particles is not constant
- Ionization energy is required to increase n

Assuming a pure H ideal gas

$$\left.\begin{aligned} n^+ &= y n_I \\ n^0 &= (1-y) n_I \\ n_e &= y n_I \\ n_I &= N_A \rho \end{aligned}\right\}$$

$$\frac{y^2}{1-y} = \frac{1}{n_I}\left(\frac{2\pi m_e kT}{h^2}\right)^{3/2} e^{-\chi/kT}$$

$$= \frac{A}{N_A \rho} T^{3/2} e^{-\chi/kT}$$

$$\boxed{\frac{y^2}{1-y} = \frac{A}{N_A} V_\rho T^{3/2} e^{-\chi/kT}}$$

Adiabatic Exponents: Ionization Zones

Pressure

$$P = \left(n^0 + n^+ + n_e\right)kT \quad = \left[\left(1-y\right)n_I + yn_I + yn_I\right]kT$$

$$= \left(1+y\right)n_I kT \quad = \left(1+y\right)N_A \rho kT \rightarrow \quad \boxed{P = \left(1+y\right)N_A k \frac{T}{V_\rho}}$$

Specific Internal Energy

$$E = \frac{3}{2}\left(n^0 + n^+ + n_e\right)\frac{kT}{\rho} + \frac{n^+\chi}{\rho} \qquad = \left(1+y\right)n_I \frac{3}{2}\frac{kT}{\rho} + \frac{yn_I}{\rho}\chi$$

$$\boxed{E = \left(1+y\right)\frac{3}{2}N_A kT + yN_A\chi}$$

Adiabatic Exponents: Ionization Zones

$$P = (1+y) N_A k \frac{T}{V_\rho}$$

$$E = (1+y) \frac{3}{2} N_A kT + y N_A \chi$$

Adiabatic change
$dQ = 0$

$$dQ = \left(\frac{\partial E}{\partial T}\right)_{V_\rho, y} dT + \left(\frac{\partial E}{\partial V_\rho}\right)_{T, y} dV_\rho + \left(\frac{\partial E}{\partial y}\right)_{T, V_\rho} dy + P dV_\rho = 0$$

$$= \left[(1+y)\frac{3}{2} N_A k\right] dT + \left[\frac{3}{2} N_A kT + N_A \chi\right] dy + \left[(1+y) N_A k \frac{T}{V_\rho}\right] dV_\rho = 0$$

$$= \left[(1+y)\frac{3}{2} N_A kT\right] \frac{dT}{T} + \left[(1+y)\frac{3}{2} N_A kT + (1+y) N_A \chi\right] \frac{dy}{1+y}$$

$$+ \left[(1+y) N_A kT\right] \frac{dV_\rho}{V_\rho} = 0$$

$$= E_{\text{ideal}} \left[\frac{dT}{T} + \frac{2}{3}\frac{dV_\rho}{V_\rho} + \left(1 + \frac{2}{3}\frac{\chi}{kT}\right)\frac{dy}{1+y}\right] = 0$$

Adiabatic Exponents: Ionization Zones

$$P = (1+y) N_A k \frac{T}{V_\rho}$$

$$E = (1+y)\frac{3}{2} N_A kT + y N_A \chi$$

Equation of State

$$dP = \left(\frac{\partial P}{\partial T}\right)_{V_\rho, y} dT + \left(\frac{\partial P}{\partial V_\rho}\right)_{T, y} dV_\rho + \left(\frac{\partial P}{\partial y}\right)_{T, V_\rho} dy$$

$$= \left[(1+y) N_A k \frac{1}{V_\rho}\right] dT - \left[(1+y) N_A k \frac{T}{V_\rho^2}\right] dV_\rho + \left[N_A k \frac{T}{V_\rho}\right] dy$$

$$= \left[(1+y) N_A k \frac{T}{V_\rho}\right] \frac{dT}{T} - \left[(1+y) N_A k \frac{T}{V_\rho}\right] \frac{dV_\rho}{V_\rho} + \left[(1+y) N_A k \frac{T}{V_\rho}\right] \frac{dy}{1+y}$$

$$dP = (1+y) N_A k \frac{T}{V_\rho}\left[\frac{dT}{T} - \frac{dV_\rho}{V_\rho} + \frac{dy}{1+y}\right] \rightarrow$$

$$dP = P\left[\frac{dT}{T} - \frac{dV_\rho}{V_\rho} + \frac{dy}{1+y}\right]$$

Adiabatic Exponents: Ionization Zones

$$f(y)=\frac{y^2}{1-y}=\frac{A}{N_A}V_\rho T^{3/2}e^{-\chi/kT}$$

Saha Equation

$$dy=\frac{dy}{df}df=\frac{dy}{df}\left[\left(\frac{\partial f}{\partial T}\right)_{V_\rho}dT+\left(\frac{\partial f}{\partial V_\rho}\right)_T dV_\rho\right]$$

$$=\frac{(1-y)^2}{2y(1-y)+y^2}\left[\frac{A}{N_A}V_\rho\left(\frac{3}{2}T^{1/2}e^{-\chi/kT}+T^{3/2}e^{-\chi/kT}\frac{\chi}{kT^2}\right)dT+\frac{A}{N_A}T^{3/2}e^{-\chi/kT}dV_\rho\right]$$

$$=\frac{(1-y)^2}{2y(1-y)+y^2}\frac{A}{N_A}V_\rho T^{3/2}e^{-\chi/kT}\left[\left(\frac{3}{2}\frac{1}{T}+\frac{\chi}{kT^2}\right)dT+\frac{dV_\rho}{V_\rho}\right]$$

$$=\frac{(1-y)^2}{2y(1-y)+y^2}\frac{y^2}{1-y}\left[\left(\frac{3}{2}+\frac{\chi}{kT}\right)\frac{dT}{T}+\frac{dV_\rho}{V_\rho}\right]$$

Adiabatic Exponents: Ionization Zones

$$dy = \frac{(1-y)^2}{2y(1-y)+y^2}\frac{y^2}{1-y}\left[\left(\frac{3}{2}+\frac{\chi}{kT}\right)\frac{dT}{T}+\frac{dV_\rho}{V_\rho}\right]$$

$$dy = \frac{(1-y)y}{2(1-y)+y}\left[\left(\frac{3}{2}+\frac{\chi}{kT}\right)\frac{dT}{T}+\frac{dV_\rho}{V_\rho}\right]$$

$$dy = \frac{(1-y)y}{2-y}\left[\left(\frac{3}{2}+\frac{\chi}{kT}\right)\frac{dT}{T}+\frac{dV_\rho}{V_\rho}\right]$$

$$\frac{dy}{1+y} = \underbrace{\frac{(1-y)y}{(1+y)(2-y)}}_{D(y)}\left[\left(\frac{3}{2}+\frac{\chi}{kT}\right)\frac{dT}{T}+\frac{dV_\rho}{V_\rho}\right]$$

Adiabatic Exponents: Ionization Zones

1st Law of Thermodynamics

$$E_{\text{ideal}}\left[\frac{dT}{T}+\frac{2}{3}\frac{dV_\rho}{V_\rho}+\left(1+\frac{2}{3}\frac{\chi}{kT}\right)\frac{dy}{1+y}\right]=0$$

Equation of State

$$dP=P\left[\frac{dT}{T}-\frac{dV_\rho}{V_\rho}+\frac{dy}{1+y}\right]$$

Saha

$$\frac{dy}{1+y}=D(y)\left[\left(\frac{3}{2}+\frac{\chi}{kT}\right)\frac{dT}{T}+\frac{dV_\rho}{V_\rho}\right]$$

Adiabatic Exponents: Ionization Zones

1st Law of Thermodynamics

$$E_{\text{ideal}}\left[\frac{dT}{T}+\frac{2}{3}\frac{dV_\rho}{V_\rho}+\left(1+\frac{2}{3}\frac{\chi}{kT}\right)\frac{dy}{1+y}\right]=0$$

$$E_{\text{ideal}}\left[\frac{dT}{T}+\frac{2}{3}\frac{dV_\rho}{V_\rho}+\left(1+\frac{2}{3}\frac{\chi}{kT}\right)D(y)\left[\left(\frac{3}{2}+\frac{\chi}{kT}\right)\frac{dT}{T}+\frac{dV_\rho}{V_\rho}\right]\right]=0$$

$$E_{\text{ideal}}\left[\left[1+\left(1+\frac{2}{3}\frac{\chi}{kT}\right)D(y)\left(\frac{3}{2}+\frac{\chi}{kT}\right)\right]\frac{dT}{T}+\left[\frac{2}{3}+\left(1+\frac{2}{3}\frac{\chi}{kT}\right)D(y)\right]\frac{dV_\rho}{V_\rho}\right]=0$$

$$E_{\text{ideal}}\left[\left[1+D(y)\frac{2}{3}\left(\frac{3}{2}+\frac{\chi}{kT}\right)^2\right]\frac{dT}{T}+\frac{2}{3}\left[1+D(y)\left(\frac{3}{2}+\frac{\chi}{kT}\right)\right]\frac{dV_\rho}{V_\rho}\right]=0$$

Adiabatic Exponents: Ionization Zones

Equation of State

$$dP = P\left[\frac{dT}{T} - \frac{dV_\rho}{V_\rho} + \frac{dy}{1+y}\right]$$

$$dP = P\left[\frac{dT}{T} - \frac{dV_\rho}{V_\rho} + D(y)\left[\left(\frac{3}{2} + \frac{\chi}{kT}\right)\frac{dT}{T} + \frac{dV_\rho}{V_\rho}\right]\right]$$

$$dP = P\left[\left[1 + D(y)\left(\frac{3}{2} + \frac{\chi}{kT}\right)\right]\frac{dT}{T} + \left[D(y) - 1\right]\frac{dV_\rho}{V_\rho}\right]$$

$$dP = \left(\frac{\partial P}{\partial T}\right)_{\text{ad}} dT = \frac{P}{T}\left(\frac{\partial \ln P}{\partial \ln T}\right)_{\text{ad}} dT = \frac{P}{T}\frac{\Gamma_2}{\Gamma_2 - 1}dT$$

$$\frac{\Gamma_2}{\Gamma_2 - 1}\frac{dT}{T} = \left[1 + D(y)\left(\frac{3}{2} + \frac{\chi}{kT}\right)\right]\frac{dT}{T} + \left[D(y) - 1\right]\frac{dV_\rho}{V_\rho}$$

$$\left[1 - \frac{\Gamma_2}{\Gamma_2 - 1} + D(y)\left(\frac{3}{2} + \frac{\chi}{kT}\right)\right]\frac{dT}{T} + \left[D(y) - 1\right]\frac{dV_\rho}{V_\rho} = 0$$

Adiabatic Exponents: Ionization Zones

1st Law

$$\left[1+D(y)\frac{2}{3}\left(\frac{3}{2}+\frac{\chi}{kT}\right)^2\right]\frac{dT}{T}+\frac{2}{3}\left[1+D(y)\left(\frac{3}{2}+\frac{\chi}{kT}\right)\right]\frac{dV_\rho}{V_\rho}=0$$

Equation of State

$$\left[1-\frac{\Gamma_2}{\Gamma_2-1}+D(y)\left(\frac{3}{2}+\frac{\chi}{kT}\right)\right]\frac{dT}{T}+\left[D(y)-1\right]\frac{dV_\rho}{V_\rho}=0$$

$$\frac{1-\frac{\Gamma_2}{\Gamma_2-1}+D(y)\left(\frac{3}{2}+\frac{\chi}{kT}\right)}{1+D(y)\frac{2}{3}\left(\frac{3}{2}+\frac{\chi}{kT}\right)^2}=\frac{D(y)-1}{\frac{2}{3}\left[1+D(y)\left(\frac{3}{2}+\frac{\chi}{kT}\right)\right]}$$

$$\frac{\Gamma_2}{\Gamma_2-1}=1+D(y)\left(\frac{3}{2}+\frac{\chi}{kT}\right)-\frac{\left(D(y)-1\right)\left[1+D(y)\frac{2}{3}\left(\frac{3}{2}+\frac{\chi}{kT}\right)^2\right]}{\frac{2}{3}\left[1+D(y)\left(\frac{3}{2}+\frac{\chi}{kT}\right)\right]}$$

Adiabatic Exponents: Ionization Zones

$$\frac{\Gamma_2}{\Gamma_2 - 1} = 1 + D(y)\left(\frac{3}{2} + \frac{\chi}{kT}\right) - \frac{\left(D(y)-1\right)\left[1 + D(y)\frac{2}{3}\left(\frac{3}{2} + \frac{\chi}{kT}\right)^2\right]}{\frac{2}{3}\left[1 + D(y)\left(\frac{3}{2} + \frac{\chi}{kT}\right)\right]}$$

…algebra…

$$\frac{\Gamma_2}{\Gamma_2 - 1} = \frac{\frac{5}{2} + D(y)\left[\frac{15}{4} + 5\frac{\chi}{kT} + \left(\frac{\chi}{kT}\right)^2\right]}{1 + D(y)\left(\frac{3}{2} + \frac{\chi}{kT}\right)}$$

$$D(y) = \frac{y(1-y)}{(1+y)(2-y)}$$

Similarly:

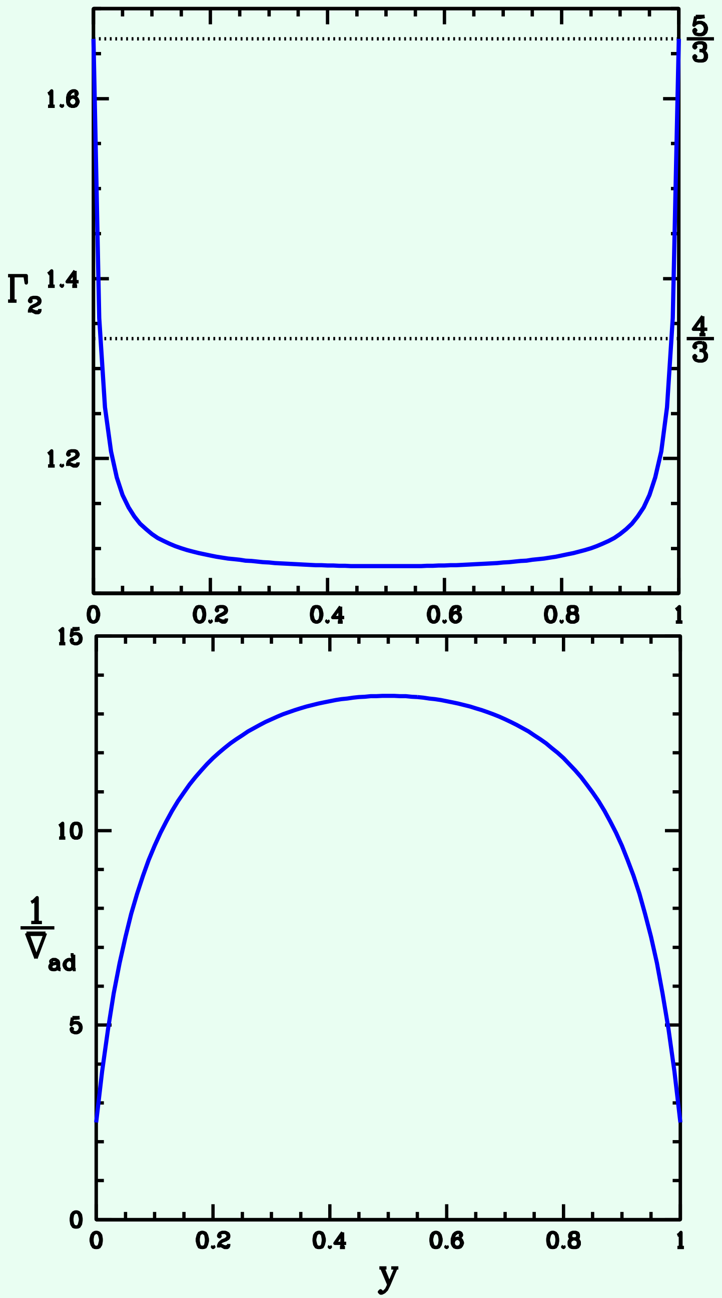
$$\Gamma_3 - 1 = \frac{2 + 2D(y)\left(\frac{3}{2} + \frac{\chi}{kT}\right)}{3 + 2D(y)\left(\frac{3}{2} + \frac{\chi}{kT}\right)^2}$$

Adiabatic Exponents: Ionization Zones

$$\frac{\Gamma_2}{\Gamma_2 - 1} = \frac{1}{\nabla_{\rm ad}} = \left(\frac{\partial \ln P}{\partial \ln T} \right)_{\rm ad}$$

Gas heats up
→ rapid increase of ionization
→ increase in number of free particles
→ pressure goes up faster than ~T

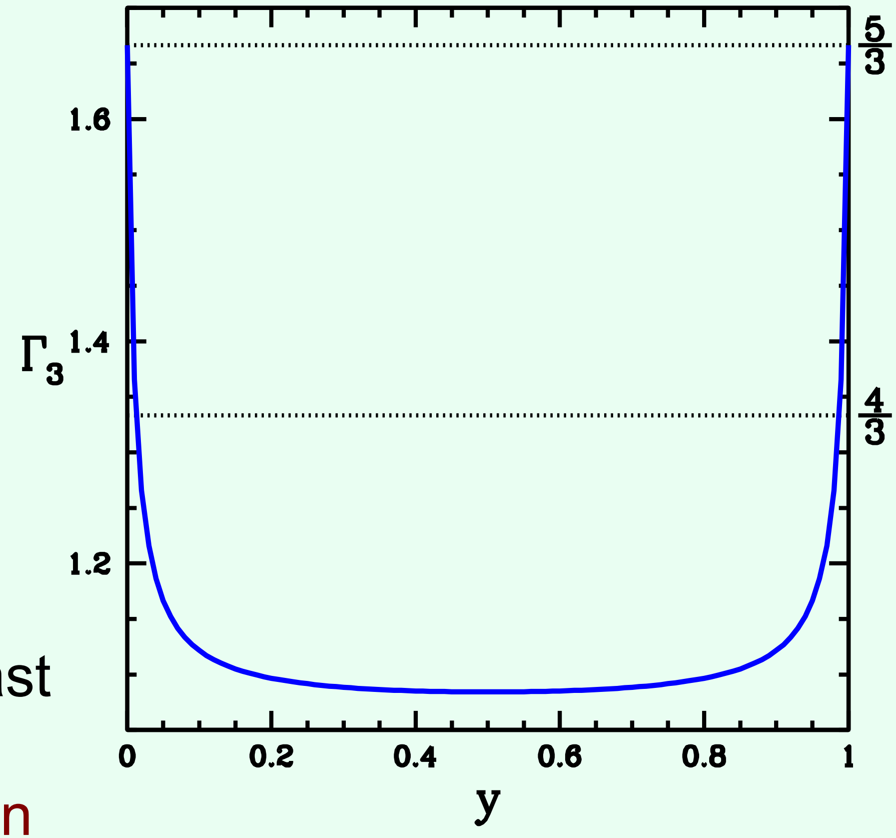
Pressure increases faster than normal
In response to temperature increase



Adiabatic Exponents: Ionization Zones

$$\Gamma_3 - 1 = \left(\frac{\partial \ln T}{\partial \ln \rho}\right)_{\mathrm{ad}}$$

Gas is compressed
→ increase in temperature
→ rapid increase in ionization
→ energy goes into ionizing gas instead of thermal energy
→ temperature does not rise as fast



Temperature increases slower than normal in response to compression

Adiabatic Exponents: Ionization Zones

Adiabatic exponents can be less than 4/3 → unstable!

Ionization zones are responsible for stellar pulsations

- second ionization of *He* provides the most significant driving
- ionization zone must be in the radiative region

Cool, luminous stars (T_{eff}<6300K) will pulsate in an "instability strip" a few hundred K wide on HR diagram