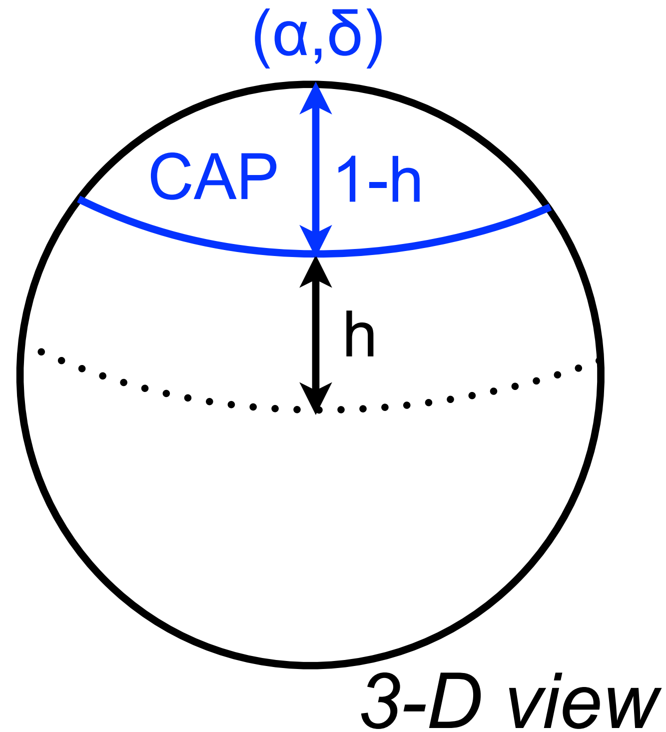
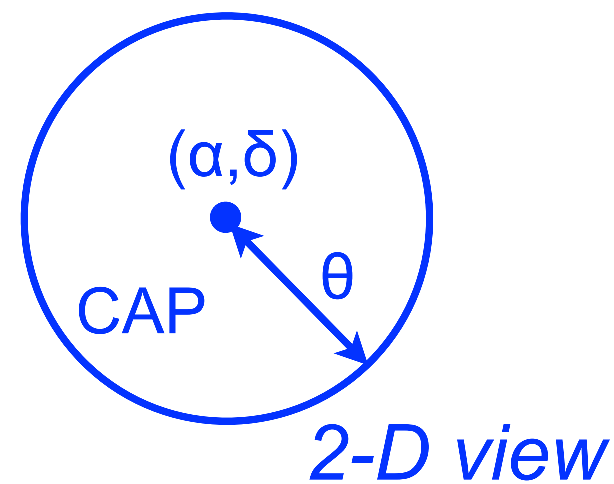
Spherical Caps

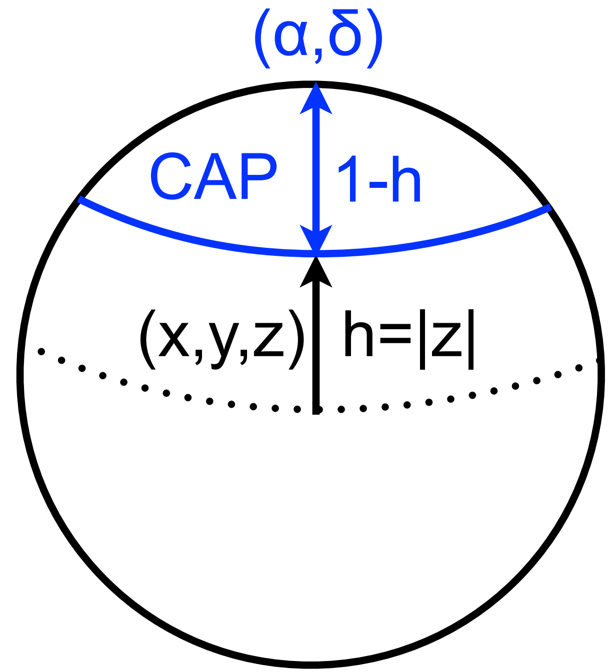
The Spherical Cap

- Spherical caps are useful models for representing arbitrary regions on the surface of the sphere
- A spherical cap is centered at a specific, Right Ascension and declination (α,δ)
- The height (1-h) of a spherical cap corresponds to some radius (θ) on the surface of the unit sphere
- By intersecting *multiple* spherical caps it is possible to construct general shapes on the surface of the sphere



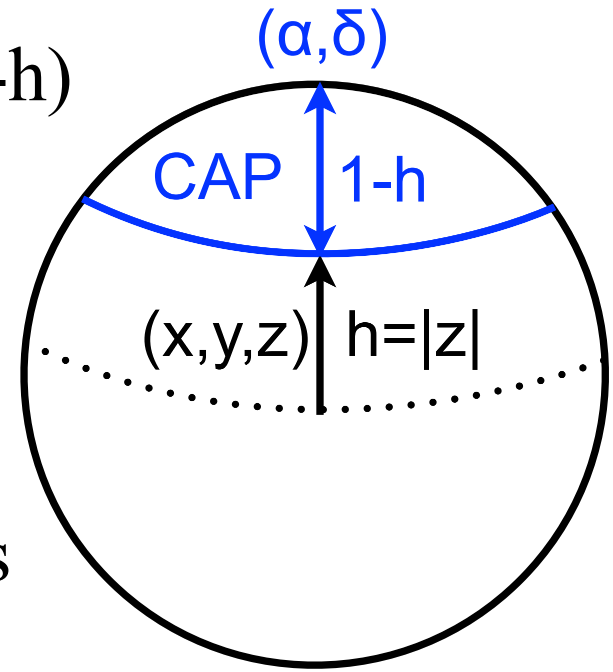


Vectorial representation of the spherical cap

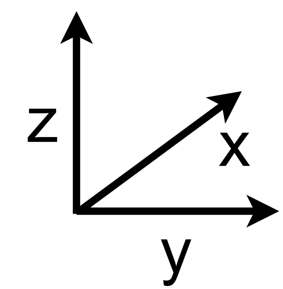


- As we know, spherical coordinates can be represented in Cartesian form (x,y,z) by the vector that points in the direction of (α,δ)
- Previously, for a *point on the surface of the sphere* we noted that $x^2 + y^2 + z^2 = 1$
- For a cap instead of a point, the *size* of the cap (which controls θ, the radius drawn on the surface of the sphere) can be controlled by $x^2 + y^2 + z^2 = h^2$
- Thus, one simple way to represent a cap is the 4-array given by x, y, z and h...we will choose the convention (x,y,z,1-h) as it is used by *Mangle* (next class' focus)

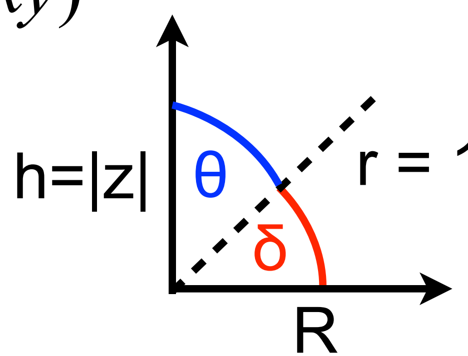
Vectorial representation of the spherical cap



- Caps can be represented by (x,y,z,1-h)
- (x,y,z) is easy to determine, it's just the Cartesian conversion from (α,δ) (e.g., using *SkyCoord*)
- For astronomers, the natural way to think about the cap size is the radius drawn on the surface of the sphere (θ from the last 2 slides, the radius we would put in *search_around_sky*)

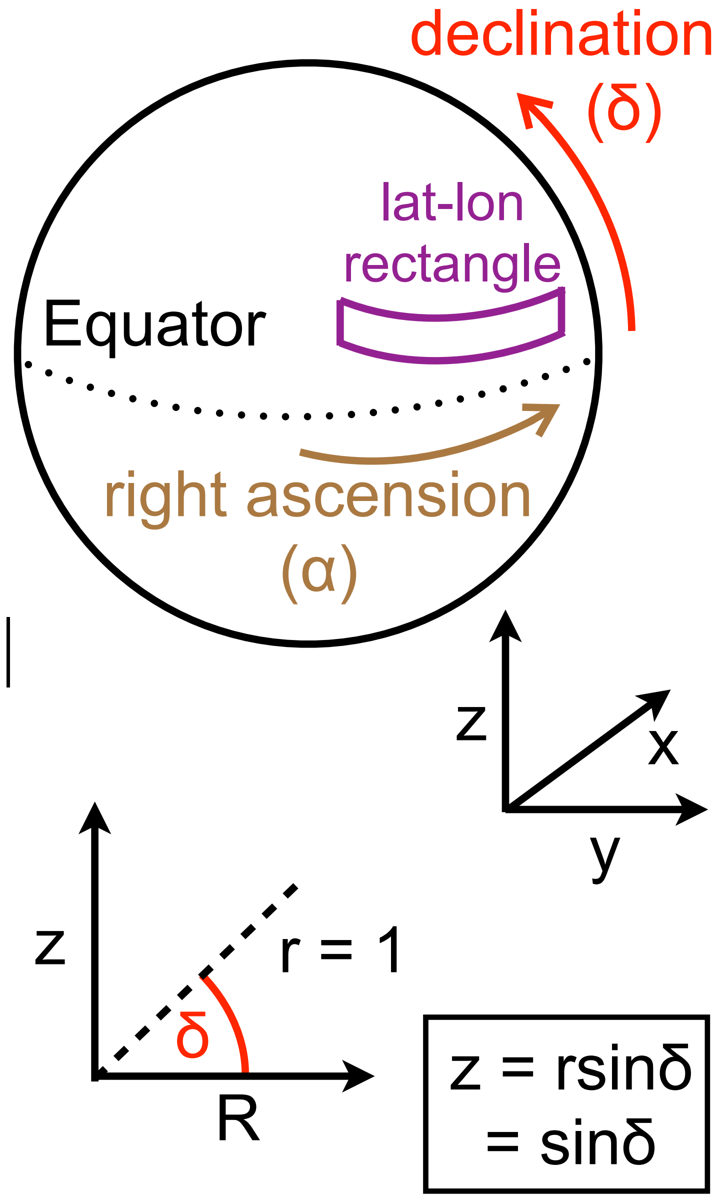


- As shown in the diagrams to the right, $1-h = 1-\cos\theta$
- So, $(x,y,z,1-h) \equiv (x,y,z,1-\cos\theta)$

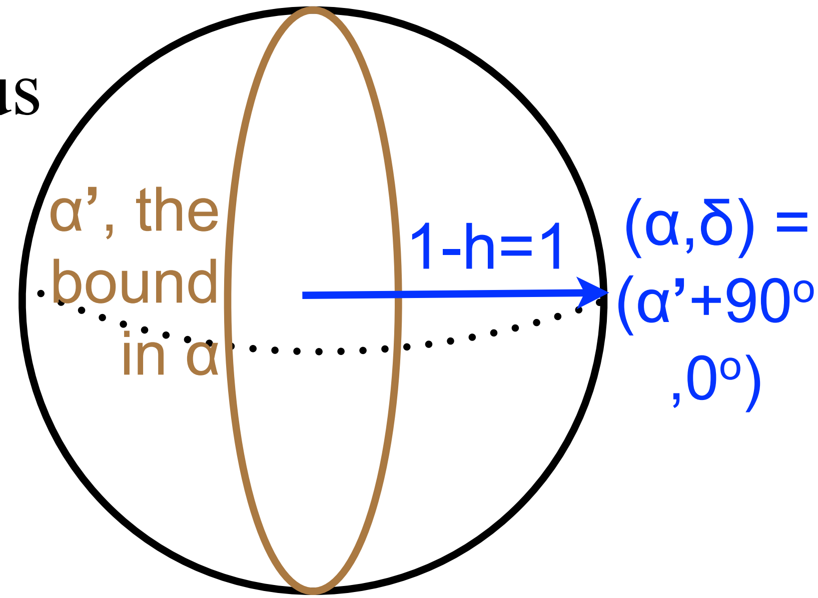


The area of a spherical cap

- Recall that the area of a *lat-lon rectangle* that runs from 0 to 2π in RA is $2\pi(z_2 - z_1)$ where
  - $z_2 - z_1 = \sin\delta_2 - \sin\delta_1$
- For a spherical cap $\delta_2 = 90^o$ and the area is then $2\pi(1 - z_1)$
- But z_1 here is just what I called $|z|$ or h in previous slides
- So, the area of a spherical cap, $2\pi(1-\cos\theta)$ is easy to determine from the vector form for a cap
  - which is (x,y,z,1-cosθ)



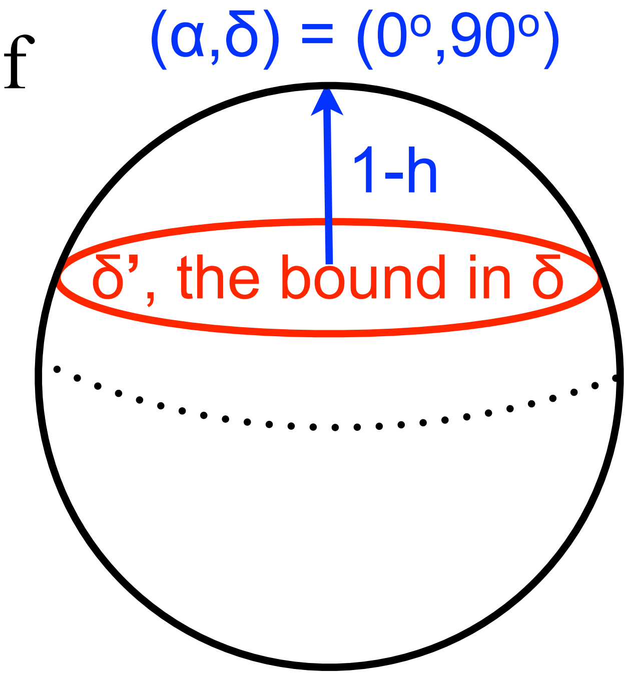
Caps bounded by Right Ascension

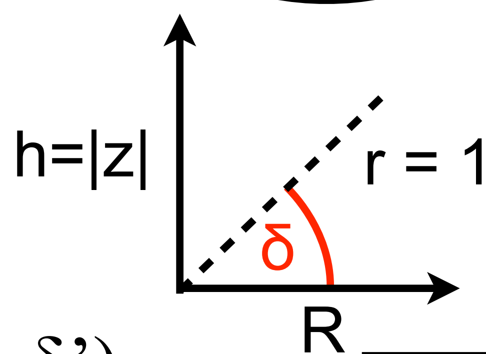


- So far we've discussed the general form, a "circle of radius θ on the surface of the sphere"
- Other main astronomy uses are fields bound by RA or dec
- Bounds in RA (α') map out a great circle on the sphere...a spherical cap that slices off exactly half of the sphere
- So, the vector representation of a bound in RA is
  - (x,y,z,1-h) = ($xyz(\alpha'+90^o,0^o)$, 1)
  - By *xyz()* I mean "conversion to Cartesian coordinates"

Caps bounded by Declination

- Bounds in dec (δ') map out lines of constant latitude on the sphere...a cap that slices off increasingly less of the sphere as δ' increases
- The (x,y,z) vector direction is always towards the north pole
- The size of the cap is given by $h = \sin\delta$
- So, the vector representation of a bound in declination is
  - $(x,y,z,1\text{-}h) = (\mathit{xyz}(0^o,90^o),\ 1 - \sin\delta')$





$z = r\sin\delta$
$= \sin\delta$

Python tasks

1. Write a function to create the vector 4-array for the spherical cap bounded by 5^h in Right Ascension
   - the answer is [-0.96592582629, 0.25881904510, 0, 1]
2. Write a function to create the vector 4-array for the spherical cap bounded by 36^oN in declination
   - the answer is [0, 0, 1, 0.41221474770752686]
3. Write a function to create the vector 4-array for the spherical cap that represents a circular field drawn on the surface of the sphere at $(\alpha,\delta) = (5^h, 36^o\text{N})$ with a radius of $\theta = 1^o$
   - the answer is [0.20938900596, 0.78145040877, 0.58778525229,0.00015230484]

Python tasks

4. Write a function that outputs your three spherical caps to a file in the following format:

```
1 polygons
polygon 1 ( 3 caps, 1 weight, 0 pixel, 0 str):
  -0.96592582629 0.25881904510 0 1
  0 0 1 0.4122147477075268 6
  0.209389006 0.781450409 0.587785253 0.00015230
```

hint:
hdr = 'this text will go first'
np.savetxt(filename, [cap1,cap2], fmt='%1.16f', header=hdr, comments='', newline='\n ')

The reason for the formatting should become clear next week