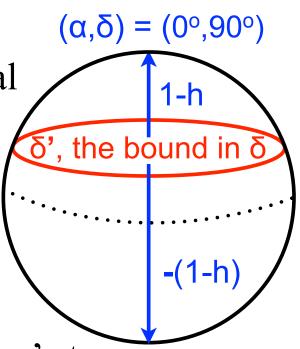


Intersections of multiple caps

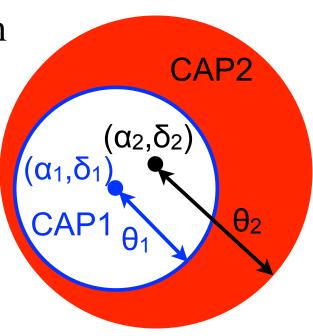
- By intersecting *multiple* spherical caps it is possible to construct general shapes on the surface of the sphere
- So, e.g., a "square" field bounded by $(\alpha_{min}$, α_{max} , δ_{min} , δ_{max}) can be represented by the intersection of 4 spherical caps



- One cap bounded by α_{min} ', one by α_{max} ' etc.
- But, for this sort of scheme, it is sometimes necessary to represent the sphere on the *other side of the cap*
- This is achieved simply in our convention by flipping the sign of (1-h) so that $(x,y,z,(1-h)) \rightarrow (x,y,z,-(1-h))$

Intersections of multiple caps - Polygons

- Consider the red area in the diagram
- It consists of CAP1 and CAP2, but we want the intersection of everything that is *in* CAP2 with everything that is *not in* CAP1
- We can create everything that is *not in* CAP1 by flipping the sign of the cap *constraint* (the cap size)



- So, if CAP2 is represented by (x₂,y₂,z₂,1-h₂) and CAP1 by (x₁,y₁,z₁,1-h₁), then the red area is represented by the intersection of (x₂,y₂,z₂,1-h₂) and (x₁,y₁,z₁,-(1-h₁))
- We will refer to shapes represented by the intersection of multiple caps as *polygons*

Basic Python implementation of Mangle

• Software called *Mangle* has been developed to apply the spherical cap formalism. The *astroconda* software stack has a basic Python implementation that can be imported with *import pymangle* (e.g. see the syllabus links)

- pip install pymangle

- Recall that *pip* is for managing Python package installations within a *conda* environment. i.e. your *conda* environment will have *pip* installed in it to manage Python packages for that one environment.
- You can also install *astroconda* (see the syllabus links), which includes *pymangle*, but it is much slower.

Basic Python implementation of Mangle

• *Mangle* expects input files with caps combined into polyons in the following format:

1 polygons
polygon 1 (3 caps, 1 weight, 0 pixel, 0 str):
 -0.96592582629 0.25881904510 0 1
 0 1 0.41221474770752686
 0.209389006 0.781450409 0.587785253 0.00015230

- The first line states how many polygons are in the file note that polygon(s) and cap(s) are *always plural*!
- In the second line, caps specifies how many caps are in the polygon and str lists the area of the polygon in steradians (which you can leave as 0 str for now)

Basic Python implementation of Mangle

- Note that if you wanted to create multiple polygons you could specify several blocks of caps, e.g.
 - 2 polygons polygon 1 (3 caps, 1 weight, 0 pixel, 0 str): -0.96592582629 0.25881904510 0 1 0 1 0.41221474770752686 0.209389006 0.781450409 0.587785253 0.00015230 polygon 2 (2 caps, 1 weight, 0 pixel, 0 str): 0.087155742747 -0.9961946980 0 1 0.785012134782 0.0686796625 0.615661475 1
- A collection of multiple polygons is typically referred to as a *mask* (or, occasionally, a *footprint*)
- The weight allows polygons to be weighted differently when creating catalogs of random coordinates (a weight=1 polygon would contain twice as many random points as a weight=0.5 polygon)

Python tasks

Please save all the plots you create as .png files in Git!

- 1. Use your *circle_cap* function from the previous lecture to create two caps, one of radius $\theta=5^{\circ}$ centered at $(\alpha,\delta) =$ $(76^{\circ}, 36^{\circ})$, which I will call *cap 1* and one of radius $\theta=5^{\circ}$ centered at $(\alpha,\delta) = (75^{\circ}, 35^{\circ})$, which I will call *cap 2*
- 2. Create a file containing these two caps as a single polygon in the *Mangle* format. The file should be called *intersection.ply*. Create a second *Mangle* file containing these two caps as two different polygons (i.e. polygon 1 should contain *cap 1* and polygon 2 should contain *cap 2*). This second file should be called *bothcaps.ply*
 - Note that the *Mangle* convention is to use an extension of *.ply* for files containing polygons

Python tasks

3. *import pymangle* and read in each of your masks, e.g.

• minter = pymangle.Mangle("intersection.ply")

then use *genrand* (see the syllabus link) to populate each mask with 10,000 random points. Plot the random points corresponding to each mask on a single plot with each mask's points plotted in a different color

• do you understand exactly how and why the masks are different? If not, then ask Jessie

4. Consider the file *intersection.ply* and its mask (*minter*)

- Flip the sign of the constraint on *cap 1* and read it in as a mask (*mflip1*)
- Plot *minter* and *mflip1* on one plot in different colors

Python tasks

- 5. Now make the constraint on *cap 1* positive again, and instead flip the constraint on *cap 2*
 - Read in this new mask as *mflip2*
 - Plot *minter, mflip1* and *mflip2* in different colors on a single plot
 - Do you understand what happened when the caps' constraints became negative? If not, then ask Jessie
 - How would the plot you've just made compare to the mask in the file *bothcaps.ply*?
- 6. Make *both* cap constraints in *intersection.ply* negative. Read in this mask and generate 1 *million* random points in it. This will take a suspiciously short amount of time.
 - Plot the random catalog. Does this mask make sense?